ADIABATIC METHOD FOR FIBRES WITH NONSEPARABLE INDEX PROFILES

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A new powerful numerical technique is proposed that quickly gives the propagation constants, group velocities and caustics of modes with specified numbers in multimode fibres of arbitrary 2-dimensional index profiles. This method is based on the adiabatic approximation of ray optics.

Introduction: Most optical fibres fabricated today have a nominally circular shape; that is, the index of refraction is supposed to depend only on radius. However, significant departures from circularity are often observed in practice,[1] and one may wonder what degradation in the fibre bandwidth results from these deviations. On the other hand, noncircular profiles may be just as good as circular profiles for high bandwidths, whether the profile dispersion is linear[2] or nonlinear.[3,4]

A great deal of interest has thus appeared recently in optical propagation in multimode fibres that have noncircular step- or graded-index profiles. Unfortunately, analytical results are limited to separable geometries.[4–7] Using numerical methods, it is easy enough to trace rays for any profile $n(x, y)$, but, in general, one does not know how to relate these rays to specific mode numbers because it is not easy to apply the quantisation conditions. Therefore, one does not know in general how to calculate the impulse response of the fibre, which is needed in applications.

We shall describe in this letter a new and powerful numerical method for obtaining the propagation constants, caustics, and group velocities associated with the various modes that can propagate in multimode fibres of arbitrary index profiles. The method, based on an adiabatic principle, was recently proposed in the field of quantum mechanics by Solov'ev.[8]

The numerical method consists in tracing rays in a medium that varies continuously and slowly from some index profile that is separable in rectangular, polar or other co-ordinate systems to the profile under study. This adiabatic principle states, in short, that mode numbers (m, n) are invariant provided that the changes in profile are slow enough. In purely geometrical terms, this amounts to stating that the ray actions are invariant (the ray action in one dimension is defined as a surface integral in the phase space $\{x, dx/dz\}$—see Reference 9). One should, however, avoid profiles that are degenerate in the sense that different modes have the same propagation constant, as is the case for circularly symmetric square-law profiles. From a numerical point of view, the index variations are considered to be slow enough if characteristic parameters such as the propagation constants or the group velocities are insensitive to any increase in slowness. One must also check that these parameters are independent of the choice of the initial profile (within limits imposed by the caustic topology), and independent of the choice of the rays that correspond to the specified mode number.

In this letter, we provide the basic equations and treat as an example some power-law, noncircularly symmetric, index profile. By using our adiabatic method, new results are discovered.

Basic equations: Because the relative index change Δ is usually small compared with unity, and the fibre is highly multimoded, it is permissible to use the laws of paraxial ray optics:

$$\dot{x} = p \qquad \dot{y} = q$$

$$\dot{p} = -\frac{\partial U}{\partial x} \qquad \dot{q} = -\frac{\partial U}{\partial y}$$

$$U(x, y, z) = 1 - \frac{n(x, y, z)}{n_0} \qquad n_0 = n(0, 0, 0) \tag{1}$$

for some arbitrary z-dependent $n(x, y, z)$ profile. Upper dots denote derivatives with respect to z.

Integration of the system of eqns. 1 starts from some profile n_i separable either in an (x, y) rectangular co-ordinate system (in order to obtain, in the final medium, caustics of the hyperbolic type), or in an (r, ϕ) polar co-ordinate system (to obtain elliptic-type caustics). For rectangular co-ordinates (x, y), one selects an initial profile $U_i(x, y) = U_x(x) + U_y(y)$, and picks up initial values of x, $\dot{x}$ (respectively, y, $\dot{y}$) such that the surface integral of the closed curve in the $\{x, \dot{x}\}$ ($\{y, \dot{y}\}$) phase space is equal to m/λ (n/λ), where $\lambda = 2\pi c/\omega n_0$.

