Optical and Quantum Electronics 9 (1977) 111–119

Optimum profiles for dispersive multimode fibres

J. A. ARNAUD*
Bell Laboratories, Crawford Hill Laboratory, Holmdel, New Jersey 07733, USA

Received 21 September 1976

A simple closed-form expression is given for the index profile of multimode fibres with arbitrary dispersion providing transmission capacities as large as $1.6/\Delta^2$ Mbit/s km, where $\Delta \equiv \Delta n/n$. Our result reduces to a previous result of Marcatili for the special case of circularly symmetric fibres. A transmission capacity of 150 Mbit/s over a 10 km long fibre appears possible with LED sources operating at optimum wavelengths for the medium.

1. Introduction

Glass fibres constitute almost ideal media for the guided transmission of information. Losses below 0.5 dB/km have been reported [1] that indicate the possibility of repeater spacings as large as 50 km. For large repeater spacings, the broadening of optical pulses as they propagate along the fibre is important, even at low or moderate data rates. The transmission capacity of systems consisting of single-mode fibres and quasi-monochromatic sources is limited only by the speed at which the source can be modulated and at which pulses can be detected. A number of practical considerations (life-time of the source, ease of cabling and splicing etc.) suggest, however, that multimode fibres are more practical than monomode fibres. We shall investigate the design of optimum multimode fibres excited by multimode sources, under the assumption that the fibre material is linear, isotropic and time invariant, using the method of ray optics. No assumption is made concerning the dispersion of the material. Ray optics methods are applicable to carefully fabricated multimode fibres. Specifically, large or fast variations of refractive index in the fibre cross-section are assumed not to be present, and mode coupling effects due to irregularities or cabling to be negligible. General concepts in optical pulse propagation are discussed in Section 2 and orders of magnitude of the effects are given. Simple analytical results based on the equalization of helical rays are derived in Section 3. We show how to determine optimum profiles and optimum materials. More general results, applicable to non-helical rays and non-circularly symmetric profiles, are derived in Section 4.

2. General results

We shall recall in this section fundamental concepts of optical pulse propagation. Consider a time-harmonic, plane, optical wave propagating in a homogeneous material. The distance between adjacent nodes of the wave field at a fixed time is the wavelength λ. We define the wavevector $\mathbf{k}$ as a vector perpendicular to the wavefronts having magnitude $k = 2\pi/\lambda$ (wavenumber). The time elapsed between successive nodes of the wavefield at a fixed point is the period T. We define the angular frequency $\omega = 2\pi/T$. Wave numbers and angular frequencies are fundamental quantities. Derived quantities, such as the refractive index $n \equiv k/(\omega/c)$ or the free-space wavelength $\lambda_0 = 2\pi c/\omega (c = 3 \times 10^8$ m/s), however, are sometimes convenient.

We shall assume in the following that the medium is isotropic. The wavenumber is independent of direction and the rays have the direction of the wave vector $\mathbf{k}$. If the medium is inhomogeneous, the

*Present address: LSS, Ecole Superieure d'Electricité, Plateau du Moulon, Gif-sur-Yvette 91190, France.

rays are curved. The phase shift ϕ suffered by the wave field along a ray is obtained by integrating k ds along the ray, where ds denotes the elementary ray length. If the wave is modulated by a pulse, the transit time t of the pulse along the ray is obtained by specifying that the phase is stationary: $t = \partial\phi/\partial\omega$. (In principle $\phi(\omega)$ and $\phi(\omega + \mathrm{d}\omega)$ are evaluated along two distinct ray trajectories. Fermat's principle, however, permits us to evaluate both quantities along the ray trajectory at ω.) Alternatively, t can be expressed as the integral of ds/u along the ray, where $u = \partial\omega/\partial k$ denotes the local group velocity. In the absence of dispersion we have $\partial\omega/\partial k = \omega/k = v$ and $t = \phi/\omega$. The local group velocity u equals the local phase velocity v. If the ratio v/u is a constant, perhaps different from unity, times of flight are proportional to phase shifts. In that special case, equalization of the phase shifts for different ray paths entails time of flight equalization. The ration v/u usually varies spatially in a complicated manner. Such a medium with variable v/u ratio is said to suffer from inhomogeneous dispersion. This turns out to be a very important effect in fibre optics.

