PULSE BROADENING IN NEAR-SQUARE-LAW GRADED-INDEX FIBRES

Indexing terms: Fibre optics, Optical-waveguide theory

A closed-form expression for pulse broadening in graded-index fibres that have small and circularly symmetric, but otherwise arbitrary, deviations from a square-law, and arbitrary $dn/d\lambda$, is applied to germania-doped fibres. The range of validity of the theoretical expression is defined by comparison with the results of numerical integration.

Numerical results are presented for the broadening of optical pulses in near-square-law fibres. These results are based on a closed-form expression previously reported by the author.[1] The r.m.s. impulse-response widths of typical germania-doped silica fibres for various doping profiles are calculated to illustrate the theory.

The broadening of pulses of quasimonochromatic light in glass fibres depends critically on the details of the refractive-index profile and on the inhomogeneous dispersion of the material, defined as the variation of the phase to group-velocity ratio as a function of distance from the axis. Three theories are at present available for evaluating the impulse response of circularly symmetric near-square-law fibres. These theories properly take into account the inhomogeneous dispersion of the material. The theory presented in Reference 2 is limited to axisymmetric modes and to r^4 perturbations of the square law. In References 3 and 4 it is assumed that the square of the refractive index is a power of the radius. The expression for the time of flight of a pulse in a particular mode follows from differentiation with respect to ω of the expression for the axial wavenumber given in Reference 3. The variation of the r.m.s. impulse width as a function of the exponent of r is given in Reference 4. For many practical fibres, however, the assumptions in References 2–4 may be too restrictive. Our result in Reference 1 is general, except for the requirement that the deviation from a square law must be small, circularly symmetric, and that ray-optics (or WKB) methods be applicable. We also assume that polarisation effects can be neglected.

Let us assume that the refractive index n of the fibre material has been measured as a function of radius r at the operating wavelength λ and at a slightly different wavelength λ'. Let the refractive-index profiles $n(r)$ and $n'(r)$ at these two wavelengths be written in the form

$$\left.\begin{aligned} (2\pi/\lambda)^2 n^2(r) &\equiv K_0 - K_\kappa r^{2\kappa} + K_2 r^4 + K_3 r^6 + \ldots(\lambda) \\ (2\pi/\lambda')^2 n'^2(r) &\equiv K_0' - K_\kappa' r^{2\kappa} + K_2' r^4 + K_3' r^6 + \ldots(\lambda') \end{aligned}\right\} \quad (1)$$

The parameters K_0, K_κ and the exponent κ are selected to best fit the given profile at the wavelength considered, and the difference is best fitted to a polynomial in r^2. For the choice $\kappa = 1$, eqn. 1 is a conventional expansion in powers of r^2. From the coefficients K_γ, K_γ', $\gamma = 0, \kappa, 2, 3 \ldots$, in eqn. 1, we define inhomogeneous dispersion parameters

$$D_\gamma \equiv K_0(K_\gamma' - K_\gamma)/K_\gamma(K_0' - K_0) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The relative time of flight τ of a pulse, defined as the ratio of the group velocity of plane waves on axis to the group velocity of a mode with radial mode number $\nu > 0$, and azimuthal mode number μ, is, from References 1, 3 and 5,

$$\tau(\alpha, \mu) = (1 - B)^{-\frac{1}{2}} \left[1 - B D_\kappa' + \sum_{\gamma=2}^{\infty} B^\gamma F_\gamma N_\gamma \right] \quad . \quad (3)$$

where

$$\left.\begin{aligned} D_\kappa' &\equiv D_\kappa/(1+\kappa) \\ B &\doteq [1 - 0{\cdot}2(\kappa - 1)](2g)^{2\kappa/(\kappa+1)} K_\kappa^{1/(\kappa+1)}/K_0 \\ g &\equiv 2\nu + |\mu| + 1 \qquad |\mu| \equiv \text{absolute value of } \mu \\ F_\gamma &\equiv \gamma! 2^{-\gamma}[D_\gamma - (\gamma+1) D_\kappa'] \varepsilon_\gamma \\ \varepsilon_\gamma &\equiv K_\gamma K_0^{\gamma-1}/K_\kappa^{2\gamma/(1+\kappa)} \\ N_\gamma &\equiv \sum_{m=0,\,2}^{\gamma} \{2^m(\gamma - m)![(m/2)!]^2\}^{-1}[1 - (\mu/g)^2]^{m/2} \end{aligned}\right\} \quad . \quad (4)$$

All the parameters on the l.h.s. of eqn. 4 are dimensionless.

