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Corpuscular theory of intensity noise with gain compression

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When light from a laser is fully absorbed by an ideal detector, the detected current exhibits a fluctuation called here "photonic noise." The spectral density of *intensity noise*, defined as the difference of the photonic-noise spectral density and a term corresponding to the shot-noise level, is negative for sub-Poissonian statistics. The usefulness of the relative-intensity-noise concept is that it is independent of any linear attenuation. A simple circuit theory of intensity noise based only on energy conservation and the Nyquist formula (zero-point fluctuation) leads to expressions of the spectral densities that agree with quantum theory even for sub-Poissonian photon statistics. When the optical gain and loss are frequency independent, the circuit theory reduces to a corpuscular theory that keeps track of the time rates of change of electron and photon numbers treated as continuous variables. Consideration is given to laser diodes in which the rate of electron-photon conversion depends nonlinearly on both the carrier and photon densities. The cross-spectral density between electrical-voltage and relative photonic fluctuations is independent of internal or external optical losses. Standard rate equations are inaccurate in the case of gain compression. Very general yet simple formulas for intensity noise are applied to room-temperature GaAs laser diodes, using recently calculated optical parameters.

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I. INTRODUCTION

When light from a laser is applied to a detector, the detector output current fluctuates. Let us assume that light is fully collected and the detector is ideal, that is, has a bandwidth much larger than the light spectral linewidth, does not introduce any spurious noise, and each photon generates one electron. The detector output electronic-rate fluctuation is called here *photonic noise.* Unlike "intensity noise," photonic noise accounts for the corpuscular aspect of light, in the same as electronic noise accounts for the corpuscular aspect of electricity. The laser *intensity-noise* spectral density is defined as the difference between the photonic (two-sided) spectral density and the average electron rate, corresponding to the shot-noise level. This quantity is negative in the case of sub-Poissonian photon statistics. This reflects the fact that "intensity noise" is not itself a directly measurable quantity. A radio-frequency spectrum analyzer following the detector measures the photonic spectrum, which extends in principle to infinity but is limited from a practical standpoint by the detector response time to perhaps 100 GHz.

The present paper offers a semiclassical theory of photonic noise that describes the time evolution of the electron number N and photon number P treated as continuous variables. The expression obtained for the fluctuation of the outgoing photonic flow agrees with Yamamoto's quantum theory within the quasilinear approximation (Chap. 11 of Ref. [1], and Ref. [2]). Our results agree also with the quantum theory of laser diodes driven by a constant voltage given by Karlsson [3] that takes gain compression into account. Previous papers based on quantum theory (Haken [4], Lax [5], and McCumber [6]) assumed injected current fluctuations at the shot-noise level, but usually the injected current is nonfluctuating.

Let us clarify what is understood by "gain compression." The optical gain $\mathcal{G}$ in general depends on both the optical-field strength (or equivalently on the number P of photons in the cavity) and the population inversion or carrier number N. The dependence of $\mathcal{G}$ on optical frequency ν or temperature T is not treated in this paper. Lasers normally operate above threshold, in which case saturation mechanisms ensure that the relative light-intensity fluctuations are small. Saturation may occur either because the optical gain decreases a result of an increase of P, or else as a result of a decrease of N caused by an increase of P through stimulated emission, with some time delay involved. In laser-diode terminology, the former saturation mechanism is called "gain compression" [7–9]. While this effect can often be treated as a small correction in laser theory, this is the main effect relevant to microwave oscillators. A tunnel diode, for example, exhibits a negative slope in its current-voltage characteristics near some appropriate bias. When this diode is connected in parallel with a load conductance smaller in absolute value that the diode negative conductance and a tuned circuit, oscillation occurs. However, as the oscillation voltage grows, the current-voltage characteristic is explored beyond its linear part and the fundamental component of the oscillating current decreases. This behavior can be described by a reduction in the absolute value of the negative conductance, that is, by "gain compression." The controlling parameter here is P, proportional to V^*V if V denotes the complex voltage across the circuit and the star complex conjugation. It is important that the two saturation effects be distinguished. One could think at first that the dependence of $\mathcal{G}$ on N is equivalent to some dependence of $\mathcal{G}$ on P at small frequencies since time delays then are unimportant. This, however, as we shall see, is not the case, and therefore the Van der Pol equation is not an appropriate models for lasers, even at low frequencies.

A key observation made by Yamamoto (see Chap. 11 of Ref. [1]) is that photonic rates from ideal laser diodes do not fluctuate at small frequencies if the injected current is a constant. This is because for a counting time interval that is long compared with the average recombination time, almost every injected electron is converted into a photon within that time interval, provided no electron is lost through spontaneous recombination and no photon is lost by the process of optical absorption. It is essential here to appreciate the difference existing between the fluctuations of the photon number P and those of the outgoing photonic rate that we denote Q, a difference that was overlooked in earlier quantum theories. The classical photonic flow from the laser cavity is correlated with the shot-noise fluctuation. It is easy to understand how this occurs: If the shot-noise rate happens to be large at some time, the optical cavity suffers an increased loss of photons and therefore the photon number gets reduced after some period of time. As a consequence, the classical emission from the laser cavity, which is proportional to the number of photons in the cavity, is reduced. This classical fluctuation may therefore compensate for the assumed shot-noise excess rate and, in some circumstances, cancel it out.

Previous semiclassical theories of laser noise culminated in a paper by Lax [10], where the reader will find references to earlier works. (Lax considered frequency-dependent conductances, but this more general situation is not discussed here. Note that what physicists call "vacuum fluctuations" is, in our work as in Lax's paper, represented by the Nyquist's currents associated with absorbers of radiation.) Lax first treated the case of pure gain compression, and obtained a valid linewidth formula (the distinction between internal and external fields is not required for evaluating the linewidth). Next, he considered the situation applicable to laser diodes where the controlling parameter is the population inversion or carrier number N. However, the noise source in the carrier rate equation was not derived from the semiclassical theory. This information is provided by Eq. (1) below.

A different kind of semiclassical theory appeared in 1982 mainly through the work of Henry (see Chap. 2 of Ref. [1]) which we call "standard rate equations" (SRE) because this is the most commonly used formalism. A handy presentation of SRE, as well as references to early quantum-theory derivations of the "Langevin forces," can be found in Agrawal and Dutta's book [7]. In his semiclassical theory, Henry does not make use explicitly of Nyquist's noise sources as Lax did, but he assumes that spontaneous emission adds photons to the oscillating field at a rate equal to the reciprocal of the lifetime of a photon in the laser cavity (for full population inversion). Fluctuation of the injected current at the shot-noise level is implicit. What the theory provides is the intensity-noise spectral density. Thus, in order to obtain the detected-current fluctuation, a shot-noise term must be added. This theory can be modified to account for injected-current fluctuations below the shot-noise level simply by modifying the Langevin term in the carrier rate equation. The price to pay for this generalization is that the spectral density of the intensity noise may be negative. But formally these modified rate equations (MRE) give the same predictions for measurable quantities as quantum [2] or circuit [11–14] theories in the absence of gain compression. When gain compression is taken into account, however, the *modified* rate equations themselves become invalid [15], and this is perhaps the major finding of this paper. Of course, when only fluctuations well above the shot-noise level are considered [16], the distinction between intensity noise and photonic noise is unnecessary.

Semiclassical theories usually describe light on the basis of a probability $\lambda(t)$ that a photon be detected at time t, the light intensity $\lambda(t)$ being an independent random function of time (doubly stochastic point process) [17]. According to this representation, the photocurrent noise cannot be below the shot-noise level. In the corpuscular theory presented in the present paper, the classical photonic flow from the laser cavity is correlated with the shot-noise fluctuation. This corpuscular theory does not make use explicitly of the concept of the self-excited point process [18] and remains very simple. It is akin to Lax's semiclassical theory [10], but gives explicitly the carrier-equation noise source.

