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Technique for Fast Measurement of Gaussian Laser Beam Parameters

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It is well known that the fundamental (Gaussian) mode laser beam intensity satisfies the equation[1]:

$$I(r,z) = [w_0/w(z)]^2 I_0 \exp[-2r^2/w^2(z)], \tag{1}$$

where

$$w(z) \equiv [(b^2 + 4z^2)/(bk)]^{\frac{1}{2}}, \tag{2}$$

$k \equiv 2\pi/\lambda$, $b \equiv kw_0^2$, $w_0 \equiv w(0)$, and $z = 0$ is taken in the plane of the beam waist. Note that b and k are the usual confocal parameter and propagation constant; respectively, as defined by Kogelnik and Li.[1]

The purpose of this paper is to describe a device that can quickly and accurately measure the value of $w(z)$ at any point along the beam and to point out how it can be used to determine the confocal parameter b of the beam with one simple measurement. A modification of the device that enables one to determine the beam waist location is then described. The Gaussian beam is, of course, completely specified by the value of b and the location of the beam waist since the diameter and phase-front radius can be calculated from this for any value of z.[1]

The value of w can be obtained in several ways. The most straightforward is to scan the beam with a pinhole (or slit), but this method is slow and tedious. A faster and simpler method is to chop the beam periodically with a straight edge traveling at a known velocity u in the plane in which one wishes to know the value of w. For a Gaussian beam, the detected power of the chopped beam is given (on the interval during which the chopper is eclipsing the beam) by

$$p(t) = p_0 \int_{-\infty}^{\infty} \int_{ut}^{\infty} \exp\left[-\frac{(ut')^2 + y^2}{w^2}\right] dt'dy. \tag{3}$$

The variation of p with time is illustrated in Fig. 1. Performing the indicated integration over y, differentiating Eq. (3) with respect to t, and solving for w gives

$$w = -(2/\pi)u\{[p(t)]_{\max}/(dp/dt)_{\max}\}. \tag{4}$$

Consider the circuit shown schematically in Fig. 2. Let $v_0(t)$ be the (amplified) voltage from the photodetector and let $v_1(t)$ be the output of a simple RC differentiating circuit when $v_0(t)$ is its input. Then since $v_0(t) \propto p(t)$, Eq. (4) becomes

$$w = (2/\pi)^{\frac{1}{2}} uRC\{[v_0(t)]_{\max}/[v_1(t)]_{\max}\}, \tag{5}$$

where R and C are the resistance and capacitance of the differentiating circuit. The value of w can be quickly determined from the circuit in Fig. 2. We have found it most convenient to choose a convenient ratio for $[v_0(t)]_{\max}/[v_1(t)]_{\max}$ and to set the gain in the two oscilloscope channels such that the two maximum values have equal height when this ratio is achieved as illustrated in Fig. 2. Then R and C are adjusted until the desired ratio is achieved, and w is calculated from Eq. (5).

The measurement of b can be reduced to a single measurement of w by first passing the Gaussian beam through a lens. Equation (57) of Ref. 2 reveals the at first somewhat surprising result

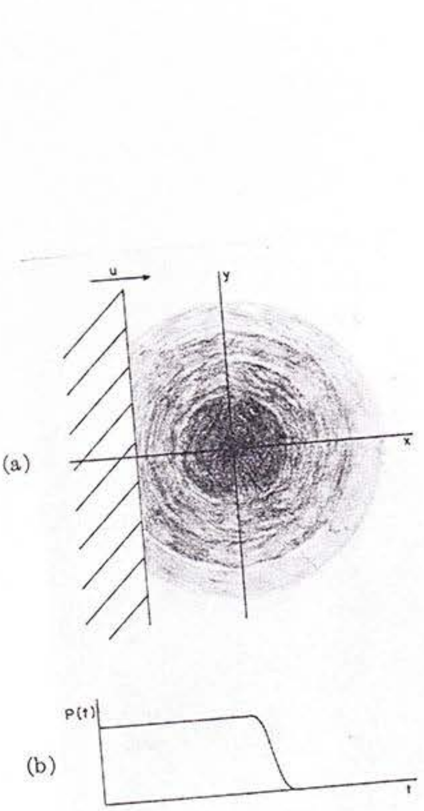

Fig. 1. (a) Relative orientation of chopper blade and Gaussian light beam. (b) Optical power of chopper beam as a function of time.

