EXP: INIST 33 83 50 47 27 1994-01-19 16:44 63-96 5 #11

Optical and Quantum Electronics **14** *(1982) Short Communication*

Short Communication

Polarization states in twisted optical fibres

Unimodal excitation and propagation of tubular modes has already been demonstrated [1]. This type of pure mode excitation is of great interest in the characterization of multimode optical fibres. Furthermore, recent experiments have shown that tubular modes can be transmitted in the same fibre with little cross-talk [2]. The injection efficiency and modal purity of these tubular modes is good only if we know their true state of polarization. However, real uniform fibres are never perfectly circularly symmetric so that the polarization eigenstates are not circular and remain to some extent unknown. A strong twist applied to the multimode fibre lifts the degeneracy and enables us to improve the injection efficiency and modal purity. As is the case in twisted monomode fibres, a twist also helps to preserve the modal purity along the fibre. This is one of the motivations for studying the state of polarization in twisted multimode optical fibres.

We shall be concerned mostly with the ray optics limit ($\lambda \to 0$). In this limit, different mode groups are coupled together. We acknowledge with thanks the observation made by an anonymous referee that, for small or moderate twists, the circularly polarized eigenstates of the untwisted fibre are not coupled by the twist and, consequently, the eigenstates remain circular. Our ray optics result (linear polarizations) applies to twists greater than about 1000 turns/metre. Moderately twisted optical fibres behave as optically active materials with circular eigenstates of polarization, both clockwise and counterclockwise [3]. We show that, for multimode optical fibres and strong twists, polarization eigenstates may become linear, i.e., radial and azimuthal (see Fig. 1). The essence of our argument is that the modes of propagation in multimode optical fibres can be accurately described by ray theory. On the other hand, it is well known that a strained piece of dielectric can be described by a symmetrical permittivity tensor, and that the states of polarization in such materials are linear [4].

Strictly speaking, this statement holds only for plane waves in homogeneous media. It holds approximately, however, whenever the scale of the inhomogeneity is large compared to the beat wavelength between the local eigenstates of polarization. Otherwise the inhomogeneities may couple the eigenstates of polarization, and the polarization states are no longer defined by the local properties of the material. In particular, when the medium is isotropic, the local state of polarization is arbitrary since the two eigenstates are degenerate and any given initial state evolves along a ray path according to the integrated torsion of that path (a purely geometrical quantity). We will show that the strong anisotropy case applies to most modes propagating in twisted multimode optical fibres whenever the twist exceeds about 1000 turns/metre. Although such a high twist rate may be unrealistic for silica fibres, it may be encountered for other materials (e.g., plastic). The required twist rate could be smaller for fibres with larger Δ-values. In any event, it seems to be of interest to understand how the wave optics result merges into the ray optics result.

We shall first recall for later reference results obtained by one of us concerning the polarization states of modes in isotropic (nontwisted) multimode fibres [5]. Next, we shall clarify the steps of our analysis of twisted optical fibres. Finally, we give the beat wavelength curve for tubular modes as a function of the twist rate.

Isotropic fibres

We are considering here graded-index, circularly symmetric fibres described by some monotonically decreasing refractive index $n(r)$. We consider modes that have an $\exp(i\mu\phi)$ dependence of the field on the azimuthal angle ϕ, μ being an integer that we shall take as positive, for definiteness. For such modes the polarization state is a function of radius only. Most authors pick up degenerate modes with opposite azimuthal numbers and consider the $\cos(\mu\phi)$ and $\sin(\mu\phi)$ azimuthal dependence of the field. This procedure is legitimate, but it complicates the matter because the polarization state in that case depends

0306-8919/82/040371-04$02.98/0 © 1982 Chapman and Hall Ltd.

