Conclusion: To improve the efficiency of the dielectric slab-line leaky-wave antenna at millimetre-wave frequencies the radiation properties of the line discontinuities have to be investigated. If the influence of shape and dimension of the discontinuities on the radiated power is clear, a desired tapering of the aperture distribution can be realised while reflection and absorber losses are minimised.

ROLE OF PETERMANN'S *K*-FACTOR IN SEMICONDUCTOR LASER OSCILLATORS

Indexing terms: Optics, Semiconductor lasers

The theory presented indicates that when the active region of a semiconductor laser is very thin the linewidth enhancement factor differs significantly from Henry's $1 + \alpha^2$ factor and can be reduced by a factor of 4. Petermann's *K*-factor may also play a role in the plane perpendicular to the junction.

The linewidth of semiconductor lasers has been found experimentally to be about 30 times the value predicted by the classical formula. This was explained by Henry,[1] who pointed out that intensity fluctuations due to spontaneous emission are essentially suppressed by changes in the number of electrons in the laser. These involve random changes of the phase of the optical field which cause an enhancement of the laser linewidth by a factor $1 + \alpha^2$, where $\alpha = \Delta n_r/\Delta n_i$ is the ratio of small changes in the real and imaginary parts of the active medium refractive index. We point out in this letter that the relevant factor $\alpha_A = \Delta\omega_r/\Delta\omega_i$, where $\omega = \omega_r + i\omega_i$ is the complex cavity frequency, differs significantly from α in the case of a thin active region.

Petermann, on the other hand,[2] found that when the wavefronts of guided modes are significantly curved as a result of gain inhomogeneities, spontaneous emission in the guided mode and the laser linewidth are very much enhanced, by a factor K. While Petermann himself was only concerned with gain-guided lasers, we noted[3] that the same effect could be found in the plane perpendicular to the junction when the active region is very thin.

In the present letter we are considering a laser which has a thin active region ($< 0{\cdot}1\ \mu$m) and exhibits both effects.

The dispersion relation of a dielectric slab of thickness $2d$ and wavenumber $k = (\omega/c)n$ in a medium of wavenumber $k_c = (\omega/c)n_c$ is given approximately by the equation

$$(k^2 - k_c^2)^2 d^2 + k_c^2 = \beta^2 \tag{1}$$

when d is small.[4] Material dispersion is neglected.

In a laser resonator the propagation constant is a fixed real number $\beta = \pi l/L$, where l is the longitudinal mode number and L the laser length.

Setting

$$\Delta n = \Delta n_r + i\,\Delta n_i \qquad \Delta\omega = \Delta\omega_r + i\,\Delta\omega_i \tag{2}$$

we find from eqns. 1 and 2

$$\alpha_A = \Delta\omega_r/\Delta\omega_i \simeq (\alpha + h)/(1 - \alpha h) \tag{3}$$

where $\alpha = \Delta n_r/\Delta n_i$ is the parameter introduced by Henry[1] and

$$h = b/a \qquad (k^2 - k_c^2)d \equiv a - ib \tag{4}$$

In the small d-limit the slab can be viewed as a reactive surface of susceptance $a - ib$, where $a > 0$ expresses normal guidance and $b > 0$ expresses the medium gain.

The line enhancement factor is not in general $1 + \alpha^2$ but $1 + \alpha_A^2$, where α_A in eqn. 3 reduces to α only when $b \ll a$.

This $1 + \alpha_A^2$ factor must be multiplied by Petermann's K factor which, for our model, is $1 + h^2$. Thus the linewidth enhancement factor F, after rearranging, is

$$F = K(1 + \alpha_A^2) = (1 + \alpha^2)[(1 + h^2)/(1 - \alpha h)]^2 \tag{5}$$

For small h-values, the laser linewidth increases according to the law

$$F/(1 + \alpha^2) = 1 + 2\alpha h \simeq 1 + 10h \tag{6}$$

if we take for α the experimental value ~ 5.

For a given value of α, F is minimum when

$$h = [1 + (1 + \alpha^2)^{1/2}]/\alpha \tag{7}$$

For $\alpha = 5$, the optimum h-value is 1·22 and the linewidth enhancement factor F is 6·2 instead of $1 + \alpha^2 = 26$.