For preliminary designs, we can assume that there is only one dopant material and that n^2 varies linearly with the dopant concentration. In that case, the $D_\gamma \equiv D$ are all equal. This is assumed in the two special cases treated below.

Let us first set $\kappa = 1$ in the previous expressions and assume that the series in eqn. 1 terminates at the r^4 term. The total impulse width for long fibres is proportional to the difference between the maximum and minimum values of τ in eqn. 3, with ν, μ restricted by the condition that the mode wavenumber k_z be larger than the cladding wavenumber $k_s \equiv (\omega/c) n_s$; that is, with the notation used in eqn. 3, $B < 1 - K_s/K_0$, where $K_s \equiv k_s^2$. We find that the particular value of ε_2 that minimises the total impulse width is

$$(\varepsilon_2)_{opt} \simeq (2/3)[1 - 2(D-1)/(1 - n_s/n_0)] \quad . \quad . \quad . \quad (5)$$

where n_s and n_0 denote the cladding and axial refractive indices, respectively. We have compared the relative time of flight predicted by eqn. 3 with $\kappa = 1$ to the exact value obtained by numerical integration of the space–time Hamilton equations. We have found that the expression in eqn. 3 is very accurate for near optimum profiles, provided that $|D - 1| \leq 0{\cdot}02$. This condition is satisfied for most fibre materials. Nevertheless, in some cases, it is useful to let κ be different from unity.

If we keep κ arbitrary in eqn. 1, but omit the series, the value of κ that minimises the total impulse width for long fibres is exactly (within the scalar ray-optics approximation)

$$\kappa = D(1 + n_s/n_0) - 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This result follows from the fact that D and κ enter in the expression of τ in eqn. 3 only as $D/(1+\kappa)$, as shown in Reference 5. Note that the coefficient in the expression of B in eqn. 4 is an approximate form of that given in Reference 3. It is applicable when $0{\cdot}8 < \kappa < 1{\cdot}2$.

Let us now go back to the general case. Usually, we are interested not in the impulse response itself, but in the r.m.s. impulse-response width, defined as

$$\left.\begin{aligned} \sigma &= 5000[\langle \tau^2 \rangle - \langle \tau \rangle^2]^{\frac{1}{2}} \qquad \text{ns/km} \\ &\equiv 5000[\langle (\tau - 1)^2 \rangle - \langle \tau - 1 \rangle^2]^{\frac{1}{2}} \qquad \text{ns/km} \end{aligned}\right\} \quad (7)$$

where, for any quantity $a(\nu, \mu)$, such as τ or τ^2,

$$\langle a \rangle \equiv \sum_{\nu,\mu} T(\nu, \mu) a(\nu, \mu) / \sum_{\nu,\mu} T(\nu, \mu) \quad . \quad . \quad . \quad (8)$$

Assuming that the material loss is independent of radius and that the fibre is only moderately overmoded, the power transmission $T(\nu, \mu)$ is, to within an unimportant constant factor,

$$T(\nu, \mu) = \begin{cases} 1 & 0 < B < 1 - K_s/K_0 \simeq 2\Delta n/n \\ 0 & 1 - K_s/K_0 < B \end{cases} \quad (9)$$

To illustrate the method, let us consider a germania-doped silica fibre, with a dopant concentration, in percent, of

$$d = 10 - 10(r_{\mu m}/40)^2 - A(r_{\mu m}/40)^4 \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where A is a small adjustable parameter. The dopant concentration d is equal to 10% on axis and 0% at the core radius $a \simeq 40\ \mu m$. The refractive index of germania-doped

