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Circuit Theory of Amplitude Noise for a Laser Diode with Electronic Feedback.

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Abstract. – The amplitude noise of a laser diode submitted to electronic feedback is evaluated using a new circuit theory. It is postulated that the electron-hole injection rate equals the photon generation rate at any time, and Nyquist-like noise sources are introduced. Previous results based on quantum mechanics are recovered. It is found that the amplitude fluctuations of an optical beam in the coherent state can be squeezed below shot noise by feeding back the driving current of a laser amplifier.

Machida and Yamamoto [1] have proposed and verified that the fluctuations of the current injected in a diode can be reduced below shot noise by use of a large resistance in series with the driving circuit. If the injected current does not fluctuate, the total optical power does not fluctuate either under ideal conditions. Fluctuations are observed, however, if only a fraction of the output power is collected. This can be understood by assuming that each photon behaves as a classical particle [2].

When the laser diode output power does fluctuate, one could hope that the fluctuations of the photons originating from one side of the laser (say, side 1) would be reduced if the optical power from the other side is detected, and the amplified detected electrical current is fed back into the diode (fig. 1*a*)). Theory and experiments, however, show that only fluctuations of classical origin can be reduced by this arrangement.

The electronic feedback problem is investigated here with the help of a circuit laser-diode theory [3] conceptually different from the semiclassical theories offered in [4, 5] that postulate initially a Poisson distribution for the laser photons. We postulate instead that (classical) Gaussian noise voltages of spectral density $|R|$ (where the bars mean: «absolute value», and appropriate units are used) are associated with (positive or negative) electrical resistances R, according to the fluctuation-dissipation theorem for quantum noise. This approach has been used before by Reynaud and others [6] for parametric oscillators and by Nilsson and others [7] for laser diodes. The particle rate equations, in which we are careful to include the noise sources, however, were not explicitly accounted for in [7]. The formulae in [4, 5] as well as quantum-mechanical results are recovered. The theory presented here is

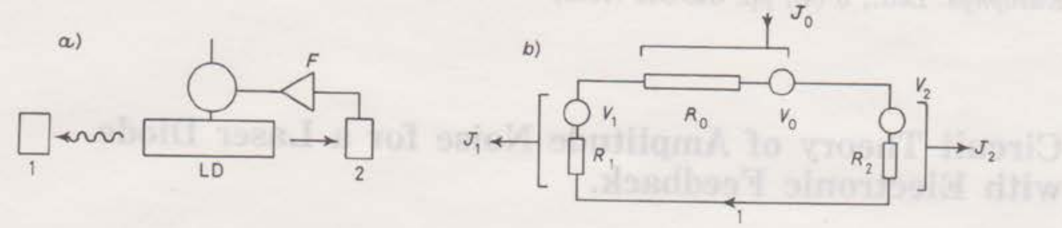

Fig. 1. – *a*) Laser diode with two outputs 1 and 2 and two absorbers of radiation (taken as ideal detectors). The current from detector 2 may be amplified and added to the laser driving current through the switch *S*. *b*) Equivalent electrical circuit. The voltages $V_k(t)$, $k = 0, 1, 2$ are white Gaussian random processes.

simpler than previous theories. Its principle will be first explained in the absence of feedback and at zero base-band (or «Fourier») frequency.

For the sake of simplicity, rather ideal laser diodes are considered that do not suffer from internal losses, operate much above threshold, have a quantum efficiency equal to unity and complete population inversion. We also assume that the diode can be represented by a single negative resistance and a reactance, that is its internal structure is ignored.

The circuit in fig. 1*b*) consists of a negative resistance $R_0 \equiv -R$ modelling the active part of the laser in series with two positive resistances R_1 and R_2 modelling perfect absorbers of radiation located on both sides of the laser. These absorbers are taken to be ideal detectors whose output currents reflect the power fluctuations of the corresponding output optical beams. An *L-C* circuit in series (not shown in the figure) resonates at the optical frequency ν_0 given by: $LC(2\pi\nu_0)^2 = 1$. It is not difficult to relate the circuit parameters L, C, R_1, R_2 to the diode length and mirror reflectivities.

Gaussian voltages whose spectra can be considered white over a small range of optical frequencies are inserted in series with the resistors according to the fluctuation-dissipation theorem [8]. For the narrow-band processes considered here, it is convenient to introduce complex random voltages (see, for example, [9]) $V_k(t) \equiv c_k(t) + is_k(t)$, $k = 0, 1, 2$. The double-sided spectral densities of c_k and s_k (expressed in root-mean-square values) are

$$(S_{cc})_k = (S_{ss})_k = h\nu_0 |R_k| \tag{1}$$

and the cross correlations vanish. The complex random voltages V_0, V_1, V_2 relating to R_0, R_1, R_2, respectively, are shown in the figure.