Classical variations will be denoted by Δ, e.g., ΔN, ΔP. The variation δR of the rate at which electron-hole pairs are converted into photons, or, conversely, photons into electrons, consists of the variation ΔR of some classical function $R(N,P)$ of N and P, plus a fluctuation $\imath(t)$ at the shot-noise level, which is simply related to Nyquist currents [19,20] as we show below. When electrons diffuse from one electron reservoir to another with small energy steps ($kT \ll h\nu$), $\imath(t)$ may be set equal to zero. In contradistinction, spontaneous Auger recombination involves fluctuations $\imath(t)$ at twice the shot-noise level because two electrons at a time recombine with holes in that process.

The key proposal [11] is that the rate R at which electrons and holes recombine in an active element is given by

$$R = \mathrm{Re}[V^*(YV + c)]/h\nu_0 , \tag{1}$$

where $h\nu_0$ is the photon energy, Y the admittance of the active element, V the complex voltage, V^* its complex conjugate, and $c(t)$ the Nyquist current in the narrow-band representation [21]. The term YV follows from Ohm's law, and the c term from Kirchhoff's law. Equation (1) thus is rather trivial in hindsight, since it simply expresses the corpuscle conservation law, the photonic rate being defined as the ratio of electromagnetic power and photon energy. Equation (1) is applicable as well to detectors, in which case Y is a constant. The fluctuation $\imath(t)$ mentioned earlier is the $\mathrm{Re}(V^* c)/h\nu_0$ term of Eq. (1). It can be considered a stationary process, provided

the variations of $|V|$ are small. Equation (1) and the Nyquist formula form the full physical basis of the theory presented in Refs. [11–15], Chap. 3 of [1], and here. Gain compression, however, was not considered in the earlier papers. It is the purpose of the present paper to present a semiclassical theory of photonic noise that takes gain compression into account. A number of observations concerning previous semiclassical theories are presented below.

Constant injected currents imply constant output photonic rates under ideal conditions, as we discussed earlier, whether or not there is gain compression. In the case of pure gain compression (that is, when $\mathcal{G}$ depends on P but not on N), however, the carrier rate equation becomes irrelevant. Then, suppressing the injected-current fluctuations apparently does not affect the output photonic rate fluctuations, in contradiction with the principle stated above which requires that photonic rate fluctuations should vanish. This observation led some authors to suggest that suppression of the injected-current fluctuations could be ascribed to a modification of the Nyquist currents associated with the active element. What is really happening is that in the limit presently considered, the fluctuations of N blow up to the point where the quasilinear approximation is no longer a valid one. In other words, the assumptions that $\mathcal{G}$ does not depend on N and that spontaneous emission is negligible are not consistent with the quasilinear approximation. Note that the term $\mathcal{G}_N \Delta N$, where $\mathcal{G}_N$ denotes the partial derivative of $\mathcal{G}$ with respect to N, and ΔN the deviation of N from its steady-state value, vanishes also when $\Delta N = 0$, that is, when a constant voltage is applied to the intrinsic laser diode [3]. But this is an entirely different situation: the injected current now necessarily fluctuates.

Even though this paper is not concerned with phase noise, a word on this subject is needed to explain why many authors found it plausible that the case of pure gain compression familiar to microwave engineers could be an acceptable laser model. The laser linewidth depends primarily on slow frequency fluctuations (phase diffusion). When a controlling parameter such as P or N deviates from its steady-state value, both the real and the imaginary parts of the active element admittance Y deviate from their steady-state values by, say, ΔG and ΔB, respectively. The (real) frequency deviation $\delta\nu$ is easily obtained by specifying that both the real and the imaginary parts of the total admittance vanishes [22]. To obtain the laser linewidth, it is not necessary to solve the carrier rate equation when the ratio $\Delta B/\Delta G$ is independent of ΔN and ΔP, that is, in the three following cases (subscripts denote partial derivatives): (a) $Y_P = 0$ (laser diodes without gain compression); (b) $Y_N = 0$ (microwave oscillators); and (c) Y_N/Y_P is a real constant, e.g., $\Delta B = 0$. In those cases, the Schawlow-Townes formula applicable to the linear regime is to be multiplied by a term of the form $(1+\alpha^2)/2$. In general, however, the carrier rate equation is required and the expression for the laser linewidth is more complicated. This is why we view the agreement in form of the above linewidth formulas as coincidental.

The paper is organized as follows. Section II presents the circuit-theory principle in a simple form. Because energy conservation is enforced at every step, it is easily verified that the detected current is equal to the injected current at low frequency and large powers. It is further shown that the independent Nyquist currents of the circuit theory are equivalent to independent shot-noise fluctuations of corpuscles.

The corpuscular theory is used in Sec. III to evaluate at zero frequency photonic noise, electrical voltage fluctuations, and cross-spectral density. The expression for the electrical-voltage–photonic-noise correlation seems to agree with recent experimental results [23,24]. Gain compression may lead to sub-Poissonian photonic noise even when the injected current suffers from fluctuations at the shot-noise level, a new result of practical interest. Arbitrary baseband frequencies and injection conditions are considered in Sec. IV. As many authors noted before, even a small amount of gain compression importantly damps the relaxation oscillations. Nonzero electrical admittances (e.g., the admittance of blocking layers) contribute to the relaxation oscillation damping. The electrical noise adds up to the noise due to spontaneous emission.

Optical losses, either internal to the laser diode or external to it, are treated in Sec. V. According to the usual optical engineering concept of "intensity noise," measurable fluctuations may be split into an intensity noise that behaves as a classical modulation and a shot-noise term. This semiclassical picture is shown to be a valid one even for sub-Poissonian photoelectron statistics, in which case a negative intensity-noise spectral density can be formally introduced. Furthermore, the cross-spectral density between electrical voltage and relative photonic noise is found to be independent of the loss. It is shown in Sec. VI that when the gain does not depend explicitly on the optical field, every measurable quantity can be obtained from the modified rate equations, but MRE are inaccurate in the case of gain compression.

Formally, the corpuscular theory is easily generalized to any number of electron and photon reservoirs connected in arbitrary manner. To ascertain whether the resulting formalism corresponds meaningfully to some physical situation, one must go back to circuit theory. We consider in Sec. VII active elements connected in parallel with a single cavity and, in Sec. VIII, two optical cavities resonating at sufficiently different optical frequencies. The two-cavity system is stable in the presence of gain compression only. An application is made in Sec. IX. The conclusion is given in Sec. X.

II. CIRCUIT AND CORPUSCULAR THEORIES

The circuit theory [11,13] rests only on the formula proposed by Nyquist [20] for the fluctuations associated with a conductance G, and on the law of energy conservation. A concise presentation is given in this section. In the case of frequency-independent gain and loss, the circuit theory can be written in the form of a rate equation for corpuscles—electrons and photons being treated equally.

As shown in Fig. 1, our laser oscillator model consists

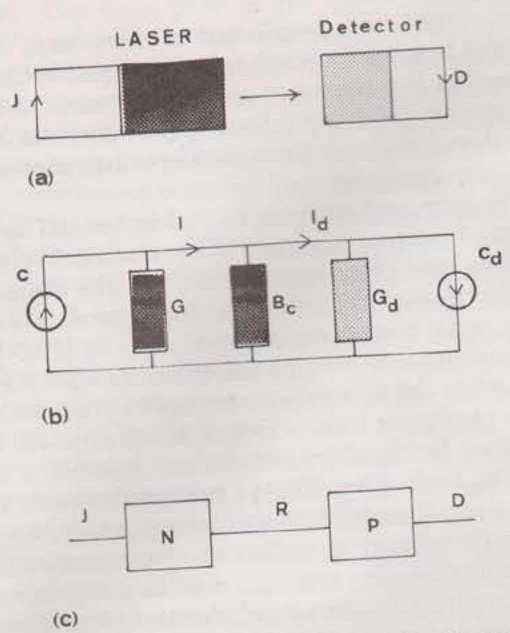

FIG. 1. (a) Laser-detector configuration showing the current source driving the laser and the detected current. (b) Schematic appropriate to the circuit theory. The laser active material is represented by a negative conductance G and a Nyquist current source c. The admittance $iB_c(\nu)$ represents the laser optical cavity. The detector is represented by a positive conductance G_d and a Nyquist current source c_d. (c) Schematic applicable to the corpuscular theory. N and P represent, respectively, the electron and photon numbers; $J(N)$ the rate at which electrons are injected into the laser diode; $R(N,P)$ the rate at which electrons are converted into photons; and $Q(P)$ the photonic rate. Spontaneous emission is not shown. The detector is assumed ideal ($Q=D$).

of a negative conductance G, a tuned circuit resonating at frequency ν_0 represented by a susceptance $B_c(\nu)$, and a positive constant conductance G_d. The subscript d stands for "detector," since in most cases the load is used to measure the laser power.

