DISPERSION IN OPTICAL FIBRES WITH STAIRLIKE REFRACTIVE-INDEX PROFILES

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In multimode circularly symmetric fibres whose index distribution is a stairlike approximation of an optimum profile, the modal dispersion increases as the number of steps decreases. For a fibre with $\Delta n/n = 0{\cdot}02$ and a core radius of 40 μm, numerical calculations based on wave optics show that the r.m.s. impulse response width at $\lambda = 1$ μm increases from 0·075 ns/km for the smooth optimum profile to 0·23 ns/km for 40 steps of equal areas. Thus an important conclusion of the analysis is that one should avoid introducing steps in the refractive-index profile of fibres for optimum results.

Scalar ray-optics techniques (WKB approximation) are usually adequate to evaluate the broadening of optical pulses propagating in highly multimoded glass fibres.[1] However, there are special cases where these techniques are not applicable. In fibres made with the vapour-phase deposition technique, the gas composition is varied in a discrete fashion, rather than continuously. The resulting refractive-index profile also varies by steps, particularly when the dopant has low diffusivity, as for germania. The main purpose of this letter is to investigate the effects that such steps may have on pulse broadening. We expect ray (or WKB) techniques to be inadequate for two reasons. First, the ray technique is difficult to apply for stepped profiles because some rays reach discontinuities very near to the critical angle and therefore travel for a long time in a homogeneous region before being reflected back toward the axis. Because this region has a low group index compared with that of the material on axis, pulses carried by these near-critical rays travel much faster than axial rays. There are not many rays that exhibit this behaviour, but a few fast rays considerably increase the r.m.s. impulse width. Whether such rays are selected or not in the numerical computation process depends critically on the details of the ray sampling procedure. Thus the numerical ray technique may exhibit instabilities. Secondly, according to the conventional ray technique procedure, partial reflections at discontinuities are neglected. Only total reflection is considered. On the contrary, partial reflections are fully accounted for by the wave-optics technique used in the present work. A similar problem has been investigated by Clarricoats and Chan[2] with the help of Maxwell's equations. These authors found that the time of flight of a mode for a profile with five steps is almost the same as for a smooth profile. However, they considered profiles that approximate square-law fibres with low V-number, while we are considering profiles that approximate optimum profiles with moderately high V-numbers. Thus their conclusions are not applicable to our problem.

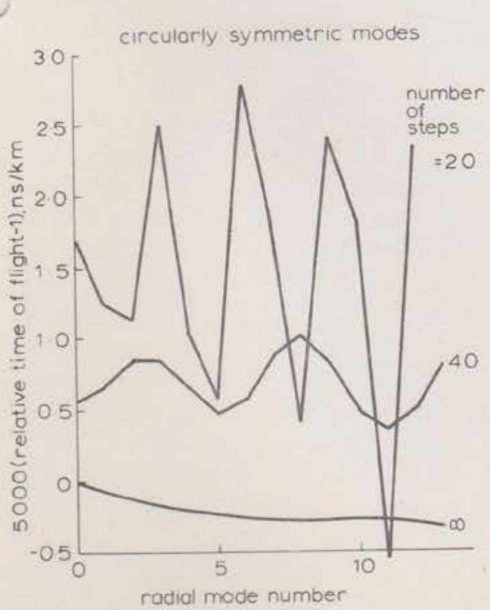

Fig. 1 *Times of flight (ns/km) of pulses in circularly symmetric modes as function of radial mode number for various numbers of steps of equal areas*
$\Delta n/n = 0{\cdot}015$, $a = 40$ μm, $\lambda = 1$ μm

We found that for the fibres considered the scalar Helmholtz equation is sufficiently accurate. This equation has therefore been used for the sake of simplicity. The numerical technique presented in this letter is applicable to almost any profile $n(r)$, continuous or discontinuous. It gives the propagation constant, time of flight and mode pattern of each of the 1300 modes carried by a typical graded or step-index fibre in 5 min on an IBM 370 computer. Good agreement is obtained with analytic expressions for step-index fibres and for square-law fibres.

When the variations of refractive index in the cross-section of a fibre are small, the transverse components of the electric field obey approximately the scalar Helmholtz equation. If the refractive index has finite discontinuities, a typical field component, denoted $\psi(x, y)$, and its first derivatives, $\partial\psi/\partial x$ and $\partial\psi/\partial y$, remain continuous.[3] For a circularly symmetric fibre, we can assume an $\exp[i(k_z z + \mu\phi - \omega t)] \times \psi(r)$ variation of the field, where the integer μ denotes the azimuthal mode number and k_z denotes the axial wavenumber (or propagation constant). The radial wave $\psi(r)$ obeys the equation

$$r^{-1}\, d(r d\psi/dr)/dr + [k^2(r) - k_z^2 - \mu^2/r^2]\psi = 0 \quad (1a)$$

where $k(r) \equiv (\omega/c)\, n(r)$ and $n(r)$ is the refractive index. We set $n(0) \equiv n$.

Eqn. 1a can be written as a pair of 1st-order equations for $\psi(r)$ and the auxiliary function $K(r)$:

$$d\psi/dr = K/r \quad (1b)$$

$$dK/dr = rA(r)\psi \quad (1c)$$

where we have set, for brevity,

$$A(r) \equiv k_z^2 + \mu^2/r^2 - k^2(r) \quad (1d)$$

as we easily verify by substituting K from eqn. 1b into eqn. 1c. Both $\psi(r)$ and $K(r)$ are continuous functions of r.