Next, let us equate particle generation rates

$$J_k/e = R_k |I|^2/h\nu_0; \qquad k = 0, 1, 2, \tag{2}$$

where the J_k denote outgoing electronic currents, the detected output currents J_1 and J_2 being positive, while the current J_0 injected into the active element is negative. «*e*» denotes the absolute value of the electronic charge and $h\nu_0$ the photon energy. «*I*» denotes the r.m.s. current flowing in the circuit.

Consider now small and slow changes j_k of J_k and r_k of R_k, and let ρ denote the relative change of $|I|^2$. The variation of eq. (2) gives

$$j_k = r_k + \rho R_k; \qquad k = 0, 1, 2, \tag{3}$$

where we have set for brevity, without loss of generality: $e = h\nu_0 = 1$, and the average value of I is also set equal to unity. For the passive resistances R_1, R_2 the variations r_1, r_2 reduce to the real parts c_1, c_2 of the random voltages. For the variation r_0 of the active resistance R_0 there is in addition to c_0 a variation due to a change in carrier number which, however, we need not calculate explicitly.

The R_k values now stand either for the average resistances or the average rates of carrier or photon generation. They satisfy the relation

$$R_0 + R_1 + R_2 = 0 \quad \text{or} \quad R \equiv -R_0 = R_1 + R_2 . \tag{4}$$

Slow changes of the oscillator frequency must be real. Otherwise the amplitude fluctuations would be unbounded. This entails that the total circuit resistance vanishes at any instant of time

$$r_0 + r_1 + r_2 = 0 \Rightarrow r_0 + c_1 + c_2 = 0 . \tag{5}$$

If r_0 from eq. (5) is introduced into eq. (3) with $k = 0$, we obtain

$$\rho = -(j_0 + c_1 + c_2)/R \tag{6}$$

which can be substituted into eq. (3) with $k = 1, 2$ to obtain the fluctuations j_1, j_2 of the currents flowing out of detectors 1, 2

$$j_1 = c_1 - (R_1/R)(j_0 + c_1 + c_2), \tag{7a}$$

$$j_2 = c_2 - (R_2/R)(j_0 + c_1 + c_2). \tag{7b}$$

Notice that $j_0 + j_1 + j_2 = 0$, as initially postulated. The c_1 and c_2 processes are uncorrelated and their spectral densities given in eq. (1) are, respectively, R_1 and R_2. In the absence of injected current fluctuations ($j_0 = 0$) and feedback the spectral density S_1 of j_1 is therefore

$$S_1 = (1 - R_1/R)^2 R_1 + (R_1/R)^2 R_2 = R_1(1 - R_1/R) \tag{8}$$

and $S_2 = S_1$. Equation (8) shows that $S_1 = 0$ only when detector 1 collects all the power ($R_1 = R$). If j_0 exhibits full shot noise, one must add according to eq. (7) a term $(R_1/R)^2 R$ on the right-hand side of eq. (8), and then $S_1 = R_1$; $S_2 = R_2$ (full shot noise).

The spectral densities calculated above apply only to vanishingly small base-band frequencies f. Let us assume for simplicity that the injected current is large so that the relaxation oscillation frequency is much larger than the cold cavity cut-off frequency: $f_0 = R/2\pi L$, where L denotes the circuit inductance. Then the term dN/dt in the carrier rate equation can be neglected, where N is the carrier number, and the photon generation rate remains equal to the carrier injection rate. However, the photon absorption rate is no longer equal to the photon generation rate because of the electromagnetic storage in the cavity modelled by the *L-C* circuit. We maintain that the total circuit impedance $Z(\nu)$, and therefore the real part of its first-order variation, vanishes at any instant of time. The optical frequency ν is here a complex function of time whose imaginary part, $\nu_i \ll \nu_0$ is related to ρ by: $4\pi\nu_i = d\rho/dt$. When $j_0 = 0$, the spectral density S_1 of j_1 at the baseband frequency f is found to be (see the Appendix; a related calculation was presented in [10])

$$S_1/R_1 = 1 - (R_1/R)/[1 + (f/f_0)^2]. \tag{9}$$

It reduces to the value given in eq. (8) when $f \ll f_0$, and to full shot-noise ($S_1 = R_1$) when $f \gg f_0$.