The tuned circuit consists of a capacitance C and an inductance L in parallel, with $LC(2\pi\nu_0)^2=1$. Let $\sqrt{2h\nu_0}V(t)$ denote the complex voltage across the circuit and $\sqrt{2h\nu_0}I_c(t)$ the complex current entering into it. The square root, where $h\nu_0$ denotes the photon energy, is introduced for later convenience. Provided the time variations are small over an optical period, we have approximately

$$\frac{dV}{dt}=\frac{I_c}{2C}. \tag{2a}$$

This expression can be justified formally from a power-series expansion of the bicomplex admittance (see Appendix A of Ref. [13]), with the ratio of baseband frequency f and average optical frequency ν_0 as a small parameter.

Let both sides of this expression be multiplied by V^*, where the star denotes complex conjugate, and add the complex conjugate. We obtain

$$\frac{dP}{dt}=\mathrm{Re}(V^*I_c),\quad P=C|V|^2 \tag{2b}$$

where P denotes the number of photons in the resonator, defined as the electromagnetic energy divided by the photon energy, a large number. Equation (2b) says that the rate of increase of the photon number P as a function of time is equal to the rate at which photons enter the resonator. The resonator losses are treated separately. The first equality in Eq. (2b) is, of course, exact, but the expression of P is valid only near the resonant frequency.

Referring to the schematic in Fig. (1b) and using Kirchhoff's law, $I_c=I-I_d$, we obtain the photonic rate equation

$$\frac{dP}{dt}=R-Q,\quad R=\mathrm{Re}(V^*I),\quad Q=\mathrm{Re}(V^*I_d) \tag{3}$$

where R denotes the rate at which photons are generated by the negative conductance and Q the rate at which photons are absorbed by the positive conductance. For slow variations, $dP/dt=0$ and, obviously, $R(t)=Q(t)$.

Consider next conductances converting light to electron-hole pairs or, conversely, electron-hole pairs to light. According to Nyquist, a fluctuating current $c(t)$ of double-sided spectral density,

$$S_c(\nu)=|h\nu G|, \tag{4a}$$

should be associated with any conductance G expressing either stimulated absorption ($G\equiv G_a>0$) or stimulated emission ($G\equiv G_e<0$). For electrons obeying the Fermi-Dirac statistics at temperature T (that is, with a single Fermi level), one easily shows that Eq. (4a) is equivalent to the thermal equilibrium form given in Nyquist's paper [19].

For the narrow-band processes considered in this paper centered at frequency ν_0, Nyquist's current may be described by a white complex random function of time,

$$c(t)=\mathrm{Re}\{\sqrt{2h\nu_0}[c'(t)+ic''(t)]\exp(-i2\pi\nu_0t)\}. \tag{4b}$$

With that understanding, the double-sided spectral densities of c' and c'' are [21]

$$\mathcal{S}_{c'}=\mathcal{S}_{c''}=|G| \tag{4c}$$

and c',c'' are uncorrelated. The normalized currents c',c'' vary slowly in comparison with the optical frequency, but may nevertheless be considered white. In a numerical simulation, the values of c' and c'' would be selected randomly and independently for each time slot T. A slot duration T of the order of 1 ps, much longer than the optical period but much shorter than any time relevant to the laser dynamics, is appropriate.

The detector is modeled by an absorbing time-independent positive conductance G_d. A physical model is a collection of atoms, all of them in the ground state. When these atoms are submitted to light at frequency ν_0, some electrons are raised from the ground state to the ionization state, taken to have an energy $h\nu_0$ above the ground level, each photon generating one electron. It is assumed that these electrons are quickly drawn out of the active region and do not affect noticeably the conductance.

Usually the detector is located many wavelengths away from the light source rather than directly connected to it,

as shown in Fig. 1. The separation between the laser diode and the detector is, however, immaterial as long as the transmission medium is single moded (focused Gaussian beam, lossless single-mode optical fiber, or coaxial line) and the detector is perfectly optically matched.

Because the positive conductance G_d models an ideal detector, the outgoing photonic rate $Q(t)$ coincides with the detected electronic rate $D(t)$ in the absence of optical losses:

$$Q(t)=\mathrm{Re}\{V^*(t)I_d(t)\}, \tag{5a}$$

$$I_d(t)=G_dV(t)+c_d(t),\quad c_d(t)\equiv c_d'(t)+ic_d''(t) \tag{5b}$$

$$Q(t)=G_d|V(t)|^2+|V(t)|c_d'(t), \tag{5c}$$

$$Q(t)\equiv(G_d/C)P(t)+q(t). \tag{5d}$$

In the final expression only the real part of the complex Nyquist current enters. In the second term of Eq. (5c) it was permissible to replace V by $|V|$ because $c_d(t)$ is a white process. The capacitance C and the photon number $P\equiv C|V|^2$ are introduced in Eq. (5d) for later convenience.

In general, $q(t)$ depends on the realization of $V(t)$. But because the relative variation of $|V|$ is small in the saturated regime considered, $q(t)$ is approximately a stationary process of spectral density

$$\mathcal{S}_q=G_d|V|^2\approx\bar{Q}. \tag{6a}$$

Upper bars denoting average values will be omitted when no confusion with instantaneous values may arise. The first-order variation of $Q(t)$ denoted δQ is from Eq. (5d):

$$\delta Q=Q_P\Delta P+q(t), \tag{6b}$$

where

$$Q_P=G_d/C\equiv1/\tau_p,\quad \Delta P\equiv P(t)-\bar{P}, \tag{6c}$$

τ_p is called the photon lifetime.

Note that δQ enters the photon-number rate equation [Eq. (3)]. Therefore, the two terms in the right-hand side of Eq. (6b) *are* correlated. This is an essential departure from conventional corpuscular theories in which δQ is written as the sum of a classical term $\Delta P/\tau_p$ and an *independent* shot-noise term q, sometimes ascribed to the detector photocurrent.

Relations analogous to Eq. (6) apply to the admittance Y modeling the active region of the laser diode, whose steady-state value is a negative conductance $\bar{G}$. The situation is more complicated than for the detector, however, because Y depends on the carrier number N, and possibly on $|V|^2$ or P in the case of gain compression. It may also be that, as N or P depart from their average values, the admittance Y acquires a small imaginary part (see the Introduction). But this imaginary part drops out from the rate equations for photonic noise and it need not be considered here. With the sign convention of Fig. 1(b), we have

$$R=\mathrm{Re}(V^*I),\quad I=-GV+c. \tag{7a}$$

This relation (given in the Introduction with a different sign convention for the current) can be written as

$$R=-G(N,|V|^2)|V|^2+r,\quad \mathcal{S}_r=-G|V|^2\approx\bar{R}. \tag{7b}$$

Here again the approximate expression of the spectral density of the shot-noise process $r(t)$ rests on the fact that the relative variation of $|V|$ is small.

In the case where G is the sum G_a+G_e of a positive conductance G_a expressing stimulated absorption and a negative conductance G_e expressing stimulated emission, the expression of the spectral density of r reads more generally

$$\mathcal{S}_r=(G_a-G_e)|V|^2\approx(2n_p-1)\bar{R},\quad n_p\equiv G_e/G \tag{8}$$

where n_p is the population inversion factor (also denoted n_s or n_{sp} and called "spontaneous emission factor").

