RADIATION FORCE ON MULTILAYER MEDIA

J.A. ARNAUD

Bell Telephone Laboratories, Inc., Crawford Hill Laboratory, Holmdel, New Jersey 07733, USA

Received 19 February 1973

It is shown that the force exerted by plane electromagnetic waves on multilayer media can be evaluated in terms of the changes in momentum flow that take place at the planes of discontinuity. The expression obtained is relevant to radiation pressure experiments.

The force exerted by light waves on dielectric bodies is small but measurable [1]. Because of the present availability of powerful laser sources this radiation pressure appears to have potential applications. The purpose of the present note is to evaluate the displacement of the center of mass of a multilayer slab under the influence of a light wave. For generality, consideration is given to media that have arbitrary transverse bi-anisotropy.

To clarify the concepts involved, let us first consider a wave packet incident on a dielectric body in vacuum. Clearly, the momentum taken up by the body is the difference between the momentum of the wave packet before and after interaction. To evaluate this quantity we only need to know the expression for the momentum m of the wave packet in free space. This is the product of the EM (electromagnetic) mass W/c^2, where W denotes the EM energy and c the speed of light in free space, and the energy velocity u, whose magnitude is c

$$m = (W/c^2)u, \tag{1}$$

$$m = W/c. \tag{1'}$$

Thus, when a wave packet is transmitted through a slab free of loss and Fresnel reflection (e.g., when $\epsilon = \mu = n$, where ϵ denotes the permittivity of the medium, μ its permeability, and n its refractive index), the momentum taken up by the slab is equal to zero, that is, the slab remains at rest if it is originally at rest. The slab, however, is *displaced* in the process by a length Δz that can be evaluated by stating that the coordinate of the center of mass of the total system: wave packet plus slab, is proportional to time [2]. One finds that the slab displacement is in the forward direction and is given by [2]

$$\Delta z = (W/c^2)(n-1)/m_0, \tag{2}$$

where m_0 denotes the mass of the slab per unit length. This displacement is perhaps accessible to measurement if a long low-loss glass fiber (wound on a drum of small weight) is used in the place of a slab. To our knowledge such an experiment has not been performed.

The displacement Δz given in eq. (2) is most easily interpreted by assuming that eq. (1), with $\boldsymbol{u}$ representing the energy velocity of the wave in the medium, is applicable to polarisable media as well as to free space. For an isotropic nondispersive medium with refractive index n the magnitude of $\boldsymbol{u}$ in eq. (1) is equal to c/n. Thus, as the EM wave enters into the medium (with $n > 1$) it loses momentum. This loss is balanced by the mechanical momentum $(W/c)(1 - 1/n)$ taken up by the medium, which is pushed in the forward direction. When the wave packet leaves the slab, opposite effects take place and the slab goes back to rest. Because of the time lag existing between the light pulse entering and leaving the slab, the slab is displaced (forward) by the length Δz given in eq. (2). On the basis of that argument (and also on the basis of microscopic arguments such as the one discussed in ref. [3] ‡) we assume that the EM momentum is always given by eq. (1)‡‡.

When the slab is not free of Fresnel reflection ($\epsilon \neq \mu$), evaluation of the mechanical momentum requires that consideration be given to waves that are reflected back and forth within the slab. An even more complicated situation arises for slabs having multilayer coatings. The purpose of this note is to show that, when the proper formalism is introduced, the evaluation of the total force $F(t)$ exerted by the wave on the medium remains in fact simple and can be effected in two ways.

Let us consider a plane EM wave propagating along the z axis of an xyz cartesian coordinate system through a medium whose parameters depend on z but not on x or y. We assume that the cross-section area is unity and that the field of the wave vanishes at $z = \pm\infty$. Assuming that the medium has only transverse bi-anisotropy, the electric and magnetic field strengths E, H, and the electric and magnetic flux densities D, B have only x, y components and can be represented by 2-vectors. The linear relation existing between E, H and D, B is written

$$\begin{bmatrix} D \\ B \end{bmatrix} = \mathsf{L}\psi\,; \quad \psi \equiv \begin{bmatrix} E \\ H \end{bmatrix}, \tag{3}$$

where the 4 x 4 matrix L, a function of z only, is symmetrical ($\mathsf{L} = \widetilde{\mathsf{L}}$) [5] †.