Most authors consider a power-division system that seems at first to be quite different from the one shown in fig. 1*a*): the laser diode has only one output port, but the output beam is split by a beam splitter. This beam splitter exhibits an unused port (labelled by 3) that does not appear in fig. 1*a*). The transmission-line analog of a beam splitter is a directional coupler. We find it convenient to set the characteristic impedance equal to unity. Thus $R = 1$. The principles outlined above show that the fluctuations of the current from the two detectors are

$$j_1 = -[R_1(1-R_1)]^{1/2} s_3 - R_1 j_0 , \tag{10a}$$

$$j_2 = [R_1(1-R_1)]^{1/2} s_3 - (1-R_1) j_0 , \tag{10b}$$

where R_1 and $1-R_1$ denote the beam splitter power reflection and transmission, respectively. Because the spectral density of the imaginary component s_3 of the noise voltage from the unused port 3 is unity (with the new convention) as well as those of the uncorrelated processes c_1 and c_2, eq. (10) gives the same observable fluctuations as eq. (7). This conclusion would not hold, however, if a squeezed vacuum were introduced at the unused port. The configuration presently discussed is the balanced homodyne optical receiver represented for example in fig. 3.8 of [11], when j_1 is subtracted from j_2 and $R_1 = R_2$.

If j_0 exhibits full shot noise, that is, if the laser generates a coherent state, j_1 and j_2 also exhibit full shot noise and are uncorrelated, a conclusion that holds as well for the configuration represented in fig. 1*a*).

Let now the current from detector 2 at baseband frequency $f \ll f_0$ be multiplied by the complex number $F(f)$ and added to the laser driving current exhibiting full shot noise ($S_0 = R$). After some algebra, the previous equations lead to

$$S_1/R_1 = 1 + (R/R_2 - 1)|1 - R/FR_2|^{-2} . \tag{11}$$

Equation (11) shows that the spectral density of the usable photons exceeds that of shot noise for any value of the complex number F. Except for a change of notation, eq. (11) is the same as eq. (90) of [4] with $\eta = 1$.

We have found, however, that when R_2 is negative and acts as a (linear, phase-insensitive) optical amplifier of gain G a similar feedback arrangement can lead to an amplitude squeezing of light initially in the coherent state. This can easily be understood intuitively for large gains: the current injected into the optical amplifier is the difference between the output optical power and the input optical power, but the latter is negligible for large gains. To prevent the output power from fluctuating it therefore suffices to suppress the injected current fluctuations with the help of a large cooled series resistance in the driving circuit following the concepts in ref. [1], or, equivalently, by introducing an optimum feedback. We find that for a gain G the optimum feedback factor is $(1 - 1/2G)^{-1}$. The power fluctuations of the outgoing beam are then those of shot noise multiplied by the factor $1/(2G-1) < 1$, that is they are squeezed.

APPENDIX

Let us first establish that the real part of the first-order variation of the impedance $Z(\nu)$ of the circuit represented in fig. 1*b*) (the reactive part is not shown) is

$$c_1 + c_2 + r_0 + 4\pi L \nu_i = 0 , \tag{A.1}$$

where $r_0 = \delta R_0$ includes both a variation due to a carrier number change and the real part of the noise source V_0 (divided by $I = 1$). A complex frequency variation $\delta\nu(t) \equiv \delta\nu_r(t) + i\nu_i(t) \ll \nu_0$ perturbs the impedance Z by the amount

$$\delta Z = \delta(R + iX) = (\mathrm{d}R/\mathrm{d}\nu + i(\mathrm{d}X/\mathrm{d}\nu))(\delta\nu_r + i\nu_i) , \tag{A.2}$$

to first order. Because $\mathrm{d}R/\mathrm{d}\nu = 0$,

$$\mathrm{Re}[\delta Z] = -(\mathrm{d}X/\mathrm{d}\nu)\, \nu_i . \tag{A.3}$$

For the LC circuit considered, $\mathrm{d}X/\mathrm{d}\nu = -4\pi L$ and we thus obtain eq. (A.1).

From the definition of the complex frequency (applicable to time variations that are small within an optical period)

$$I(t) = I(0) \exp\left(-2\pi i \int \delta\nu(t)\, \mathrm{d}t\right). \tag{A.4}$$

Therefore $4\pi\nu_i = \mathrm{d}\rho/\mathrm{d}t$, where $\rho \equiv \delta|I|^2/|I|^2$.

By Fourier-transforming the slow fluctuations, $\mathrm{d}\rho/\mathrm{d}t = -2\pi i f \rho$. The relation in eq. (A.1) becomes

$$c_1 + c_2 + r_0 = 2\pi i L f \rho . \tag{A.5}$$

The derivation now follows as for the case $f = 0$, since eq. (3) is unchanged. Namely

$$j_0 + c_1 + c_2 = (-R + 2\pi i L f)\rho . \tag{A.6}$$

When ρ from this equation is substituted into

$$j_1 = c_1 + \rho R_1 , \tag{A.7}$$

we obtain

$$j_1 = c_1 - (R_1/R)(j_0 + c_1 + c_2)/(1 - if/f_0) , \tag{A.8}$$

where $f_0 = R/2\pi L$. The spectral density of j_1 given in eq. (9) of the paper follows readily from the above eq. (A.8).

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