The classical part of the photonic rate R [the first term in Eq. (7b)] may be written as the product of P and a gain $\mathcal{G}$ according to

$$R(N,P)\equiv\mathcal{G}(N,P)P, \tag{9a}$$

where

$$\mathcal{G}(N,P)\equiv-G(N,|V|^2)/C,\quad P=C|V|^2. \tag{9b}$$

Because the relative variations of N and P are small in the saturated regime, it is permissible to consider first-order expansions with respect to N and P and write

$$\Delta R\equiv R(N,P)-R(\bar{N},\bar{P})=R_N\Delta N+R_P\Delta P, \tag{10a}$$

$$\Delta N=N-\bar{N},\quad \Delta P=P-\bar{P}, \tag{10b}$$

where the subscripts indicate partial derivatives, evaluated at the average values $\bar{N},\bar{P}$. The first-order expansion in Eq. (10) is valid if $\langle\Delta N^2\rangle\ll\langle N\rangle^2$ and $\langle\Delta P^2\rangle\ll\langle P\rangle^2$. The variances of N and P can be evaluated by integrating over frequency the detailed expressions given later for the spectral densities. These integrals always exist.

Our main objective is to express the rate fluctuation $\delta R\equiv\Delta R+r$ in terms of ΔP plus independent noise terms. To achieve that goal, we need to eliminate ΔN by using the electron-rate equation

$$\frac{dN}{dt}=J(t)-S(t)-R(t). \tag{11}$$

The three rates J,S,R on the right-hand-side of Eq. (11) consist of an average part, a classical variation, and a fluctuation. The rate J at which electrons are injected may have a specified part (modulation) and depend on N if the electrical source has a nonzero admittance. It may be affected by random fluctuations, thermal or shot noise, depending on the kind of electrical driver used. For generality the spectral density of the injected-current fluctuation is written $\mathcal{S}_j=\xi\bar{J}$, where ξ would be unity for fluctuations at the shot-noise level, but is usually negligibly small.

The spontaneous emission rate S is some function $S(N)$ of the carrier density N and is affected by a fluctuation $s(t)$. If we assume for a moment that the diode is constant-voltage driven, N is a constant and the fluctuation δS of S reduces to $\mathscr{s}(t)$. The word "spontaneous" means that the recombination events are independent of each other. The spectral density of the $\mathscr{s}(t)$ process, however, depends on the nature of the recombination events. If the elementary event is the recombination of one electron and one hole (radiative spontaneous recombination), the spectral density is at the shot-noise level. But in the Auger effect, two electrons recombine at a time. Since N must not change, the electrical driver instantaneously replaces these two electrons and the spectral density of $\mathscr{s}(t)$ is at twice the shot-noise level (the same fluctuation results when an electron generates exactly two electrons by secondary emission). The spectral density of $\mathscr{s}$ is written $\mathcal{S}_{\mathscr{s}}=\xi_s\bar{S}$.

Finally, as we discussed above, $R(t)$ is the rate at which electrons and holes recombine in the active element by the process of stimulated emission to generate photons in the oscillating mode. R is a function of N and P, plus a fluctuation $\mathscr{r}(t)$ whose spectral density is given in Eq. (8).

Before going into further detail, let us consider the steady-state and the low-frequency expressions. For the steady-state values we have from Eqs. (3) and (11)

$$\bar{J}-\bar{S}=\bar{R}=\bar{Q}\ . \tag{12}$$

At low frequency the output photonic rate $Q(t)$ is equal to the photon generation rate $R(t)$, as we have seen earlier. On the other hand, the carrier rate equation, Eq. (11), shows that at low frequency ($dN/dt=0$) and large injected currents ($S \ll J$), $R(t)$ is equal to the injection rate $J(t)$. The theory therefore predicts that for slow variations and high injected currents, the photonic rate $Q(t)$ is equal to $J(t)$. In this proof it was unnecessary to introduce explicitly the Nyquist currents or the dependence of the optical gain $\mathcal{G}$ on the carrier number N or on $|V|$ (nonlinear gain). The conclusion that $Q(t)=J(t)$ under ideal conditions (Chap. 11 of Ref. [1]) follows here from energy conservation, enforced at every step of the theory independently of the specific form of the noise sources.

Let us now consider small deviations at angular frequency Ω ($d/dt \to j\Omega$). We obtain from Eqs. (3), (6), and (11)

$$\delta J-\delta S-j\Omega\Delta N=\delta R=\delta Q+j\Omega\Delta P\ . \tag{13a}$$

Equation (13a) can be written in a more detailed form:

$$\Delta J+\mathscr{j}-\Delta S-\mathscr{s}-j\Omega\Delta N=\Delta R+\mathscr{r}=\Delta Q+\mathscr{q}+j\Omega\Delta P\ , \tag{13b}$$

if we separate the classical parts and the fluctuations, where

$$\Delta J=J_N\Delta N,\quad \Delta S=S_N\Delta N\ , \tag{14a}$$

$$\Delta R=R_N\Delta N+R_P\Delta P\ , \tag{14b}$$

$$\Delta Q=Q_P\Delta P,\quad Q_P\equiv 1/\tau_p\ , \tag{14c}$$

$$\mathcal{S}_{\mathscr{j}}=\xi J,\quad \mathcal{S}_{\mathscr{s}}=\xi_s S\ , \tag{14d}$$

$$\mathcal{S}_{\mathscr{r}}=(2n_p-1)R,\quad \mathcal{S}_{\mathscr{q}}=(2n_t-1)Q\ , \tag{14e}$$

and $\mathscr{j},\mathscr{s},\mathscr{r},\mathscr{q}$ are independent. For simplicity we will take in the following $\xi_s=1$ (radiative spontaneous recombination) and n_t-1 because the detector temperature T_d is such that $kT_d \ll h\nu_0$, but ξ and n_p remain as parameters.

The system of equations in Eq. (13) enables one to evaluate the photonic noise $\delta Q=\Delta Q+\mathscr{q}$ and the fluctuation ΔU of the voltage across the diode from $\Delta U=U_N\Delta N$.

If Eq. (14) is introduced into Eq. (13b), we obtain

$$(J_N-S_N-j\Omega)\Delta N+\mathscr{j}-\mathscr{s}=R_N\Delta N+R_P\Delta P+\mathscr{r} =(Q_P+j\Omega)\Delta P+\mathscr{q}\ . \tag{15}$$

Let Eq. (15) be divided through by $\bar{R}=\bar{Q}$, and introduce the following dimensionless parameters:

$$x\equiv g\Delta N/N,\quad p\equiv \Delta P/P\ , \tag{16a}$$

$$(N/S)S_N\equiv s,\quad (N/R)R_N\equiv g,\quad (P/R)R_P\equiv 1-\gamma\ . \tag{16b}$$

s is between 1 and 2 for radiative spontaneous recombinations but may reach the value of 3. At room temperature, the differential gain factor g is usually between 1 and 4, but at low temperatures g is almost equal to zero below the peak gain frequency (because an increase of N raises the quasi-Fermi level without affecting much the gain at a constant frequency) and almost infinite above the peak-gain frequency. The relative gain compression γ is not accurately known. It may be of the order of 0.1.

Let us further introduce the spontaneous-to-stimulated rate ratio ζ and the spontaneous lifetime τ_s,

$$\zeta=\frac{\bar{S}}{\bar{R}},\quad \tau_s=\frac{\bar{N}}{\bar{S}}\ , \tag{17a}$$

and the complex parameter

$$\chi\equiv\frac{\zeta}{g}(s-\tau_s J_N+j\tau_s\Omega)\equiv\chi'+j\chi''\ . \tag{17b}$$

If the intrinsic diode is loaded by an electrical admittance $Y_e(\Omega)$, the term $\tau_s J_N$ in Eq. (17b) may be written, from Ohm's law,

$$\tau_s J_N\equiv -y(\Omega),\quad y(\Omega)\equiv R_d Y_e(\Omega),\quad R_d\equiv\frac{NU_N}{eS}\ , \tag{17c}$$

where U denotes the electrical voltage across the intrinsic diode and NU_N is of the order of $3kT/e \approx 0.075$ V. The denominator eS in the expression of R_d, where e denotes the absolute value of the electron charge, is the threshold current in the absence of gain compression because the spontaneous recombination rate is then clamped at its threshold value. With gain compression, S increases slightly above threshold. The y parameter is real at low frequencies.

