PHOTONIC NOISE WITH NONLINEAR GAIN

For frequency-independent gain and loss, the circuit theory of photonic noise based on the Nyquist noise sources reduces to a corpuscular theory that postulates independent shot-noise fluctuations of the particle rates. For linear gain, the corpuscular theory coincides with standard rate equations (SREs). When the relative gain compression is comparable with unity, however, SREs are in error by a large factor. The exact theory shows that in the presence of nonlinear gain, photonic noise may be less than shot noise, even when the injected current exhibits shot-noise fluctuations. This new effect cannot be predicted by SREs.

When light from a laser is applied to a detector (Fig. 1a), the detector current fluctuates. For an ideal detector these fluctuations are attributed to photonic noise, i.e. to the random arrival of photons on the detector, each photon generating one electron. Photonic noise can be split into an 'intensity noise' of a classical nature and a shot-noise term expressing the independent generation of photoelectrons. If a linear attenuator is introduced between the laser and the detector the relative-intensity-noise (RIN) remains the same.[1] This conclusion can be proven from eqn. 2 to hold even when photonic noise is below shot noise, in which case a negative RIN can be formally introduced.
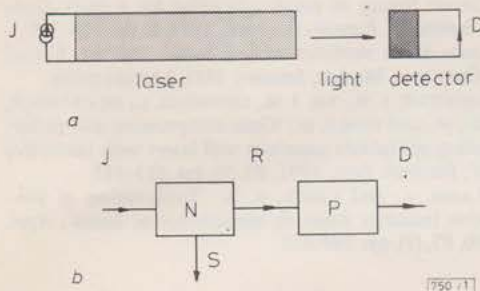

Fig. 1 *Laser-detector arrangement and schematic diagram applicable to rate-equation theories*

J, S, R, D denote particular rates, N the number of electrons and P the number of photons in the laser cavity
a Laser-detector arrangement
b Schematic diagram applicable to rate equation theories

Laser intensity noise has been investigated in the 1960s on the basis of quantum mechanics in the form of rate equations that describe the time evolution of the number of electrons N and number of photons P. In the following the approximations and terminology relevant to laser diodes are used. Standard rate equations (SREs) are written below, for simplicity, at zero baseband frequency. Full population inversion and shot-noise fluctuations of the injected current are implied. We have[2,3] (see Fig. 1b)

$$\bar{J} = \bar{S} + \bar{R} \qquad\qquad \bar{R} = \bar{D} \quad (1a)$$

$$0 = \Delta S + \Delta R + n \qquad \Delta R = \Delta D + p \quad (1b)$$

$$\Delta R = R_N\,\delta N + R_P\,\delta P \qquad \Delta S = S_N\,\delta N \quad (1c)$$

$$\delta D = \Delta D + d \qquad \Delta D = (D/P)\,\delta P \quad (1d)$$

$$\mathcal{S}_n = 2S + 2D \quad \mathcal{S}_p = 2D \quad \mathcal{S}_{np} = -2D \quad (1e)$$

J denotes the injected electronic rate that is supposed to be independent of N, $S(N)$ the spontaneous emission rate, $R(N, P) \equiv G(N, P)P$ the rate at which electrons are converted into photons in the oscillating mode by the process of stimulated emission, and D the outgoing photonic rate. Subscripts denote partial derivatives. Upper bars that denote average values are omitted when no confusion with instantaneous values may arise. The Langevin terms $n(t)$, $p(t)$ in eqn. 1b are white processes whose double-sided spectral densities $\mathcal{S}$ are given in eqn. 1e. The spectral density of the independent shot-noise process $d(t)$ that accounts for photoelectron shot-noise is equal to the average photonic rate D. SREs are obtained semi-classically by considering that spontaneous emission adds to the oscillating field a small random field, the spontaneous rate in the mode being equal to $D/P \equiv 1/\tau_p$, where τ_p denotes the photon lifetime. An expression for photonic noise δD is easily obtained from the system in eqn. 1. It is shown below, however, that strictly speaking, eqn. 1 is valid for linear gain only, that is when the gain G does not depend on the photon number P.

A different theory of photonic noise has been proposed[4,5] called 'circuit theory'. This theory rests only on the Nyquist formula that associates noise voltages to resistors, and on the law of energy conservation. It has been found to agree with quantum mechanical results even for nonclassical states of light and electrical feedback,[6] but is simpler. When the loss and the gain are frequency-independent, this circuit theory is equivalent to a corpuscular theory in which particle rates consist of a classical fluctuation plus an independent shot-noise fluctuation.[7] Each shot-noise fluctuation corresponds to a Nyquist noise voltage.