In order to give a feel for the parameters involved let us consider a laser with a 0·05 μm-thick active region, and a free-space wavelength of 1 μm. Let the active slab have a relative dielectric constant equal to $13{\cdot}69 + i\varepsilon_i$, where the parameter $-\varepsilon_i$ is proportional to the medium gain. The confining layers have a real dielectric constant equal to 13·25 (n_c = 3·64).

The values of our a and b parameters follow from eqn. 4. Expressed in (micrometres)$^{-1}$, and with $2d = 0{\cdot}05$, we have $a = 0{\cdot}434$, $b = -0{\cdot}987\varepsilon_i$, $h = b/a = -2{\cdot}27\varepsilon_i$ and $K = 1 + h^2$.

The gain in decibels/millimetre is equal to $165b$. Using for α the experimental value ~ 5, we obtain Table 1.

Table 1

$-\varepsilon_i$	0·01	0·05	0·088	1·86
$h = b/a$	0·027	0·113	0·2	1·22
G, dB/mm	1·63	8·14	14·3	302
K	~ 1	~ 1	1·04	2·5
α_A	5·8	12	∞	$-1{\cdot}2$
$\Delta f_A/\Delta f$	1·27	5·3	∞	0·23

This Table gives the ratio of the laser linewidth calculated by the formulas in this letter: Δf_A and Henry's result Δf. The active region thickness is 0·05 μm and the free-space wavelength is 1 μm. The gain G (dB/mm) is also listed. h, Petermann's K-factor and $\alpha_A = \Delta\omega_r/\Delta\omega_i$ are intermediate results.

As we can see from Table 1, even for the usual gain values (8 dB/mm), our $\alpha_A = \Delta\omega_r/\Delta\omega_i = 12$ is considerably larger than Henry's $\alpha = 5$. This α_A parameter is even infinite when $h = 1/\alpha$, but then the theory clearly needs revision. Going to even higher pumping rates, the laser linewidth Δf_A decreases and may be four times smaller than Δf. Note that even for such high pumping rates the relative index change remains small and the weakly guiding approximation remains valid.

All of these analytical results have been checked against exact numerical results for active slabs. The numerical method has been applied also to the usual case of 0·1 μm-thick active layers, where the accuracy of the thin-slab theory is poor. The same effects are found to be significant. For the gain appropriate to a 100 μm-long laser ($G = 48$ dB/mm), for example, we find $\Delta f_A/\Delta f = 2{\cdot}5$. Thus the admitted theory is then in error by a large factor.

The theory presented thus involves a re-examination of the laser linewidth data when the active region is thin ($< 0{\cdot}05$ μm) or the laser is short (< 100 μm). This theory is also relevant to the junction plane in the case of gain guidance with narrow stripes. In order to be able to apply our thin-slab approximation it suffices that the field extent be a few times the active region width. Finally we pointed out circumstances where the linewidth is four times smaller than according to existing theories.

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Appendix: This Appendix sketches the proof of the validity of eqn. 5 for laser modes well above threshold. The details will be given elsewhere.

We consider a laser model with perfectly reflecting mirrors, the coupling taking place sideways, in order to have exactly $\beta = \pi l/L$ a real constant. With that coupling mechanism only one longitudinal mode can be coupled to a single-mode fibre. The theory should nevertheless be approximately applicable to the more usual end-mirror coupling.

The total electric field is written

$$E(t) = E_0 \exp[i\phi(t)] + f(t)$$

where the first term represents the oscillating mode with $\phi = \phi_r + i\phi_i$ the complex phase. $f(t)$ represents the field accumulated from spontaneous emission from time 0 to t. This is a Wiener–Levy process analogous to the field in a capacitor driven by a white Gaussian electric current. If we specify that the complex phase $\phi(t)$ varies in such a way that $|E(t)|$ is independent of time we find

$$\text{phase}(E) = (Y + \alpha_A X)/E_0 \qquad f = X + iY$$

where $\alpha_A = \Delta\omega_r/\Delta\omega_i$; β = constant.