It is also useful to define a normalized baseband frequency,

$$f_n\equiv\frac{f}{f_0}\equiv\tau_p\Omega\ , \tag{18}$$

where f_0 denotes the cold cavity linewidth. In terms of f_n, $\chi\equiv\chi'+i\chi''$ reads

$$\chi'=\frac{\zeta}{g}(s+y'),\quad \chi''=\frac{N}{Pg}f_n+\frac{\zeta}{g}y'' \tag{19}$$

where $y\equiv y'+jy''$ may depend on f_n.

With this notation, Eq. (15) is simply

$$-\chi x+\mathscr{l}^0=x+(1-\gamma)p+\mathscr{r}^0=(1+jf_n)p+\mathscr{q}^0\ . \tag{20a}$$

The spectral densities of the reduced independent noise sources introduced in Eq. (20a),

$$\mathscr{l}^0\equiv(\mathscr{j}-\mathscr{s})/R,\quad \mathscr{r}^0\equiv\mathscr{r}/R,\quad \mathscr{q}^0\equiv\mathscr{q}/Q \tag{20b}$$

($\bar{R}=\bar{Q}$, but the distinction between R and Q is maintained for later convenience), are

$$R\mathcal{S}_{\mathscr{l}^0}=\zeta+\xi(1+\zeta)\ ,$$

$$R\mathcal{S}_{\mathscr{r}^0}=2n_p-1\ , \tag{20c}$$

$$Q\mathcal{S}_{\mathscr{q}^0}=1\ .$$

If x from the first equality in Eq. (20a) is substituted in the middle term, we obtain an expression for $\delta R/R$ proportional to p plus noise terms. The relative photon-number fluctuation p follows by comparison with the last expression in Eq. (20a). Relative photonic noise then follows from

$$\frac{\delta Q}{Q}=p+\mathscr{q}^0\ . \tag{20d}$$

In this expression, p is a weighted sum of the independent noise sources $\mathscr{l}^0,\mathscr{r}^0,\mathscr{q}^0$ and is therefore correlated with $\mathscr{q}^0$.

A useful concept is that of relative intensity noise ($N_{\rm ri}$) defined according to

$$QN_{\rm ri}\equiv Q\mathcal{S}_{\delta Q/Q}-1\ . \tag{20e}$$

We allow the relative intensity noise to be negative in some cases for the reason discussed later in Sec. V.

Equivalent electrical circuits are often convenient as visual aids and they may be constructed to simulate laser diodes in systems. An equivalent electrical circuit can almost be read off Eq. (15), ΔN and ΔP being represented by electrical voltages, while the rates are represented by electrical currents. The optical cavity is represented by a capacitor of value unity submitted to a voltage ΔP. The electrical current flowing through this capacitor is indeed the rate $j\Omega\Delta P$ given in the last term of Eq. (15). The rate δQ going from the cavity to the absorbing load is represented by a conductance Q_P in parallel with a current source $\mathscr{q}$. The active part of the laser diode delivering a rate δR to the cavity is described by the first two terms in Eq. (15). The middle term can be represented by a quadripole whose output voltage is ΔP (see above) and the input voltage $(-R_N/R_P)\Delta N$ is proportional to the electrical voltage $\Delta U=U_N\Delta N$ across the intrinsic diode. The quadripole consists simply of a series conductance $-R_P$ and current source $\mathscr{r}$. It is easy to verify from Ohm's law that the input and output current of this quadripole are indeed equal to δR. Electron storage corresponding to the term $-j\Omega\Delta N$ is represented by a capacitance $-R_P/R_N$ connected at the input port of the quadripole. The other terms ($J_N,S_N,\mathscr{j},\mathscr{s}$) are conductances and current sources at the input port of the quadripole

In this equivalent electrical circuit, independent noise sources enter only once, and generalization to multiple elements connected to the optical cavity is straightforward. One drawback is that negative conductances and capacitances (usually $\partial R/\partial P>0$) can be realized only with active electronics. By duality (current-voltage) transformation, this circuit can be reduced in the absence of gain compression to the one given in Chap. 11 of Ref. [1] that involves only positive reactive elements.

To conclude this section, the circuit theory based on Nyquist's currents [13] can be expressed in the case of frequency-independent gain and losses in the form of a corpuscular theory that postulates that the rates of electron-hole recombination into photons are classical functions of N (number of electrons) and P (number of photons) plus independent fluctuations whose spectral densities are equal to the average rates. Gain compression is accounted for. This theory is in full agreement with quantum theory but *does not* coincide with the particle-rate theories usually presented in which the photon-number fluctuations are independent of the "detector shot noise."

III. PHOTONIC NOISE AT ZERO FREQUENCY

It is assumed in this section that the frequency is equal to zero, that is, we set $f_n=0$ in Eq. (20), and the χ parameter is real. The formulas derived in this section are valid for laser diodes from about 1 to 50 MHz. Optical losses are neglected here but they are easily restored, as we shall see in Sec. V.

Without gain compression, $\gamma=0$, $p\equiv\Delta P/P$ drops out from the last two equations in Eq. (20a) and thus

$$x\equiv g\frac{\Delta N}{N}=\mathscr{q}^0-\mathscr{r}^0 \tag{21}$$

and from the first and last expressions in Eq. (20a)

$$\frac{\delta Q}{Q}=\chi(\mathscr{r}^0-\mathscr{q}^0)+\mathscr{l}^0\ . \tag{22}$$

The spectral densities are, therefore, using Eq. (20c),

$$Q\mathcal{S}_{\Delta N/N}=\frac{2n_p}{g^2}\ , \tag{23a}$$

$$Q\mathcal{S}_{\Delta N/N,\delta Q/Q}=-\frac{2n_p\chi}{g}\ , \tag{23b}$$

$$QN_{\rm ri}=\zeta+\xi(\zeta+1)-1+2n_p\chi^2\ . \tag{23c}$$

Let us recall our notation: Q denotes the average photonic rate with dimension s^{-1} and $\mathcal{S}$ double-sided spectral densities. Because the carrier number N is clamped to its threshold value in the case of linear gain, the parameters g, n_p, and s are constant above threshold. The population inversion factor n_p is infinite at zero optical gain, but usually comprised between 1 and 3. The dimensionless differential gain g is usually between 1 and 4, and $s\approx 2$. Usually, $\xi\approx 0$. ζ denotes the spontaneous-to-stimulated emission ratio whose reciprocal is the ratio

I_e/I_{th} of injected to threshold current minus one. Finally, $\chi=\zeta s/g$ if the electrical driver has infinite internal impedance.

Equation (23a) gives the fluctuations of the electrical voltage U across the diode since $\Delta U=U_N\Delta N$. The spectral density of these voltage fluctuations decreases as the laser output increases and becomes negligible compared with the thermal voltage fluctuation associated with the diode series resistance (not considered here) at large powers.

Equation (23c) gives the relative intensity noise. For large constant injected currents, we find by taking the appropriate limit that the spectral density of δQ tends to a constant value equal to S.

Finally, Eq. (23b) gives the cross spectral density between electrical voltage fluctuation and relative photonic noise. The quantity $S_{\Delta N/N,\delta Q/Q}$ can be shown to be independent of the optical attenuation. The correlation is, for a constant injected current [13],

$$C_{UQ}\equiv\frac{S_{\Delta U,\delta Q}}{\sqrt{S_{\Delta U}S_{\delta Q}}}=-\left[1+\frac{g^2}{2n_p\zeta s^2}\right]^{-1/2}. \tag{24}$$

Measurements of C_{UQ} have been reported in Refs. [23] and [24].