Using the rationalized Gauss system, Maxwell's equations can be written, for plane waves

$$\psi_z^+ + \mathsf{L}\psi_t = 0, \tag{4}$$

where the z and t subscripts denote partial differentiations, and $\psi^+ \equiv c\gamma\psi$, where

$$\gamma = \widetilde{\gamma} \equiv \begin{bmatrix} 0 & \sigma \\ -\sigma & 0 \end{bmatrix}; \qquad \sigma \equiv \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \tag{5}$$

Within a uniform section of the medium (L being a constant) the wave equation (4) becomes an eigenvalue equation

$$\psi_\alpha^+ - u_\alpha \mathsf{L}\psi_\alpha = 0 \qquad \alpha = 1, 2, 3, 4, \tag{6}$$

where the u_α denote the phase velocities of the four waves that can propagate in the medium; e.g., the forward and backward ordinary and extraordinary waves. Because the medium is free of dispersion we do not distinguish between phase and energy velocities. The ψ_α, ψ_α^+ in eq. (6) denote eigenstates of polarization. The energy flow $S = cE \times H$ is directed along z. Its magnitude is

$$S = \tfrac{1}{2}\widetilde{\psi}^+\psi. \tag{7}$$

According to eq. (1), the EM momentum flow is equal to the EM energy flow S divided by c^2 and multiplied by the energy velocity u. The total EM momentum flow $\mathcal{M}(z, t)$ is therefore the sum of the products $c^{-2}\, S \cdot u$ relative to the four waves. Note that in the present case the energy flow, the energy velocity and the momentum flow can be treated as scalar quantities. We have

$$c^2 \mathcal{M}(z, t) = \sum_{\alpha=1}^{4} S_\alpha u_\alpha = \tfrac{1}{2} \sum_{\alpha=1}^{4} \widetilde{\psi}_\alpha^+ \psi_\alpha u_\alpha. \tag{8}$$

It turns out that $\mathcal{M}(z, t)$ can be expressed as a function of the *total* field $\psi^+ \equiv \Sigma_\alpha \psi_\alpha^+$ and the material matrix L only. We have indeed

$$c^2 \mathcal{M}(z, t) = \tfrac{1}{2}\widetilde{\psi}^+ \mathsf{L}^{-1} \psi^+, \tag{9}$$

because of eq. (6) and of the orthogonality conditions

$$\widetilde{\psi}_\alpha^+ \mathsf{L}^{-1} \psi_\beta^+ = 0, \qquad \alpha \neq \beta, \tag{10}$$

which hold between any two nondegenerate eigenstates of polarization [6] ††.

Let us consider next a multilayer medium. The total force exerted by the wave on the medium is equal to the sum of the changes in EM momentum flow at each plane of discontinuity. Because $\mathcal{M}(z, t)$ can be written in the form given in eq. (9), we have simply

$$c^2 F(t) = -\tfrac{1}{2} \sum_i \Delta_i (\widetilde{\psi}^+ \mathsf{L}^{-1} \psi^+)$$

$$= -\tfrac{1}{2} \sum_i \widetilde{\psi}_i^+ \Delta_i (\mathsf{L}^{-1}) \psi_i^+, \tag{11}$$

where the $\Delta_i(\ )$ denotes variations at the ith plane of discontinuity, and ψ_i^+ the value of ψ^+ at that plane (ψ^+ is a continuous function of z).

This result, eq. (11), can alternatively be obtained by evaluating the sum of the Lorentz force acting on the polarization current P_t and the magnetic dual of the Lorentz force [7] acting on the magnetization current M_t.

$$c^2 F(t) = \int_{-\infty}^{+\infty} c(P_t \times H - M_t \times E)\mathrm{d}z, \tag{12}$$

where

$$\begin{bmatrix} P \\ M \end{bmatrix} = \begin{bmatrix} D \\ B \end{bmatrix} - \begin{bmatrix} E \\ H \end{bmatrix} = (\mathsf{L} - 1)\psi. \tag{13}$$