In our examples, we have selected square-law media:

$$U_x = \Delta(x/x_c)^2 \tag{2}$$

and the initial conditions

$$p(0) = 0$$
$$x(0) = \{(m + \tfrac{1}{2})\lambda/\pi\Omega_x\}^{1/2}$$
$$\Omega_x = \frac{\sqrt{(2\Delta)}}{x_c} \tag{3}$$

Similar relations hold in the (y, z)-plane with $y_c \neq x_c$.

In polar co-ordinates (r, ϕ), the most interesting case is the one where the radial mode number α is equal to zero. The corresponding rays are circular in projection, with a radius r_0 such that $r_0 = \{\mu\lambda r_c^2/2\pi\sqrt{(4\Delta)}\}^{1/3}$, where the integer μ denotes the azimuthal mode number. If $U(r) = 1 - n(r)/n_0$ denotes the profile, the initial values of x, y, p, q may be taken as

$$x(0) = r_0 \qquad y(0) = 0$$
$$p(0) = 0 \qquad q(0) = \sqrt{(4\Delta)}(r_0/r_c)^2 \tag{4}$$
$$\mu = 2\pi r_0^3\sqrt{(4\Delta)}/\lambda r_c^2$$

In the forthcoming examples we select $U(r) = \Delta(r/r_c)^4$.

It is remarkable that, after adiabatic transformation, the projected ray remains a closed curve. The z-integral of $p^2 + q^2$ over that closed curve must remain equal to its initial value. This is, like μ, an adiabatic invariant.

Once the ray has arrived at the medium under study with index $n_f(x, y)$, or $U_f = 1 - n_f/n_0$, we evaluate the propagation constant of the mode labeled m, n (or μ, α) from the relations:

$$\beta = 2\pi(1 - E_f)/\lambda$$
$$E_f = U_f(x, y) + \tfrac{1}{2}(p^2 + q^2) \tag{5}$$

The times of flight could be obtained by considering a small change of ω, keeping m, n constant, and looking at the corresponding change of β. But it is easier numerically to evaluate the average value $\bar{U}_f$ of U_f, defined as the limit of the integral along a ray from 0 to z of U_f, divided by z, when $z \to \infty$. We have verified that this limit exists. The ratio τ of the time of flight of a pulse carried by the mode considered divided by that of a pulse carried by the fundamental mode (modelled as an axial ray), is then[9]

$$\tau - 1 = E_f - 2\bar{U}_f \tag{6}$$

For the sake of simplicity we have neglected the material dispersion. An alternative method consists in evaluating E_f not just for the mode m, n of interest, but also for $m \pm 1$, $n \pm 1$, and using the formula

$$\tau - 1 = -E_f + m\frac{\partial E_f}{\partial m} + n\frac{\partial E_f}{\partial n} \tag{7}$$

in finite-difference form.

Example of application: We have studied the modes of propagation in the nonseparable noncircular profile:

$$U_f(x, y) = \Delta[(x/x_c)^2 + (y/y_c)^2]^2 \tag{8}$$

with $\Delta = 0{\cdot}01$, $x_c = 50\ \mu m$, $y_c = 33\ \mu m$ and $\lambda = 1\ \mu m$.

The change from the initial square-law profile U_i to U_f in eqn. 8 is effected by writing the exponent of the bracket in eqn. 8 in the form $1 + z/Z_0$, with $Z_0 \approx 10^5$, and terminating the integration at $z = Z_0$. For the numerical integration of the ray equations (eqn. 1), we used the Euler method, with an integration step of 5 μm. The computing time is of the order of 15 min on a HP 9835 desk computer.

For $m = 2$, $n = 3$, for example, we found a normalised propagation constant $E = 1{\cdot}9825 \times 10^{-3}$. The relative time of flight $\tau - 1$ is found from eqn. 6 to be $0{\cdot}66 \times 10^{-3}$ and, from eqn. 7, $0{\cdot}77 \times 10^{-3}$. A theoretical formula for profiles such as the one in eqn. 8 that are homogeneous functions of degree 2κ in x and y is:[2]

$$\tau - 1 = [(\kappa - 1)/(\kappa + 1)]E \tag{9}$$

Thus $\tau - 1$ should equal $0{\cdot}66 \times 10^{-3}$ since here $\kappa = 2$. This is in very good agreement with our numerical result from eqn. 6. The agreement is not as good with eqn. 7 because we have there replaced a derivative by a finite difference. A typical ray trace is shown in Fig. 1.