Let us now consider the effect of nonlinear gain, $\gamma\neq0$. We obtain by solving Eq. (20):

$$\frac{\delta Q}{Q}=\frac{\wp+\chi\mathscr{r}^0+(\gamma-1)\chi\mathscr{d}^0}{1+\gamma\chi}, \tag{25}$$

$$QN_{ri}=\frac{\zeta+\xi(1+\zeta)-1-2\gamma\chi+2(n_p-\gamma)\chi^2}{(1+\gamma\chi)^2}. \tag{26}$$

Equation (26) gives the relative intensity noise. Modified rate equations lead to an expression which is identical to Eq. (26) for linear gain ($\gamma=0$), but in the general situation γ does not appear in the numerator of Eq. (26) as it should.

According to Eq. (17), the case of a zero impedance electrical drive corresponds to the limit $\chi\to\infty$ in Eq. (26). We obtain

$$QN_{ri}=\frac{2(n_p-\gamma)}{\gamma^2}. \tag{27}$$

This is exactly the result obtained by Karlsson [3] from quantum theory. It differs from the result obtained from MRE unless γ is negligible compared with unity. The current fluctuation is $\mathscr{s}+\delta Q$.

The result in Eq. (27) also applies to the case of pure gain compression ($R_N=0$ or $\mathcal{G}_N=0$), e.g., to laser diodes operating below the peak-gain frequency at $T=0$ K [see, e.g., Eq. (11.46) in Chap. 11 of Ref. [1], where $s/2$ corresponds to our γ factor and only the case $n_p=1$ is considered. The maximum output power corresponds to $\gamma=1$.] However, the quasilinear approximation breaks down if the diode is driven by a constant current and spontaneous recombination is neglected.

It is interesting that Eq. (26) predicts that photonic fluctuations below the shot-noise level can be obtained even when the injected-current fluctuations are at the shot-noise level ($\xi=1$) by a combination of gain compression and spontaneous emission and/or finite drive impedance. For example, if we set in Eq. (26) $n_p=\gamma=\chi=1$, we obtain a photonic spectral density relative to shot noise equal to $(1+\zeta)/2$. At large injected currents ($\zeta\approx0$), photonic noise is half the shot-noise level, that is, sub-Poissonian photon statistics has been generated. (Under the same conditions, standard rate equations predict a photonic noise at 1.5 the shot-noise limit). If the electrical drive impedance is infinite, we have explicitly $\chi=\zeta s/g$, and a small ζ value implies a large s/g value that may not be plausible at room temperature. But an alternative is to reduce the electrical drive impedance (thermal noise remains negligible). Equation (17) shows that $\chi=1$ and $\zeta=0.2$ are consistent with $s=2$, $g=1$, if the series resistance R_s is equal to 25 Ω, for a threshold current of 1 mA and $NU_N\approx3kT/e$. Since the internal resistance of a laser diode is of the order of 10 Ω, the 25-Ω total value can be achieved without negative resistances. The γ value of unity assumed in this example is not plausible, however. But there are spatial inhomogeneity effects (lateral diffusion, induced grating, ...) that may lead to a large effective gain-compression factor (it is not entirely clear at the moment, though, whether the present noise theory applies also to *effective* gain compression). Equation (26) predicts sub-Poissonian photon statistics for small γ values at large injected currents and small R_s values, but the noise reduction is too small to be of practical interest.

IV. PHOTONIC NOISE AT ARBITRARY BASEBAND FREQUENCIES

From the first two expressions in Eq. (20a), x can be obtained and substituted in the middle term to get

$$\frac{\delta R}{R}=\frac{(1-\gamma)\chi p+\chi\mathscr{r}^0+\ell^0}{1+\chi}. \tag{28}$$

This relation incidentally is applicable to the linear load as well, in which case, $g=\gamma=0$, and thus $\chi\to\infty$, with the change of notation, $\mathscr{r}\to-\mathscr{q}$, $R\to-Q$.

If we specify that $\delta R/R$ in Eq. (28) is equal to $(1+jf_n)p+\mathscr{q}^0$, we find that p is multiplied by the factor

$$\mathcal{D}\equiv1+jf_n+(\gamma+jf_n)\chi, \tag{29}$$

where χ is defined in Eq. (19). The relaxation frequency f_r is the real part of a root of $\mathcal{D}(f)$. We have approximately

$$2\pi f_r=\left[\frac{d\mathcal{G}}{dN}Q\right]^{1/2}. \tag{30}$$

Gain compression strongly enhances the relaxation oscil lation damping but does not affect their frequency much.

The relative photonic noise is, from the above equa tions,

$$\delta Q/Q=p+\mathscr{q}^0$$
$$=\{\wp+\chi\mathscr{r}^0+[jf_n+(jf_n+\gamma-1)\chi]\mathscr{q}^0\}/\mathcal{D}. \tag{31}$$

Using the expressions for the spectral densities in E (20c), we finally obtain after much simplification t

double-sided relative intensity noise N_{ri},

$$QN_{ri}=\frac{\zeta+\xi(\zeta+1)-1-2\gamma\chi'+2(n_p-\gamma)|\chi|^2}{(1+\gamma\chi'-\chi''f_n)^2+[\gamma\chi''+(1+\chi')f_n]^2}, \tag{32}$$

where Q denotes the *total* generated photonic rate. The relative intensity noise is independent of internal or external linear losses even if it is negative. Thus the result in Eq. (32) is fully general for cold detectors.

V. LINEAR LOSS: CONCEPT OF INTENSITY NOISE

The concept of intensity noise, or that of relative intensity noise (N_{ri}), is often used in optical engineering. In this context, "intensity" refers to classical rates and does not include fluctuations relating to the corpuscular behavior of light flows ($\mathscr{q}$). This intensity-noise concept is shown here to be a valid one provided the losses are linear. It is immaterial whether the losses are internal or external to the laser cavity. However, if losses are introduced in the cavity, it is understood that the mirror reflectivities are modified to keep the photon lifetime unchanged. The average rate Q of *total* photon generation (not the output power) must also remain the same. Intravalence-band absorptions do not qualify as linear losses, while free-carrier absorption does, approximately. In the following discussion only external losses are considered explicitly to simplify the wording.

Let $Q(t)$ be the photonic rate leaving the laser diode and $\mathscr{q}(t)$ the corresponding shot-noise fluctuations. Once all the equations have been solved, the fluctuation ΔP of the photon number P is expressed as a weighted sum of elementary noise sources. We have, therefore, in the most general situation,

$$Q_P\Delta P\equiv a(t)+B\mathscr{q}(t), \tag{33}$$

where $a(t)$ is a noise term independent of $\mathscr{q}(t)$ and B a generally complex constant.

According to the corpuscular or circuit theories presented previously, the total rate fluctuation is

$$\delta Q=Q_P\Delta P+\mathscr{q}(t) \tag{34}$$

and the spectral density of δQ is therefore

$$S_{\delta Q}=S_a+|1+B|^2Q \tag{35a}$$

or, using the definition in Eq. (20e),

$$N_{ri\,Q}\equiv S_{\delta Q/Q}-\frac{1}{Q}=\frac{S_a}{Q^2}+\frac{|1+B|^2}{Q}-\frac{1}{Q}. \tag{35b}$$

Consider now the situation in which the total flux $Q(t)$ is split into $L(t)+D(t)$, where L may represent the loss rate and D the detected rate. In particular, for the average rates, $\overline{Q}=\overline{L}+\overline{D}$. The corpuscular equations remain unchanged except that the right-hand side of Eq. (34) is now

$$L_P\Delta P+\ell(t)+D_P\Delta P+\mathscr{d}(t). \tag{36}$$

Thus, since for linear losses ($L_P=L/P$, $D_P=D/P$) the condition

$$L_P+D_P=Q_P \tag{37}$$

holds, $Q_P\Delta P$ is unaffected, and $\mathscr{q}(t)$ can be replaced by the equivalent term $\ell(t)+\mathscr{d}(t)$.