Next, to obtain $\langle ff^* \rangle$ in comparison to $E_0 E_0^*$ we observe that the power accumulated from spontaneous emission must be compensated on the average by a decay of the power in the oscillating mode. The small positive difference between coupling loss and gain is evaluated by considering the laser output as amplified spontaneous emission. We find that well above threshold $\langle ff^* \rangle / E_0 E_0^*$ is proportional to the K-factor.

The ambiguity pointed out in eqn. 5 concerning the concept of 'power in the mode' does not arise in our model because the fields in the active slab plane are real, irrespective of any loss or gain. The theory gives simple results for the coherent power for any drive current. The K-factor is obtained only for far-above-threshold modes. We have checked that the slab radiation involving both TE and TM modes is isotropic when the slab guidance is weak.

Aside from the distinction between α_A and α, and the introduction of the K-factor, the differences between the theory sketched above and Henry's theory are only formal.

FREQUENCY DEPENDENCE OF SOURCE ACCESS RESISTANCE OF HETEROJUNCTION FIELD-EFFECT TRANSISTOR

Indexing terms: Semiconductor devices and materials, Field-effect transistors

The determination of the source resistance in a heterojunction field-effect transistor is investigated. Classical methods as well as zero drain-source impedance measurements give a frequency dependence of the equivalent access resistance. This effect is due to the particular configuration of the access zone which is constituted by the GaAlAs layer and the two-dimensional electron gas. It may be represented by a distributed circuit with the equivalent resistances of these layers and the heterojunction capacitance. The noise factor dependence on R_s is considered and comments on the influence of technological parameters of the structure are made.

Introduction: Field-effect transistors based on the dynamic properties of the GaAs/GaAlAs heterojunction have been studied in many papers[1-3] during the last few years, especially for ultra-high-speed logic or low-noise amplifier applications. The knowledge of their high-frequency equivalent circuit is then crucial: one of the most important parameters for the calculation of the expected performances of the TEGFETs and a better understanding of their behaviour is the equivalent source access resistance. Several elements of the equivalent circuit are classically deduced from the $I_{gs}(V_{gs})$, $C_{gs}(V_{gs})$, $R_{ds}(V_{gs})$ characteristics and from the scattering parameter measurements in the 0·1–2 GHz frequency range. A more precise equivalent circuit can be derived from the S-parameters measured in the 6 to 18 GHz frequency range by using an optimisation procedure. The main result is then the small value of the source resistance which is quite different from the estimated R_s value deduced from the above-mentioned DC measurements (typically 7 Ω). Moreover, such a discrepancy is not observed with conventional FET structures.

Source resistance study: For simplicity, the evaluation of the equivalent access resistance is studied at zero V_{DS} bias voltage. The channel configuration under the gate of the field-effect transistor can then be described by a distributed RC dipole and expressed in terms of intrinsic impedance parameters Z_{ij}.[4]

Each real term Re (Z_{ij}) is represented as a function of frequency in Fig. 1a, the parasitic elements of the equivalent

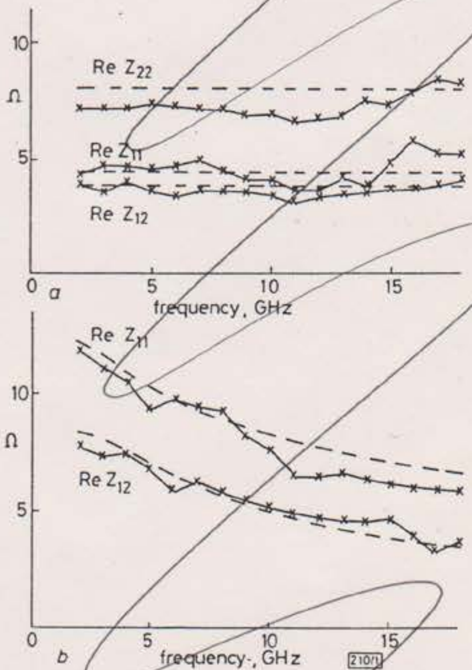

Fig. 1 *Experimental (——) and theoretical (– – – –) frequency dependence of real terms of impedance parameters*
a FET: $V_{DS} = 0$ V, $V_{GS} = 0$ V
b TEGFET: $V_{DS} = 0$ V, $V_{GS} = 0{\cdot}5$ V