The corpuscular theory reads as eqn. 1 except that eqn. 1b and 1d are replaced by the most transparent relations

$$j = \Delta S + s + \Delta R + r \qquad \Delta R + r = \Delta D + d \quad (2a)$$

$$\mathcal{S}_j = J \quad \mathcal{S}_s = S \quad \mathcal{S}_r = R \quad \mathcal{S}_d = D \quad (2b)$$

The white-noise processes j, s, r and d are independent. Note that $d(t)$ enters in both eqn. 2a and eqn. 1d. This is why sub-Poissonian photon statistics may be obtained from this formulation. Let us recall that at zero frequency the spectral density of the weighted sum $ax + by$ of two processes $x(t)$, $y(t)$ is given by

$$\mathcal{S}_{ax+by} = a^2\mathcal{S}_x + b^2\mathcal{S}_y + 2ab\mathcal{S}_{xy} \quad (3)$$

In the linear gain approximation ($G_P = 0$) it can be shown that the expressions obtained from eqns. 1 and 2 for all measurable quantities are identical.[5] However, let us now compare the two formulations in the case of nonlinear gain. It is convenient to introduce nondimensional parameters

$$(N/R)R_n \equiv g \qquad\qquad (P/R)R_P \equiv 1 - \gamma \quad (4a)$$

$$S/D \equiv \zeta \quad (N/S)S_N \equiv s \quad \chi = \zeta s/g \quad (4b)$$

In eqn. 4a, g is the differential gain factor and γ the relative gain compression. ζ is the spontaneous-to-stimulated rate ratio. When $\gamma = 0$, the parameters g and s are constant above threshold and the reciprocal of ζ is the ratio of injected current to threshold current minus one. When $\gamma > 0$, the carrier number N slowly increases above threshold and the parameters g, s may vary slightly.

The SRE, eqn. 1, and the corpuscular rate equations, eqn. 2, lead, respectively, to the following expressions for the photonic noise spectral density $\overline{S_{\delta D}}$ relative to the shot-noise level D

$$D^{-1}\mathcal{S}_{\delta D} = 1 + 2(\zeta + \chi^2)/(1 + \gamma\chi)^2 \qquad \text{(SRE)} \quad (5)$$

$$D^{-1}\mathcal{S}_{\delta D} = 1 + 2(\zeta + \chi^2 - \gamma\chi - \gamma\chi^2)/(1 + \gamma\chi)^2 \quad \text{(exact)} \quad (6)$$

These two expressions coincide for linear gain ($\gamma = 0$) as stated earlier. If the injected current is much larger than the threshold current the ζ and χ parameters vanish and photonic noise reduces to the shot-noise level from either formulation. However in general they obviously do not agree.

For long wavelength lasers the recombination rate is dominated by Auger effects and $s = 3$. If the differential gain factor g is taken as unity, $\chi = 3\zeta$. As long as the rate compression factor γ is small compared with unity, the error made in using SREs remains small. But γ values comparable with unity have been considered recently, in which case the error made in using SREs may be large.

Rather large gain compression factors have been measured for quantum wells.[8] For existing lasers γ may be of the order of 0·05. Larger γ values would occur if the refractive index

step and the mirror reflectivities were made larger, because the optical field in the active layer would in that case be enhanced for a given output power. The exact mechanism for nonlinear gain and the numerical values of γ are not yet well established. Measured values of low-frequency photonic noise are larger than expected because of the perturbating effect of side-modes.[9]

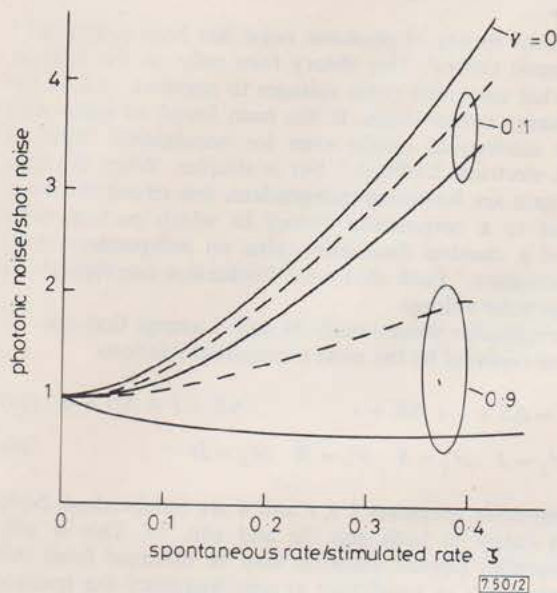To exemplify the difference between the theoretical results

Fig. 2 *Variation of photonic noise (double-sided) spectral density relative to shot-noise level D as function of spontaneous to stimulated rates ratio ζ*

Three values of relative gain compression γ are considered: $\gamma = 0$ (linear gain), $\gamma = 0.1$ and $\gamma = 0.9$

——— exact from eqn. 2

- - - - standard rate equations eqn. 1

in eqns. 5 and 6 the photonic spectral density is represented in Fig. 2 as a function of spontaneous-to-stimulated rates ratio ζ for $\gamma = 0$ (linear gain), $\gamma = 0.1$ and $\gamma = 0.9$. It is remarkable that the corpuscular (or circuit) theory presented in this Letter predicts that photonic noise may be reduced below the shot-noise level by nonlinear gain. This may occur for any γ value provided g/s and ζ are small enough.

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