The detected rate fluctuation $\delta D(t)$ can be written

$$\delta D\equiv D_P\Delta P+\mathscr{d}$$
$$=\frac{D_P}{Q_P}(Q_P\Delta P)+\mathscr{d}=\frac{D}{Q}[a+B(\ell+\mathscr{d})]+\mathscr{d}. \tag{38a}$$

Because a, ℓ, and $\mathscr{d}$ are uncorrelated, the spectral density of δD is

$$S_{\delta D}=(D/Q)^2S_a+(D/Q)^2|B|^2L+|1+(D/Q)B|^2D \tag{38b}$$

and the relative intensity noise is, according to the above expression,

$$N_{ri\,D}\equiv S_{\delta D/D}-\frac{1}{D}=\frac{S_a}{Q^2}+\frac{|1+B|^2}{Q}-\frac{1}{Q}=N_{ri\,Q}. \tag{39}$$

The above result also holds when photonic noise is below shot noise, in which case a negative intensity-noise spectral density can be formally introduced. This, of course, reflects on the fact that "light intensity" is not a valid concept of the quantum level.

The relative intensity noise is measured by subtracting from the detected current spectral density the shot-noise contribution equal to the average rate D and dividing the result by D^2. When the attenuation is large, the intensity noise must be extracted from a much larger shot-noise level and in that case it is preferable to employ a beam splitter and two detectors. The shot-noise term disappears when the correlation between the two photocurrents is measured (Hanburry-Brown and Twiss experiment).

Using the previous equations, it can be shown that the cross spectral density between the electrical fluctuations and the *relative* detected noise $\delta D/D$ is independent of the attenuation. The detector is supposed to be perfectly matched optically. If some reflexion occurs, the reflected light should not be reflected to the laser diode. Room-temperature detectors do not radiate appreciably at optical and near-infrared wavelengths. But at wavelengths larger than approximately 4 μm, the thermal photonic flow propagating from the detector to the laser may importantly affect the laser operation. Similarly, an optical-amplifier spontaneous emission may flow back to the laser diode unless this is prevented by a large optical fiber attenuation or an optical isolator.

VI. LINEAR ACTIVE MEDIUM

When the active medium is linear (at a fixed carrier density), the electron-hole–to–photon conversion is proportional to the square of the optical-field strength or, equivalently, for a given laser structure, to the photon number. It turns out that for linear active media ($\gamma=0$) the modified rate equations give results that agree exactly for every measurable quantity with the corpuscular and quantum theories.

Let the corpuscular equations be written in the form usually given to rate equations, lumping together the noise sources relating to N and P:

$$j\Omega\Delta N = -\Delta S - \Delta R + F_n, \quad F_n \equiv \mathscr{j} - \mathscr{s} - \mathscr{r} \tag{40a}$$

$$j\Omega\Delta P = \Delta R - \Delta Q + F_p, \quad F_p = \mathscr{r} - \mathscr{q} \tag{40b}$$

$$\Delta S = S_N \Delta N, \quad \Delta R = R_N \Delta N + R_P \Delta P\ , \tag{40c}$$

$$\Delta Q = (Q/P)\Delta P, \quad \delta Q = \Delta Q + \mathscr{q}\ . \tag{40d}$$

From the above definitions, the nonzero spectral densities of the noise sources F_n, F_p, and $\mathscr{q}$ are (remembering that $\bar{J} - \bar{S} = \bar{R} = \bar{Q} \equiv Q$)

$$\begin{aligned} \mathcal{S}_n &= (\xi\zeta + \xi + \zeta + 2n_p - 1)Q\ ; \\ \mathcal{S}_p &= 2n_p Q\ , \\ \mathcal{S}_q &= Q\ , \\ \mathcal{S}_{np} &= (1 - 2n_p)Q\ , \\ \mathcal{S}_{pq} &= -Q\ . \end{aligned} \tag{40e}$$

MRE are based on the concept of a classical field intensity. Spontaneous emission is supposed to add a power n_p/τ_p to the oscillating field. They differ from the above equations only by the expressions of the spectral densities given in the second line of Eq. (40e). According to MRE,

$$\mathcal{S}_{np} = -2n_p Q, \quad \mathcal{S}_{pq} = 0 \quad \text{(MRE)}\ . \tag{41}$$

It is remarkable that in spite of these drastic differences MRE give an exact result in the absence of gain compression (see Appendix B of Ref. [13]). The proof is lengthy but straightforward. The standard rate equations given, e.g., in [7] are the same as the above MRE except that only the case in which $\xi = 1$ is considered. The observation made earlier concerning the concept of intensity noise applies here as well: MRE are applicable formally to sub-Poissonian statistics if the concept of negative intensity-noise spectral densities is accepted.

If MRE spectral densities in Eq. (41) were used for the case of gain compression: $\gamma > 0$, we would obtain Eq. (26) without the terms proportional to γ in the numerator. The error made in using MRE is therefore significant unless γ is very small compared with unity.

VII. MULTIPLE ACTIVE ELEMENTS

It is easy to generalize formally the corpuscular theory to any number of electron and photon reservoirs connected in arbitrary manner. One must ascertain, however, that the resulting expression for photonic noise is applicable to some specific situation. In this section we consider n active elements connected in parallel with a single cavity. We consider n electron reservoirs with electron numbers N_k, $k = 1, 2, \ldots, n$. It is assumed that they are driven by constant electronic rates J_k and that the carriers do not diffuse from one reservoir to another. This formalism (extended to the continuum) is applicable for example, to vertical-cavity surface-emitting diodes [25–27], if they are radially inhomogeneous, the diffusion length of the order of 1 μm being much smaller than the active area diameter. The parameter $p \equiv \Delta P/P$ is the same for all the elements.

Equation (28),

$$\frac{\delta R}{R} = \frac{(1-\gamma)\chi p + \chi r^0 + t^0}{\chi + 1}\ , \tag{42}$$

applies to each element with the subscripts $k = 1, \ldots, n$ omitted for brevity. Therefore, the fluctuation of the total photonic rate entering into the cavity, relative to the total generation rate Q, is

$$\sum_k \delta R_k / Q \equiv \sum_k \Gamma_k\, \delta R_k / R_k \equiv Ap + a\ , \tag{43a}$$

where $\Gamma_k \equiv R_k/Q$, and

$$A = \sum_k \Gamma_k \frac{(1-\gamma k)\chi k}{1 + \chi k}\ , \tag{43b}$$

$$z \equiv Q\mathcal{S}_a = \sum_k \Gamma_k \frac{(2n_{pk} - 1)|\chi k|^2 + \zeta_k}{|1 + \chi k|^2}\ . \tag{43c}$$

It remains to solve the relation

$$(1 + jf_n)p + q^0 = Ap + a\ , \tag{43d}$$

which is the same as for a single element, namely,

$$QN_{\mathrm{ri}} = \frac{z + 2A' - 1}{(1 - A')^2 + (f_n - A'')^2}\ , \tag{44}$$

where z is given in Eq. (43c) and $A \equiv A' + iA''$ in Eq. (43b).

When all the active elements have the same parameters (but, of course, not the same noise sources), the result in Eq. (44) is the same as for a single active element. We further verify that for a single active element and at zero frequency, Eq. (44) coincides with Eq. (26), with $\xi = 0$. After much simplification, we find that for a single active element, Eq. (44) also coincides with Eq. (32).

When the active elements are connected in series rather than in parallel, the result is quite different from the one given above unless the elements are identical. If diffusion occurs between the active elements, the algebra becomes complicated, but it remains true that for a large constant total injected current, the photonic rate does not fluctuate at low frequencies.

VIII. MULTIPLE MODES

Modern laser diodes oscillate on a single mode because the cavity is short or because frequency-selective Bragg reflectors have been introduced. Low-power side modes are excited, nevertheless, by spontaneous emission. Their total power is often 30 dB below the main-mode power. These low-power side modes can be treated in the linear approximation and are therefore exponentially distributed. The total power does not fluctuate much, but the main-mode power fluctuations may be large since the side-mode rates are subtracted from a constant injected rate.