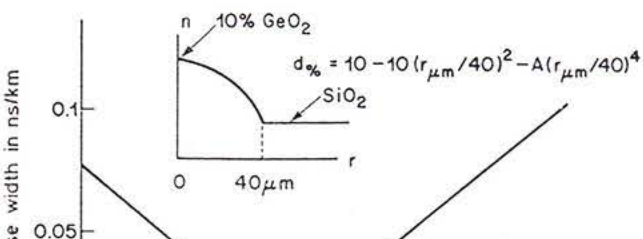

Fig. 1 *Variation of r.m.s. impulse-response width of germania-doped fibre with 10% doping on axis as function of coefficient of r^4 term in profile*

silica has been measured by Fleming[6] for 0% and 13·5% GeO_2 concentration. Linear interpolation and use of eqn. 10 gives the refractive-index laws at λ = 0·88 and 0·9 μm, respectively.

For the special value A = 0·051, for example, we obtain

$$\left.\begin{aligned} n^2(r) &= 2{\cdot}15439+2{\cdot}7281\times10^{-5}r^2-9{\cdot}193\times10^{-11}r^4 \\ n'^2(r) &= 2{\cdot}15352+2{\cdot}87075\times10^{-5}r^2-9{\cdot}186\times10^{-11}r^4 \end{aligned}\right\} \quad (11)$$

where r is in micrometres. The coefficients K_0, K_1, K_2, D_1 and D_2 are obtained from eqn. 11 and the definitions in eqns. 1 and 2. Substituting the values in eqns. 7 and 3, we obtain, with $1-K_s/K_0 \simeq 2\Delta n/n = 0{\cdot}017$, the r.m.s. impulse width of σ = 0·016 ns/km. Fig. 1 shows how the r.m.s. impulse width varies as a function of the coefficient A in eqn. 10 for $d(r)$.

In the foregoing, the optical carrier was assumed monochromatic. If this is not the case, the sign $\langle\rangle$ in eqn. 7 should be replaced by

$$\langle\!\langle\ \rangle\!\rangle \equiv \int \langle\ \rangle_\lambda f(\lambda)\,d\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where $\langle\ \rangle_\lambda$ is evaluated by the procedure outlined above at various wavelengths λ within the (normalised) spectral curve $f(\lambda)$ of the source. This conclusion, based on space–time ray optics, agrees with the result in Reference 7.

In conclusion, we have described a procedure that permits the accurate evaluation of the impulse response of large-transmission-capacity multimode fibres. The computing time needed is very small because closed-form expressions are used.

J. A. ARNAUD

Bell Laboratories
Crawford Hill Laboratory
Holmdel, NJ 00733, USA

10th August 1975

References

1 ARNAUD, J. A.: 'Pulse broadening in multimode graded-index fibres', *Electron. Lett.*, 1975, **11**, pp. 8–9
2 GAMBLING, W. A., and MATSUMURA, H.: 'Pulse dispersion in a lens-like medium', *Opt. Electron.*, 1973, **5**, p. 429
3 STREIFER, W., and KURTZ, C. N.: 'Scalar analysis of radially inhomogeneous guiding media', *J. Opt. Soc. Am.*, 1967, **57**, p. 779
4 OLSHANSKY, R., and KECK, D. B.: Topical meeting on optical fiber transmission, Williamsburg, Va., 1975 [Notation $y \equiv 4(D-1)$]
5 ARNAUD, J. A.: 'Pulse broadening in multimode optical fibres', *Bell Syst. Tech. J.*, 1975, **54**, Sept. (to be published)
6 FLEMING, J. W.: 'Measurements of dispersion in GeO_2–B_2O_3–SiO_2 glasses'. Fall meeting of American Ceramic Society, Pa., Oct. 1975
7 PERSONICK, S. D.: 'Optimal trade-off of mode-mixing optical filtering and index difference in digital fibre optic communication systems', *Bell Syst. Tech. J.*, 1974, **53**, pp. 785–800