To solve eqn. 1 for some given $n(r)$, we use the following straightforward integration procedure: a value of k_z is selected in the range $k(0)$ to $k(a)$, where a denotes the core radius. The integration of eqn. 1 proceeds from the initial condition

$$(K/\psi)_{r=0} = \mu \quad (2)$$

which follows from the fact that, near the axis, $A(r) \approx \mu^2/r^2$, and therefore $\psi(r) = r^\mu$, $K(r) = \mu r^\mu$ is solution of eqns. 1b and c. The axial wavenumber k_z is varied until the condition $\psi \to 0$ as $r \to \infty$ is reached. Usually, five iterations are sufficient.

Once the correct values of k_z and $\psi(r)$ have been obtained, the relative time of flight $\tau(\alpha, \mu)$ defined as the ratio of the time of flight of a pulse in a mode, with radial number α and azimuthal number μ, to the corresponding time for free waves on axis, is evaluated by application of the Hellmann–Feynman theorem. The relative time of flight is (Reference 4, Appendix A, with a slightly different notation)

$$\tau(\alpha, \mu) = \int_0^\infty D(r)\, k^2(r)\psi^2(r)\, r\, dr \times \left[D(0)\, k(0)\, k_z \int_0^\infty \psi^2(r)\, r\, dr\right]^{-1} \quad (3)$$

where $D(r) = (\omega/k)(\partial k/\partial\omega)$ is a material dispersion parameter.* For simplicity, we shall assume that the material is free of dispersion. Then $D(r) = D(0) = 1$. Once $\tau(\alpha, \mu)$ has been obtained for all propagating modes and various optical wavelengths, the r.m.s. impulse width is evaluated from

$$\sigma \equiv (\langle\tau^2\rangle - \langle\tau\rangle^2)^{1/2} \quad (4)$$

where $\langle\ \rangle$ denotes an average over all propagating modes and over the normalised spectral width of the source.[1] For simplicity, the latter is subsequently assumed to be zero.

* In eqn. 3, ψ^2 can be interpreted as the axial power density.[4] Eqn. 3 should not be confused with the expression proposed by Brown[5], and, more recently, by Kawakami[6], which is restricted to nondispersive media

Pulses on modes whose k_z is only slightly larger than the wavenumber k_s in the cladding (near-cutoff modes) are travelling very fast compared with the other modes, particularly for small μ. We found that these fast modes may increase the r.m.s. impulse width by almost an order of magnitude, compared with the value predicted by ray or WKB methods. However, they carry little power and are likely to be attenuated by materials adjacent to the fibre. Therefore we have chosen to ignore their contribution to the r.m.s. impulse width when the field intensity at $r = 1{\cdot}25a$ exceeds $0{\cdot}1\%$ of the maximum field.

Let us consider first a fibre that has the following (smooth) refractive-index profile:

$$[n(r)/1{\cdot}5]^2 = \begin{cases} 1-2(\Delta n/n)(r/a)^{2\kappa} & r < a \\ 1-2(\Delta n/n) & r \geqslant a \end{cases} \quad . \quad . \quad (5)$$

$\Delta n/n$ is the relative difference of refractive index between the fibre axis and the cladding. The exponent κ of eqn. 5 that minimises the r.m.s. impulse width is $\kappa = 1-1{\cdot}2(\Delta n/n)$.

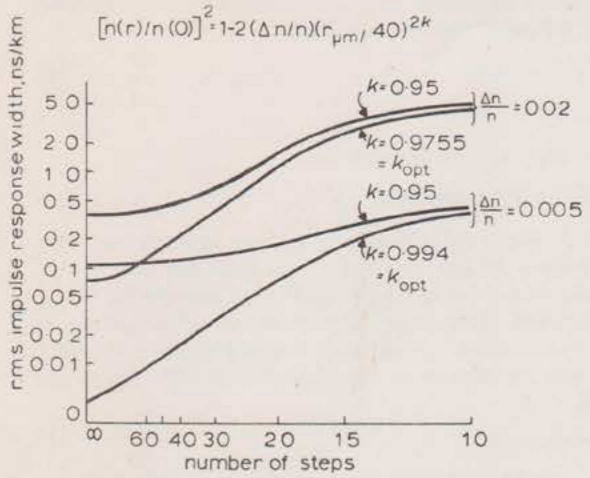

Fig. 2 *Root-mean-square impulse response width σ of fibres with stairlike profiles that approximate $r^{2\kappa}$ as function of number of steps along radius*

The steps have equal areas, the core radius is 40 μm and $\lambda = 1$ μm. κ_{opt} denotes the value of κ that minimises the r.m.s. impulse width for the smooth profile. Material dispersion and the effect of the nonzero linewidth of the source are neglected

Next, let us consider fibres with a stairlike profile that approximates the smooth profile in eqn. 5. The assumption (made in the following) that the successive rings in the fibre cross-section have equal areas seems to model correctly the fibres that are at present fabricated.[7] The relative times of flight of circularly symmetric modes are shown in Fig. 1 as functions of the radial mode number, for the smooth profile, for 40 steps and for 20 steps. We have assumed that $\Delta n/n = 0{\cdot}015$, $a = 40$ μm and $\lambda = 1$ μm. The relative time of flight exhibits oscillations of very large amplitude when there are 20 steps or fewer.

Fig. 2 gives the variation of the root-mean-square impulse response width σ as a function of the number of steps for a core radius of 40 μm, $\Delta n/n = 0{\cdot}02$ and $0{\cdot}005$. Two values of κ were considered: the value given earlier that minimises σ for the smooth profile and $\kappa = 0{\cdot}95$. This figure shows that stairlike profiles in multimode fibres with steps of equal areas may cause large degradation of the transmission capacity, even if there are as many as 40 steps. The r.m.s. impulse width is then almost four times larger for the stairlike profile than for the smooth optimum profile.

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J. A. ARNAUD
W. MAMMEL

Bell Laboratories
Crawford Hill Laboratories
Holmdel, NJ 07733, USA

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