Figure 1 Polarization eigenstates of tubular modes in strongly twisted multimode fibres.

on ϕ, as well as on r. Let us thus emphasize here our choice of an exp $(i\mu\phi)$, $\mu > 0$, dependence. For any μ-value, there are two eigenstates of polarization. They are circular, respectively clockwise (HE modes) and counterclockwise (EH modes). While this is true for most high-order modes, the argument is simpler for tubular modes because such modes can be represented by helical rays, that is by rays that remain at a constant distance r from the axis. Consider any initial noncircular eigenstate of polarization. The laws of ray optics recalled earlier state that the field rotation (with respect to the principal normal to the ray path) is in an opposite sense to the integrated torsion of the ray path. Because this integrated torsion over a period of the helix is never a multiple of π (as geometry easily shows), we conclude that the polarization state depends on z. But this contradicts the concept of mode recalled earlier. The only way out is to assume that the state is initially, and remains, circular. In that case, a geometrical rotation merely amounts to a phase shift, which splits apart the two propagation constants.

The situation is schematically summed up in Fig. 2a. It is useful to quote here the expression for the beat wavelength, for the special case of a tubular mode at cut-off in a square-law fibre:

$$\text{Beat wavelength } 4\pi r_c/(2\Delta)^{3/2} \tag{1}$$

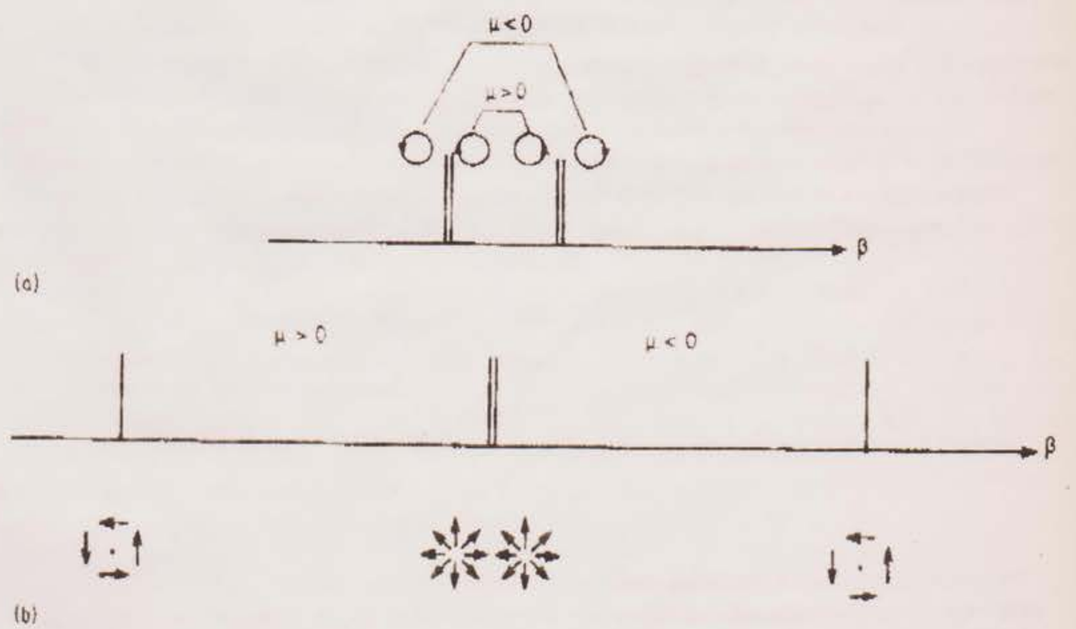

Figure 2 Schematic representation of the constants of propagation and states of polarization for $\mu > 0$ and for $\mu < 0$. (a) For isotropic square-law fibres; (b) for strongly twisted square-law fibres.

where r_c is the core radius and Δ the relative index change. Note that the optical wavelength does not enter into Equation 1. Note also that, by combining degenerate $\pm\mu$ modes we can generate the true mode with linear polarizations, but the polarization is not uniform (that is, not pointing to the same direction at every point in the cross-section). Thus the widely used LP modes, which are supposed to be linearly and uniformly polarized, have no physical existence in circularly symmetric fibres.