Let the previous definitions be applied to uniform optical fibres, with axial coordinate z and transverse coordinates x, y. The transit time t per unit length of a pulse is usually a function of the initial position x, y, of the pulse, its initial direction, characterized by the transverse components k_x, k_y of the wavevector, and the carrier angular frequency ω. For clarity, we shall omit the dependence of t on the coordinates x, y and combine the rectangular components k_x, k_y into a transverse component $k_t \equiv (k_x^2 + k_y^2)^{1/2}$. The symmetry of the fibre implies that t depends only on the square of k_t. Thus t is a function of k_t^2 and ω. The maximum value that k_t^2 can attain is proportional to the relative change of index $\Delta = \Delta n/n$ in the fibre cross section. Indeed, a wave can be trapped within the fibre core only if the axial component k_z of the wavevector (propagation constant) is larger than the cladding wavenumber k_c. We have

$$(k_t^2)_{\max} = k_0^2 - (k_z^2)_{\min} = k_0^2 - k_c^2 \tag{1a}$$

and

$$\Delta \equiv \tfrac{1}{2}(k_0^2 - k_c^2)/k_0^2 \approx (n_0 - n_c)/n_0. \tag{1b}$$

The maximum variation Δt of the time of flight can consequently be written in the form of a power series in Δ and f, where f denotes the departure of the optical frequency from some reference frequency. The terms exhibited in the series

$$\Delta t = a\Delta + b\Delta^2 + cf + df^2 + ef\Delta \tag{2}$$

are sufficient to discuss the most important effects. The coefficients a, b, c, d, e, in Equation 2 are determined by the wavenumber distribution $k(x, y, \omega)$ of the fibre in a manner that will be discussed in subsequent sections.

Let us consider quasi-monochromatic sources and set $f = 0$. The dominant term in Equation 2 is $a\Delta$. For a step-index fibre, for example, we have

$$\Delta t\ (ns/\mathrm{km}) = 5000\Delta. \tag{3}$$

The numerical factor 5000 in Equation 3 represents the time of flight of axial pulses for typical materials (axial group velocity $\approx 3\times 10^8/1.5$, m/s). We shall prove in Section 3 that it is always possible to find index profiles such that the term $a\,\Delta$ vanishes. We are then left with the term $b\Delta^2$. Numerically, this quadratic term is

$$\Delta t\ (\mathrm{ns/km}) = (5000/8)\Delta^2. \tag{4}$$

Some sources, particularly light-emitting diodes (LEDs), have a nonzero linewidth. For clarity we shall first consider the effect for axial rays, i.e. when $\Delta = 0$. The dominant term in Equation 3 is then cf, a term often referred to as the material dispersion effect. At optical frequencies of the order of 250 terahertz (free-space wavelength $\approx 1.2\ \mu$m), however, the term cf vanishes and the dominant term becomes df^2. Measurements on bulk samples [2] show that the coefficient d is almost the same for all materials. We have

$$\Delta t\ (\mathrm{ns/km}) \approx 0.0016f^2 \tag{5}$$

where f is expressed in terahertz. (1 terahertz = 1000 gigahertz.)

The last term $ef\Delta$ in Equation 2 exists because index profiles that are optimum at one particular frequency are not necessarily optimum at neighbouring frequencies. For LED sources and most dopants (particular phosphoric oxide) the term $ef\Delta$ is, however, negligible compared with the term df^2. For phosphor oxide, $\lambda_0 = 1.27\ \mu$m and $\Delta = 0.01$ the term $ef\Delta$ is significant only when $f \lesssim 0.2$ terahertz. In any event, the term $ef\,\Delta$ can usually be eliminated, as we shall see, if two dopant materials are used. Elimination of the term $ef\,\Delta$ provides a degree of freedom concerning the operating frequency. It also allows the transmission capacity of the fibre to be increased by frequency multiplexing.

In the above discussion, we considered only the maximum possible variation of the time of flight. The transmission capacity of a fibre is more intimately related to its impulse response $P(t)$. To evaluate $P(t)$ we need to know the distribution of the pulses emitted by the source in position, direction and angular frequency.

The source is represented in general by some distribution $f(x, y, t, k_x, k_y, \omega)$ at the input plane of the fibre. A well known theorem asserts that the distribution f does not vary along ray trajectories if the loss is uniform. (This theorem is equivalent to saying that the determinant of paraxial ray matrices is unity, a result established by Luneburg in 1944. It is sometimes referred to as Liouville's theorem.) When both the distribution f and the time of flight t are known, the impulse response $P(t)$ can be obtained by integration. The data-transmission capacity of the system is approximately given by

$$T = 1/(4\sigma) \tag{6a}$$

where σ denotes the r.m.s. impulse response width

$$\sigma \equiv [\langle t^2\rangle - \langle t\rangle^2]^{1/2}; \qquad \langle a\rangle = \int_{-\infty}^{+\infty} aP(t)\,\mathrm{d}t \tag{6b}$$

and the integral of $P(t)$ over time is assumed to be unity.