In this section we consider the case where two modes have almost the same power but are sufficiently far apart frequency wise that the carrier number cannot follow the beat frequency, as is usually the case for adjacent longitudinal modes. The two-mode case occurs when a longitudinal mode is about to stop oscillating while another one is coming up (mode hopping). Photons at frequency ν_1 and photons at frequency ν_2 may in that case be considered as two different corpuscles. The two-mode situation considered may also correspond to two polarization states, TE and TM, which do not interfere even if the frequencies are similar.

From the point of view of the present corpuscular theory, the configuration with two identical modes is stable when gain compression is taken into account but photonic noise goes to infinity as $\gamma \to 0$. We find indeed from the previous equations,

$$Q_1 N_{\mathrm{ri}\,1} = Q_2 N_{\mathrm{ri}\,2} = \frac{n_p}{\gamma^2} \tag{45}$$

for large constant injected currents, $\gamma \ll 1$, and identical modal parameters. Of course, $\delta Q_1 + \delta Q_2 = 0$.

The linearization procedure leading to Eq. (45) would be invalid if photonic noise were too large. Equation (45) offers perhaps a way of measuring the gain compression factor γ.

IX. APPLICATION

The theory of photonic noise presented in this paper is applied to a GaAs vertical-cavity surface-emitting laser diode of the type proposed by Iga [25–27] at room temperature. Because of short cavity lengths and frequency-selective mirrors, such diodes exhibit high main–to–side-mode power ratios. (More than 50 dB have been measured [26]. In conventional diodes, side modes may render the present single-mode theory inapplicable.) Furthermore, the high mirror reflectivities ensure that the field fluctuations are essentially the same everywhere in the active material. Because of the high internal field, the gain-compression factor γ is likely to be higher than in conventional diodes for the same output power, particularly in the case of quantum wells.

Let us assume that the electrical current is injected with the help of a circular coating of 5 μm in diameter that conducts electricity but lets light go through. If confinement in the radial direction is ensured by chemical etching, it is reasonable to assume that the active area is radially homogeneous. This area contains about 20 cells of the size of the diffusion length, but we have seen that a multiple-active-element diode exhibits the same photonic noise as a single-element diode if it is homogeneous. Let the active layer thickness d be 0.5 μm and the mode volume 10 μm^3. The mirror spacing is supposed to be such that the laser operates at peak gain. The voltage applied to the intrinsic diode is the band-gap energy divided by the electron charge, $U \approx 1.42$ V. We assume that the rear mirror reflectivity is unity and that the front mirror reflectivity $R = 0.96$. With these values of d and R, the gain in the active medium is $g_p = 400\ \mathrm{cm}^{-1}$. According to Ref. [14], this gain value corresponds at low fields to a carrier density of $2 \times 10^{18}\ \mathrm{cm}^{-3}$ and a threshold current $I_{\mathrm{th}} = 1.1$ mA. Internal losses (mainly due to free-carrier absorption in the doped Bragg reflectors) are neglected.

To go further, we need to know the specific form of gain compression. Agrawal [8] has proposed that the active medium gain be of the form

$$g_p \propto \frac{N - N_0}{\sqrt{1 + I_d/I_s}}\ . \tag{46}$$

The ratio of threshold to transparency carrier numbers is $N_{\mathrm{th}}/N_0 = 1.66$. $I_d = eQ$ is the detected current (for an ideal detector and no optical losses) and I_s the corresponding saturation value. Somewhat arbitrarily, we take $I_s = 7$ mA. According to this expression, the relative gain compression γ is

$$\gamma = \tfrac{1}{2}\frac{1}{1 + I_s/I_d}\ . \tag{47}$$

We further assume that the spontaneous-recombination current is proportional to the square of N,

$$\frac{I_e - I_d}{I_{\mathrm{th}}} = \left[\frac{N}{N_{\mathrm{th}}}\right]^2, \tag{48}$$

where $I_e = eJ$ denotes the injected current. Then we have $s = 2$. Reference [14] shows that this "bimolecular" approximation is not very accurate, but it will suffice for our purposes. From Eqs. (47) and (48) we easily obtain the carrier density for various ratios of injected to threshold currents. The values of the differential gain factor g and the population inversion factor n_p [14] are reproduced in Table I, together with the detected current I_d and the ratio ζ of spontaneous to stimulated emission.

Next we evaluate from Eq. (26) the photonic noise relative to shot noise at low frequencies for injected-current fluctuations at the shot-noise level ($\xi = 1$):

$$I_d^{-1}\mathcal{S}_{\delta I_d} = 1 + 2\frac{\zeta - \gamma\chi + (n_p - \gamma)\chi^2}{(1 + \gamma\chi)^2}\ . \tag{49}$$

The χ parameter is, according to Eq. (17) and with $s = 2$,

TABLE I. Photonic noise of a GaAs room-temperature laser diode. This table gives for various ratios of injected to threshold currents (I_e/I_{th}) the photonic noise relative to the shot-noise level as given in Eq. (26) of this paper, and from modified rate equations (MRE). R_s is the electrical driver resistance.

I_e/I_{th}	1	5	15	75
N/N_{th}	1	1.1	1.3	2
ζ	∞	0.33	0.13	0.055
g	2.5	2.2	1.85	1.4
n_p	1.7	1.5	1.2	1
I_d (mA)	0	4	14	78
χ		0.3	0.2	0.4
$I_d^{-1}\mathcal{S}_{\delta I_d}$ [Eq. (26)]		1.7	1.16	0.94
$I_d^{-1}\mathcal{S}_{\delta I_d}$ MRE		1.75	1.3	1.3
R_s (Ω)		∞	48	2.1

$$\chi = \frac{2\xi}{g} + \frac{NU_N}{gI_d R_s}, \tag{50}$$

where $NU_N \approx 0.075$ V and the electrical resistance R_s includes the laser-diode series resistance. For each value of the injected current, R_s is adjusted to minimize the noise. The optimum R_s is given in Table I, as well as the noise value. The noise calculated with the modified rate equation is also shown.

Table I shows that for sufficiently high injected currents, sub-Poissonian photon statistics can be obtained, even for injected-current fluctuations at the shot-noise level. Because γ according to Eq. (47) never exceeds 0.5, photonic noise, admittedly, cannot be reduced much below the shot-noise level (factor 0.94). More favorable conditions can perhaps be found at lower temperatures. In any event, gain compression should be considered in precise evaluations of photonic noise.

X. CONCLUSION

The general theory of laser-diode noise can be based on the Nyquist formula only (circuit theory). The expressions obtained appear to coincide with those obtained from quantum theory in all cases. When the gain and losses are frequency independent, quantum or circuit theories can be expressed in the form of a corpuscular theory, electron or photon numbers (N and P) being treated as continuous classical variables. The rate of electron-hole–to–photon conversion (or the converse) consists of a classical function of N and P, plus a shot-noise fluctuation. The shot-noise fluctuations, which are equivalent to the Nyquist currents of the circuit theory, are independent of each other. Previous simple corpuscular theories postulate that the classical flow from the optical cavity to the detector is uncorrelated with the shot-noise term, from which it follows that the detected fluctuation is at best at the shot-noise level.

The corpuscular theory has been applied to a simple laser model with gain compression (explicit dependence of the gain on the optical field, or photon number). We have shown that sub-Poissonian photon statistics can be obtained even if the injected-current fluctuation is at the shot-noise level. Without gain compression, the results obtained coincide with those obtained from modified rate equations. The actual mechanism behind gain compression remains a matter of investigation. One should distinguish "effective gain compression" that may express transverse diffusion or induced longitudinal gratings and have inherent time constants of the order of 1 ns, and the fundamental gain-compression mechanism that has time constants of the order of 0.2 ps [28]. The theory presented in this paper is of practical interest when γ is of the order of 0.1 and above.

Frequency-dependent losses are encountered, in particular, in gain-guided and external cavity lasers. In that case MRE may be in error by large factors. The theory of photonic noise in the case of frequency-dependent losses has been given by Lax in 1967 [10], but is restricted to the case of pure compression. Further discussion concerning the effect of frequency-dependent loss or gain, including gain compression, will be presented elsewhere.

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