Twisted fibre

By 'twisted' fibre, we understand a fibre whose axis remains straight, but with one end rotated by τ turns/metre with respect to the other end. Clearly, the strain thus induced preserves the rotational symmetry of the fibre. Therefore, modes whose fields have an exp $(i\mu\phi)$ azimuthal variation continue to exist (while cos $(\mu\phi)$, sin $(\mu\phi)$ modes no longer exist).

The theory of photoelasticity shows that under such a strain, the permittivity tensor at a distance r from axis takes the form

$$\epsilon_r = \epsilon/\epsilon_0 = n^2(r)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2\pi g\tau r \\ 0 & 2\pi g\tau r & 1 \end{bmatrix} \tag{2}$$

where τ is, as before, the twist in turns/metre and g is a photoelastic constant of the order of 0.15 for silica; $n(r)$ is the index of the unstrained fibre. The electric field E of a plane wave with vector k in such a medium obeys the equation

$$(k{\times}k{\times} + (\omega/c)^2\epsilon_r)E = 0 \tag{3}$$

where $k\times$ is the 3×3 antisymmetrical matrix defined in [6]. Nontrivial solutions of Equation 3 exist only if the determinant of the 3×3 matrix in the bracket in Equation 3 vanishes. This provides us with the value of $k_z \equiv \beta$ as a function of k_r, the radial component of k, μ/r, the azimuthal component of k, and r. By considering only paraxial rays, and by assuming that the perturbation due to the twist is small. we have obtained the following approximation for the two βs:

$$\beta_\pm = k - (k_r^2 + \mu^2/r^2)/2k_0 - \pi g\tau r\{(\mu/r) \pm [k_r^2 + (\mu/r)^2]^{1/2}\}. \tag{4}$$

Here, $k = (\omega/c)n(r)$, while $k_0 = (\omega/c)n(0)$. One must exclude only extremely small angles (of order $g\tau r$) of the rays with respect to axis, and rather large angles ($\gg 0.1$ radians). We have checked by comparison with an exact numerical solution of Equation 3 that Equation 4 is valid in practical cases.

Application of the standard Hamiltonian equations of ray optics enables us to determine the contants of motion $\beta_\pm$ for the helical rays that are of special interest to us and correspond to r = constant. In general, $k_r \neq 0$ because the medium is not isotropic. In the present case, however, it turns out that $k_r = 0$ for both states. We thus arrive at the beat wavelength:

$$\text{Beat wavelength} = 2\pi/(\beta_+ - \beta_-) = 2/g\tau V \tag{5}$$

again for helical rays at cut-off in square-law media. The two expressions in Equations 1 and 5 are equal when

$$\tau = \Delta/\pi g k_0 r_c^2.$$

Going back to Equation 3, we find that β_+ corresponds to a radial electric field, as shown on Fig. 2b, while β_- corresponds to an azimuthal electric field. It is reasonable to assume that if $\tau \ll 5$ turns/metre, the electric field is circularly polarized, as is the case in isotropic fibres.

According to a previous footnote, this situation is preserved for larger twist rates, until different mode groups couple together. When the beat wavelength between the material local eigenstates is less than about $\pi r_c/(2\Delta)^{1/2}$ (the beat wavelength between the $\mu - 1$ and $\mu + 1$ scalar modes) the ray optics result with linear polarizations applies. Note that in the limit $\mu \to \infty$, scalar modes with $\mu - 1$ and $\mu + 1$ azimuthal numbers overlap radially to the point of being almost coincident. The electromagnetic components associated with these scalar modes have in part the same azimuthal variations $[(\mu - 1) + 1 =$