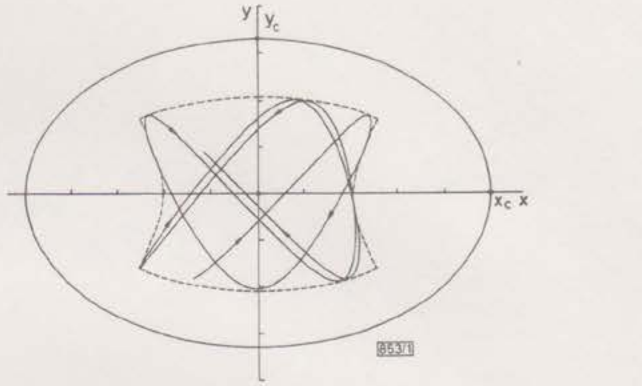

Fig. 1 *Ray and caustic (dotted line) for the profile in eqn. 8 and m = 2, n = 3*

To obtain modes with elliptical-like caustics, we used as the initial medium $U_i = \Delta(r/r_c)^4$, with $r_c = 50\ \mu m$, and traced rays with the initial conditions given in eqn. 4 and $\mu = 10, 20, 30$ (see Fig. 2). We have verified that there is nothing peculiar with profiles homogeneous in x and y, and that similar results can be obtained with other profiles as well.

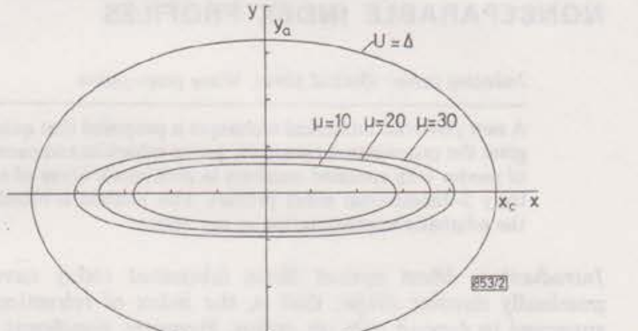

Fig. 2 *Closed ray traces for the profile in eqn. 8 (radial mode number $\alpha = 0$) various values of the azimuthal mode number $\mu = 10, 20, 30$*

Corresponding normalised propagation constants are $E_f = 1{\cdot}17 \times 10^{-3}$, 3×10^{-3}, $5{\cdot}2 \times 10^{-3}$; relative times of flight are $\tau - 1 = 3{\cdot}9 \times 10^{-4}$, 1×10^{-3}, $1{\cdot}7 \times 10^{-3}$

Some of the modes obtained by our ray technique have a normalised propagation constant larger than Δ, and thus should exhibit leaks. The calculation of the modal field and the leak (if any) from the ray traces is possible, but this will not be discussed here.

We have limited ourselves to the scalar approximation, but it is possible to obtain the split in degeneracy between the two electromagnetic modes associated with a given scalar mode by using the ray theory of electromagnetic modes proposed in Reference 10, which consists of the calculation of the integrated torsion of the ray path. To conclude, the new adiabatic method proposed in this letter appears to be a powerful technique for treating multimode optical fibres that have almost arbitrary index profiles. The computer time that this method requires is quite moderate. Using the method, we have found that, in fibres with nonseparable profiles, closed ray trajectories exist, which correspond to modes with zero radial order. This seems to be a new result. The profiles of noncircularly symmetric fibres can be measured by the method discussed in Reference 1. It is then an easy matter to calculate the ray trajectory for each mode, using eqns. 1 to 4, and calculating the time of flight, preferably from eqn. 6. From these times of flight, the impulse response is obtained in a straightforward manner.

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