USE OF PRINCIPAL MODE NUMBERS IN THE THEORY OF MICROBENDING

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The modal theories of microbending that have been proposed so far postulate that modes that have the same principal mode number carry the same optical power. This assumption is shown to be incorrect. The exact result is given.

The precise calculation of the effect of random bends on the transmission characteristic of multimode optical fibres remains of great practical interest. Following a proposal made by Gloge,[1] the modal theories of microbending available today postulate that modes that have the same principal mode number carry the same optical power. The principal mode number is defined as twice the radial mode number plus the azimuthal mode number. (See, for example, References 2 and 3.) The purpose of this letter is to discuss the validity of this assumption.

It is asserted[3] that modes that have the same principal mode number have nearly equal propagation constants, and that therefore they are tightly coupled to each other and behave as one mode group, labelled m. We will argue that modes that have the same principal mode number do not have the same propagation constant, even approximately, with the sole exception of fibres with a square-law profile, and secondly that even if the modes were degenerate they would not couple to each other. Finally, we will give the exact result.

Let us first consider the assertion that modes that have the same principal mode number are nearly degenerate, restricting our attention, as most authors do, to fibres that have a power-law profile

$$1 - n(r)/n(0) = \Delta(r/r_c)^{2\kappa} \tag{1}$$

where the parameter κ is equal to unity for square-law profiles and to infinity for step-index fibres. The other quantities have their usual meaning. It is well known that, within the WKB approximation, the spacing in propagation constant between two adjacent modes that have the same azimuthal mode number is equal to $2\pi/Z$, where Z denotes the axial ray period, and the spacing in propagation constant between two adjacent modes that have the same radial mode number is equal to Φ/Z, where Φ denotes the azimuthal ray period (see Reference 5, eqn. 4-298). Thus degeneracy occurs only if $\Phi = \pi$. But consider, for example, a step-index fibre. A ray trajectory, projected on a plane perpendicular to the fibre axis, obeys the usual law of reflection at the circular core-cladding interface, and clearly the azimuthal ray period can take any value from 0 to π. The latter value is applicable only to the exceptional case of meridional rays. It is only for square-law profiles that $\Phi = \pi$ for all rays.

Let us see now whether, at least for square-law profiles, the concept of mode group can be used. Degenerate modes would carry the same optical power if they were tightly coupled to each other as a result of the random bending of the fibre axis. But in fact they are not coupled at all because of the existence of selection rules. The curvature C of the fibre axis in the x-z plane, where z denotes a co-ordinate along the fibre axis, is equivalent to a perturbation of the refractive index[5]

$$\Delta n = Cx = Cr\cos\phi = Cr(e^{i\phi} + e^{-i\phi})/2 \tag{2}$$

in an r, ϕ, z cylindrical co-ordinate system. It follows from eqn. 2 that two modes are coupled by the bends only if their azimuthal mode numbers differ by plus or minus one. But this is never the case for two degenerate modes. Indeed, let μ, α denote the aximuthal and radial mode numbers of one mode, and μ', α' denote the azimuthal and radial mode numbers of the other mode. If these two modes are degenerate

$$2\alpha + \mu = 2\alpha' + \mu' \tag{3}$$

and therefore

$$\mu - \mu' = 2(\alpha' - \alpha) \tag{4}$$

is never equal to plus or minus one.

Degenerate modes may be coupled by fibre distortions other than bending. However, even for such distortions, the tight-coupling approximation requires that the power spectral density of the curvature process at zero spatial frequency be infinite, or at least very large. We are unaware of plausible defomation mechanisms that would satisfy this condition. We thus conclude that the concept of degenerate mode groups in the theory of microbending should not be used.

The exact result is exceedingly simple.[6] From either ray theory or by applying the WKB approximation to the coupled mode equations, we have found that the power Q in mode μ, α at z obeys the following partial differential equation:

$$\frac{\partial Q}{\partial z} = \frac{\partial}{\partial \mu} d_0 \frac{\partial Q}{\partial \mu} + \frac{\partial}{\partial \alpha} d_1 \frac{\partial Q}{\partial \mu} + \frac{\partial}{\partial \mu} d_1 \frac{\partial Q}{\partial \alpha} + \frac{\partial}{\partial \alpha} d_2 \frac{\partial Q}{\partial \alpha} \tag{5}$$

where d_i, $i = 0, 1, 2$ are functions of μ and α. If the fibre distortion is due to random bending, the d_i are given by the following expression:

$$d_i = \tfrac{1}{2}k^2 \sum_s s^i F_s G_s \tag{6}$$

where k denotes the propagation constant on axis: $k = (\omega/c)\,n(0)$. The summation is over all integral values of $s: 0, \pm 1, \pm 2, \ldots$, and G_s denotes the spectral power density of the curvature process at the spatial frequency

$$u \equiv (s + \Phi/2\pi)/Z \tag{7}$$

The curvature process is assumed to be stationary and to have the same statistical properties in every meridional plane. The F_s are Fourier coefficients defined with respect to a ray trajectory in the straight fibre, $r(z)$, $\phi(z)$:

$$F_s \equiv \left| \int_0^Z \rho(z) \exp(-2\pi i u z)\, dz/Z \right|^2, \qquad \rho \equiv re^{i\phi} \tag{8}$$

and u is defined in eqn. 7. Both F_s and G_s are functions of μ, α and the integer s. The above results are valid for any curvature spectra and any index profile $n(r)$.

For the special case of square-law profiles, the F_s are all equal to zero, except

$$F_{-1} = (Z/k\pi)\alpha; F_0 = (Z/k\pi)(\mu + \alpha) \tag{9}$$

In the present case, the axial and azimuthal ray periods Z, Φ are constants, equal, respectively, to $\pi r_c/(2\Delta)^{1/2}$ and π. It follows that, in the expression in eqn. 6 for the diffusion coefficients, only the value of the spectral power density G at the spatial frequency $1/2Z$ enters. We shall call this value γ. This result confirms our previous argument based on the selection rule, that the value of the curvature spectral density at zero spatial frequency, whether large or small, is irrelevant.

To make our argument more precise, it is convenient to write the diffusion equation eqn. 5 with the d_i as given above for the square-law profiles, as an equation for $P(m, \mu, z)$, where $m = 2\alpha + \mu$ is the principal mode number. The result of the transformation is

$$(2\sqrt{(2\Delta)}/\gamma k r_c) \frac{\partial P}{\partial z} = mP_{mm} + 2P_m + 2\mu P_{m\mu} + mP_{\mu\mu} \tag{10}$$

where subscripts denote partial differentiation with respect to m or μ. Statistical modes are obtained by setting the derivative of P with respect to z equal to $-\lambda P$, where λ represents the microbending loss, with the boundary condition

$$P(\sqrt{(2\Delta)}kr_c/2, \mu) = 0.$$

Some of the statistical modes are in fact independent of μ; this is the case, in particular, for the fundamental solution. But most of the statistical modes do depend on μ. The exact form will be given elsewhere. In other words, the statistical modes in bent square-law fibres do not satisfy the condition postulated in most recent works that degenerate modes carry equal power. The fact that this condition holds for some of the statistical modes is coincidental.

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