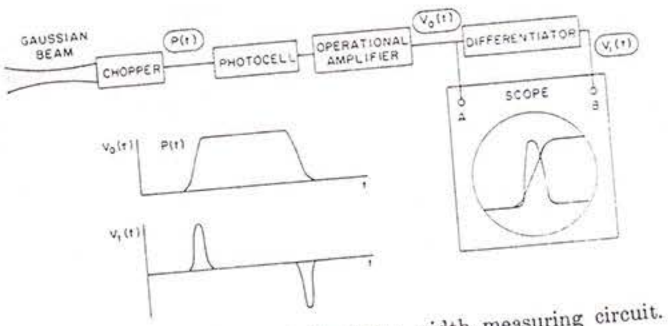

Fig. 2. Block diagram of the beam width measuring circuit.

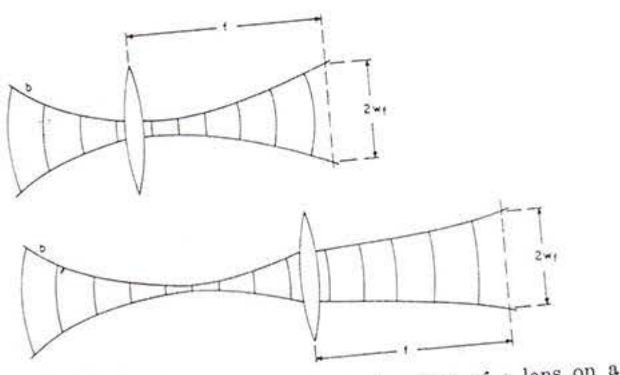

Fig. 3. Schematic representation of the effect of a lens on a Gaussian light beam.

that if a beam of confocal parameter b is passed through a lens of focal length f, the beam radius w_f measured in the focal plane of the lens is independent of the location of the lens relative to the beam waist and is given by the simple expression

$$w_f = 2f/(kb)^{\frac{1}{2}}. \qquad (6)$$

This can be readily understood if we recall that the field at the focal plane of a lens is proportional to the far field pattern of the incident beam, which depends on the beam waist radius but obviously not on its location. Therefore, a single measurement of w in the focal plane of the lens gives the value of b directly. This is illustrated in Fig. 3.

The beam waist location can, in principle, be determined from a measurement of b as described in the preceding section and a measurement of w at an arbitrary point on the beam by using Eq. (2). In practice, this method often turns out to be rather inaccurate. An alternative method involves chopping the beam alternately at two different places. The differentiated output consists in general of alternate pulses of unequal height. By sliding the chopper along the beam until the heights of the pulses are equal, the beam waist is located at the midpoint between the two chopping planes.

It is convenient to construct the device using a chopper with a photocell mounted behind it and provision to mount lenses at a distance equal to their focal length in front of the blade. The technique has been used for values of w from 5 μm to more than 1 mm with an accuracy of 2%. This range in w allows one to measure b over a range from 1.0 mm to over 100 m (using a 5-cm focal length lens for b less than about 1 m and a 25-cm lens for larger b) with an over-all accuracy of better than 10%.

The accuracy of the measurement of w was determined by comparison with measurements made with a scanning pinhole. The accuracy of the confocal parameter measurement was determined by estimating the error in the values of f, w, and the position of the chopper relative to the focal plane of the lens individually. This estimate was verified by measuring the value of b for the output of several lasers with known cavity dimensions.

It can be shown that the uncertainty (Δz) in the beam waist location using the two-chopper technique is given by $\Delta z = \epsilon b^2/4S$, where ϵ is the uncertainty in the ratio of the two *equal* beam radii and S is the separation between the two chopping points. Consider both Δz and S as a fraction of the confocal parameter in question. Set $\zeta = \Delta z/b$ and $\sigma = S/b$; then $\zeta = \epsilon/4\sigma$. Since ϵ can be kept below about 1%, the beam waist can be located quite accurately provided the separation between the chopping planes is not too small relative to the confocal parameter of the beam.

The beam measuring technique that we have described could be used in conjunction with a laser to measure the focusing properties of lenses or mirrors with long focal lengths, such as those employed in optical waveguides and gas lasers. This technique, which would consist of measuring the beam parameters before and after the lens under test, would be faster than conventional interferometry and probably as accurate if the laser beam were to possess a truly Gaussian intensity distribution. Measurements made with pinholes indicate that this condition is adequately fulfilled with many lasers. Nevertheless, a significant improvement in accuracy of measurement of lenses and mirrors may require the development of lasers with apodized internal apertures, capable of delivering almost ideal Gaussian beams.

This Letter was the subject of a talk given at Philadelphia, 10 April 1970 (OSA Spring Meeting).

References

1. H. Kogelnik and T. Li, Appl. Opt. 5, 1550 (1966)
2. H. Kogelnik, Bell. Syst. Tech. J. **44**, 455 (1965).