LASER LINEWIDTH WITH GAIN COMPRESSION

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An exact yet simple expression for the linewidth of laser diodes based on the Nyquist formula is given. The expression applies to the case where the optical gain depends on both the carrier and photon numbers and differs from the expression derived from standard rate equations. Gain compression in conjunction with an electrical conductance may reduce both the linewidth and the intensity noise.

It is desirable to reduce the linewidth of laser diodes, particularly in phase-shift keyed optical communication systems. Gain compression importantly affects the dynamical properties of laser diodes.[1] A linewidth formula which is accurate when the optical gain depends on both the carrier and photon numbers is reported in this Letter for the first time to our knowledge. Without gain compression the linewidth enhancement factor is of the form $1 + \alpha^2$.[1] When the optical gain depends on the photon number P but not on the carrier number N, the Lax semiclassical result[2] reads: $1 + \beta^2$, α and β being defined below. The general situation is usually treated on the basis of standard rate equations (SREs).[1] It has been shown, however, that SREs are rigorous for linear gain only.[3]

Most singlemode laser diodes can be modelled by a susceptance $B_c(\nu) \simeq -4\pi C(\nu - \nu_0)$, in parallel with a constant load conductance G_d modelling the detector or radiation into free space, and an admittance $Y(N, P) \equiv G(N, P) + iB(N, P)$ modelling the active element. The dependence of Y on frequency or on controlling parameters such as temperature or strain is not considered. The steady-state oscillation condition requires that $Y(\bar{N}, \bar{P})$ be opposite to G_d. Complex Nyquist currents are associated with both negative and positive conductances.[2-4]

We define nondimensional differential parameters

$$g \equiv (N/G)G_N \qquad \gamma \equiv -(P/G)G_P \tag{1a}$$

$$\alpha \equiv B_N/G_N \qquad \beta \equiv -B_P/G_P \tag{1b}$$

where subscripts N, P denote partial derivatives. The differential gain factor g is of the order of unity and the α factor of the order of -5 at $1{\cdot}55\,\mu$m. The γ factor may be of the order of $0{\cdot}1$ but is not accurately known and β is usually neglected.

In the special case where the ratio $\Delta B/\Delta G$ of the deviations of B and G from their steady-state values is independent of $\Delta P/\Delta N$, there is no need to solve th carrier rate equation. The (real) frequency deviation $\delta\nu$ is obtained by eliminating ΔG. The expression of the laser (full-width at half power) linewidth $\Delta\nu$ is

$$F \equiv 4\pi\tau_p^2 Q\,\Delta\nu = \begin{bmatrix} 1 + \alpha^2 & (\gamma = 0) \\ 1 + \beta^2 & (g = 0) \\ 1 + \alpha^2 = 1 + \beta^2 & (\alpha + \beta = 0) \end{bmatrix} \tag{2}$$

where $P_0 \equiv h\nu_0 Q$ denotes the generated optical power and $\tau_p \equiv C/G_d$ the photon lifetime. In the first case (linear gain) the electrical voltage across the diode can also be obtained. In the second case (pure gain compression) the photon number fluctuation can be obtained.

The frequency deviation $\delta\nu$ is in any case proportional to

$$f(t) + \alpha n + \beta p \tag{3a}$$

where $f(t)$ is an independent white noise process of spectral density $2/Q$, and we have defined

$$n \equiv g\,\frac{\Delta N}{N} \qquad p \equiv \gamma\,\frac{\Delta P}{P} \tag{3b}$$

The second term in eqn. 3a expresses the fact that the optical frequency is affected by refractive index changes due to carrier number variations. The last term represents the Kerr effect.

In the general situation, the carrier rate equation is required. n and p satisfy (see eqn. 2 of Reference 3)

$$n - p = d - r \qquad cn + p = \gamma(t - d) \tag{3c}$$

where

$$c \equiv \frac{\gamma}{g}\left(s\zeta + \frac{NU_N}{eQ}\,G_{ext}\right) \qquad s \equiv \frac{N}{S}\,S_N \qquad \zeta = S/Q \tag{3d}$$

and $S(N)$ denotes the spontaneous emission rate, $NU_N \simeq 0{\cdot}07$ V at room temperature and e is the electron charge. For generality we have considered the electrical conductance G_{ext} across the intrinsic diode.