Using eq. (4) and integrating by parts, eq. (12) becomes

$$c^2 F(t) = \int_{-\infty}^{+\infty} \widetilde{\psi}_t (\mathsf{L} - 1)\psi^+ \mathrm{d}z$$

$$= \int_{-\infty}^{+\infty} \widetilde{\psi}_z^+ (\mathsf{L}^{-1} - 1)\psi^+ \mathrm{d}z$$

$$= -\tfrac{1}{2} \int_{-\infty}^{+\infty} \widetilde{\psi}^+ \mathsf{L}_z^{-1} \psi^+ \mathrm{d}z. \tag{14}$$

Note incidentally that, because the force exerted by the wave on the medium is just opposite to the force exerted by the medium on the wave, an alternative expression of $F(t)$ is the negative of the time derivative of the integral over all space of the EM momentum density $c^{-2}S$

$$c^2 F(t) = -\frac{\mathrm{d}}{\mathrm{d}t} \int_{\text{all space}} S\, \mathrm{d}V, \tag{15}$$

which is easily shown to coincide with eq. (14) for the case considered.

If L varies as a function of z by steps, as assumed before, eq. (14) becomes

$$c^2 F(t) = -\tfrac{1}{2} \sum_i \widetilde{\psi}_i^+ \Delta_i (\mathsf{L}^{-1}) \psi_i^+, \tag{16}$$

in agreement with eq. (11).

In the special case where the medium is isotropic with permittivity ϵ and permeability unity, eq. (16) becomes

$$F(t) = -\tfrac{1}{2} \sum_i H_i^2 \Delta_i (\epsilon^{-1}). \tag{17}$$

Once the total force, eq. (16) or (17), is found, the position of the center of mass of the slab is readily obtained by integration.

In conclusion, we have shown that the force exerted by an electromagnetic wave on a multilayer medium with transverse bi-anisotropy can be expressed in a simple way in terms of the changes in momentum flow at the planes of discontinuity. This result generalizes a result previously obtained by Haus [8] applicable to EM waves entering from free space into a homogeneous isotropic medium. It should be noted that the momentum transferred to the medium is partly in the form of the material being dragged along with the light wave and partly associated with surface forces at planes of discontinuity [8]. In order to discuss the spatial distribution of the force, electrostrictive and magnetostrictive phenomena have to be taken into account. In the present note, we have addressed ourselves only to the problem of evaluating the total force and ignored its spatial distribution.

‡ This paper shows that for a simple artificial dielectric (a meander line) the EM momentum is unambiguously given by eq. (1).

‡‡ The momentum of the medium can be ignored in directions where the medium has translational invariance, if we make use of a canonical momentum $p = (W/\omega)k$, where k denotes the wave vector. In particular, at a plane interface, the tangential component of p is continuous (Descartes–Snell law of refraction). Györgyi [4] has shown that the law of refraction can alternatively be based on the expression, eq. (1), of the true momentum. In Györgyi's formulation the momentum of the medium is taken into account. Generally speaking, p describes interactions taking place in the medium; it is not directly related to the kinetic momentum of the medium itself.

† Note that the medium that we are considering is perhaps non-reciprocal.

†† Note that eq. (10) holds also in the degenerate case (isotropic media) if the total field is properly decomposed.

References

[1] A. Ashkin, Appl. Phys. Letters 19 (1971) 283.

[2] O. Costa de Beauregard, Compt. Rend. Acad. Sci. (Paris) 274 (1972) 164;
N.L. Balasz, Phys. Rev. 91 (1953) 408;
W. Shockley, Proc. Natl. Acad. Sci. 60 (1968) 807.

Volume 7, number 4 OPTICS COMMUNICATIONS April 1973

[3] J.A. Arnaud, Electron. Letters 8 No. 22 (1972) 541.
[4] G. Györgyi, Am. J. Phys. 28 (1960) 85.
[5] P. Penfield and H.A. Haus, Electrodynamics of continuous media (The M.I.T. Press, Cambridge, 1967) p. 101.
[6] J.A. Arnaud, J. Opt. Soc. Am. 63 (Feb. 1973), to be published.
[7] O. Costa de Beauregard, Compt. Rend. Acad. Sci. (Paris) 263B (1966) 1007; Il Nuovo Cimento Serie 10, 63B (1969) 611.
[8] H.A. Haus, Physica 43 (1969) 77.