EXP: INIST   33 83 50 47 27   1994-01-19   16:46   65-96 S   #14

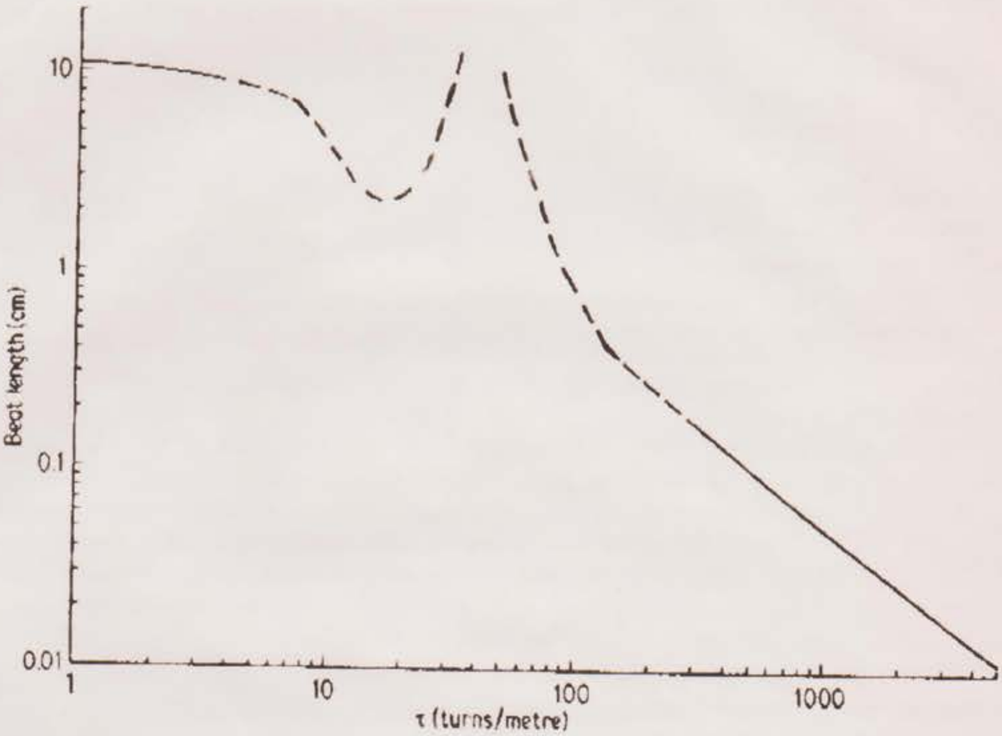

Figure 3 Beat wavelength between the eigenstates of polarization as a function of torsion for a multimode square-law fibre ($2r_C = 50\ \mu m$; $\Delta = 0.01$; $\lambda_0 = 1\ \mu m$) at cut-off. Under 2 turns/metre the effect of torsion is negligible; below 1000 turns/metre the eigenstates of polarization are those of unstrained multimode fibres. Beyond 1000 turns/metre the eigenstates are linear, as shown in Fig. 1. The broken line is qualitative and requires further investigation.

$(\mu + 1) - 1]$ and, therefore, they are coupled by the (circularly symmetric) perturbation introduced by the twist. In this way, linear polarizations may be generated out of circular polarizations. Fig. 3 shows the evolution of the beat length for a square-law fibre, at cut-off, when the torsion increases. We have not yet analysed in detail how the situation described in this short communication merges with the situation for small or moderate twist rates.

The results given here were first presented at the Theoretical Workshop on Optical Waveguide, Norway in September 1981.

References

1. P. FACQ, P. FOURNET and J. ARNAUD, *Elect. Lett.* 16 (1980) 648–50.
2. S. BERDAGUE and P. FACQ, to be published.
3. R. ULRICH and A. SIMON, *Appl. Opt.* 18 (1979) 2241.
4. M. BORN and E. WOLF, 'Principle of Optics' (Pergamon Press, New York, 1975) pp. 681–682.
5. J. ARNAUD, *Elect. Lett.* 11 (1975) 554.
6. Idem, 'Beam and Fiber Optics' (Academic Press, New York, 1976) p. 169.

Received 23 November 1981; revised 22 January 1982

A. BARTHELEMY
J. ARNAUD
Laboratoire d'Electronique des Microondes
UER des Sciences
123 rue A. Thomas
Limoges 87060
Cédex
France