For injected-current fluctuations at the shot-noise level, full population inversion and *radiative* spontaneous recombination, the spectral densities of the independent noise sources d, r, t (proportional to the noise sources d, r and $j - s$ introduced in Reference 3) are

$$QS_d = QS_r = 1 \qquad QS_t = 2\zeta + 1 \tag{4}$$

The linewidth enhancement factor F follows from eqns. 3 and 4.

$$F = 1 + \left(\frac{\alpha - c\beta}{1 + c}\right)^2 [\gamma^{\circ 2}(\zeta + 1) - \gamma^\circ + 1] \tag{5a}$$

where

$$\gamma^\circ \equiv \frac{\alpha + \beta}{\alpha - c\beta}\,\gamma \tag{5b}$$

In the three cases considered in eqn. 2, $\gamma^\circ = 0$ and the simple factors are readily recovered from eqn. 5 (for gain compression $c \to \infty$).

The linewidth $\Delta\nu$ is independent of *linear* external or internal losses. In the latter case, it should be remembered that Q denotes the total generated (not output) photonic rate.

If SREs are used, the '1's following ζ and the γ° term in eqn. 5a do not appear. The circuit and SRE formulations coincide in the special cases exhibited in eqn. 2 only.

To illustrate the theoretical result in eqn. 5, we consider a

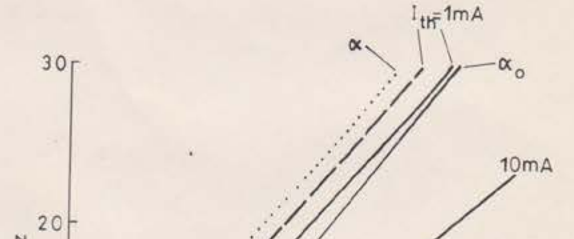

Fig. 1 *Variation of laser linewidth as function of reciprocal of optical power*

Line labelled α_0 applies to linear gain ($P_s = \infty$)
——— exact for saturation power $P_s = 100$ mW and three values of threshold current I_{th}
– – – – curve following from standard rate equations
······· curve following from simple $1 + \alpha^2$ factor with power dependent α

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short laser diode at $1{\cdot}55\,\mu$m. We assume that N is much larger than the transparency value

$$-G_d^{-1}G(N, P) = \frac{N/N_{th}}{1 + P_0/P_s} \qquad g = 1 \qquad \gamma = \frac{1}{1 + P_s/P_0} \quad (6a)$$

For linear gain the saturation power P_s should be taken as infinite. Subscript *th* denotes threshold values.

If spontaneous recombination is dominated by the Auger effect and B_N by the plasma effect

$$S = \frac{I_{th}}{e}\left(\frac{N}{N_{th}}\right)^3 \qquad s = 3 \qquad \alpha = \alpha_0 \frac{N}{N_{th}} \quad (6b)$$

In the Auger events two electrons recombine at a time and the spectral density of the corresponding noise term is twice the shot-noise level. We therefore multiply ζ in eqn. 5 by 3/2.

The laser linewidth $\delta\nu$ is plotted in Fig. 1 as a function of reciprocal power for $\tau_p = 1$ ps, $\beta = 0$, $\alpha_0 = 5$, $P_s = 100$ mW, and three values of the threshold current (1, 10 and 30 mA). For small powers and large threshold currents the laser linewidth is much smaller than predicted by the $1 + \alpha^2$ factor (dotted curve labelled 'α'). This is because spontaneous emission (or external electrical conductances G_{ext}) tends to clamp N at its threshold value and thus minimise the α-factor effect. Gain compression is required to prevent ΔP from blowing up. At powers comparable with P_s the linewidth increases because of increased α values as noted in References 5 and 6. For comparison, linewidth values derived from SREs at a threshold current of 1 mA are shown in the Figure. The simple $1 + \alpha_0^2$ factor corresponds to the straight line labelled α_0 going through the origin.

In conclusion, the exact linewidth formula has been written in a simple form. It differs somewhat from the SRE result. The Auger spontaneous recombination is twice the shot-noise level. Gain compression and carrier number clamping may lead to much reduced linewidths.

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