Let us assume that the source distribution f is independent of k_x, k_y, (Lambertian source), independent of x, y within the fibre-core cross section, and has a Gaussian power spectrum of r.m.s. width f

$$P(\omega) = \exp\{-[(\omega - \omega_0)/2\pi f]^2\}. \tag{7a}$$

On the basis of the previous discussion one can show that the r.m.s. impulse response width of a multimode fibre can be reduced to

$$\sigma\ (\mathrm{ns/km}) = \{[150\Delta^2]^2 + [0.0018f^2(\text{terahertz})]^2\}^{1/2} \tag{7b}$$

if we are free to select the radial variation of two dopant materials and the central frequency of the source.

For a typical fibre, $\Delta \approx 0.01$. For a typical light emitting diode, $f \approx 9$ terahertz. Thus, according to Equation 7, $\sigma \approx 150$ ps/km. The maximum transmission capacity is therefore, Equation 6a, about 150 Mbit/s over a 10 km length.

3. Helical rays in circularly symmetric fibres

A wealth of information can be simply obtained by comparing the times of flight of pulses on helical rays, that is, on rays having a constant radius. It turns out that, for all practical purposes, it is sufficient to minimize the variation of t for helical rays.

The ratio of the axial length dz and ray length ds is given by [3]

$$(\mathrm{d}z/\mathrm{d}s)^2 = 1 + (r/k)\,\partial k/\partial r. \tag{8}$$

The time of flight per unit length is therefore

$$t(r, \omega) = [1 + (r/k)(\partial k/\partial r)]^{-1/2}(\partial k/\partial\omega). \tag{9}$$

Because k varies with r and ω, the time of flight t is, in general, a function of r and ω.

We shall investigate whether the following conditions

$$(1)\ t = t_0, \text{ at some } \omega$$
$$(2)\ t = t_0 \text{ and } \partial t/\partial\omega = \partial t_0/\partial\omega$$
$$(3)\ t = t_0 \text{ and } \partial t/\partial\omega = \partial t_0/\partial\omega = 0$$

optimizing the transmission capacity of the fibre for various sources of interest can be satisfied. In the above expressions, $t_0(\omega) \equiv t(0, \omega)$ denotes the time of flight of pulses propagating along the fibre axis.

3.1. Quasi-monochromatic sources

If the source is quasi-monochromatic, we seek to maintain the time of flight t as constant when the radius r of the helical path varies from zero (fibre axis) to the core radius a, the angular frequency ω being kept a constant: $t = t_0$. This condition is, from Equation 9

$$r\,\partial k/\partial r = k\{[(\partial k/\partial\omega)(dk_0/d\omega)]^2 - 1\}. \tag{10}$$

For any one-parameter class of material, the r.h.s. side of Equation 10 is a known function of k. Indeed, the dispersion $dn/d\lambda_0$ and the refractive index n can be measured for various concentrations of the doping material considered, for example germanium oxide, at some given optical frequency. On the basis of these measurements, we can plot $dn/d\lambda_0$ as a function of n, the dopant concentration playing the role of a parameter. We choose to consider $dn/d\lambda_0$ a function of n, or, equivalently, $\partial k/\partial\omega$ (the partial derivative of k with respect to ω at a fixed radius or dopant concentration) a function of k. The material on axis has been selected beforehand. The solution of Equation 10 is straightforward. The dependence of r on k must be

$$r/a = \exp\left[-\int_{k_c}^{k} \{1 - [(\partial k/\partial\omega)/(dk_0/d\omega)]^2\}^{-1}\, dk/k\right] \tag{11}$$

where we have introduced the cladding wavenumber $k_c \equiv (\omega/c)n_c$ from the conditions $r = 0$ when $k = k_0$ and $r = a$ when $k = k_c$. An equivalent form of Equation 11 is

$$(r/a)^2 = \exp\int_{2\Delta}^{N} [2\bar{N} - \bar{N}^2 - N]^{-1}\, dN \tag{12}$$

where we have defined relative phase and dispersion indices

$$N \equiv 1 - k^2/k_0^2 \tag{13a}$$
$$\bar{N} \equiv 1 - (\partial k^2/\partial\omega)/(\partial k_0^2/\partial\omega), \tag{13b}$$

which are more closely related to the index profile than k and $\partial k/\omega$. Other convenient forms for $\bar{N}$ are given in Appendix A.

Optimum profiles have been calculated according to Equation 12 for a fibre with 17% germanium oxide on axis and a boron-oxide doped cladding. The variation of $\bar{N}$ and N at $\lambda_0 = 1.06\ \mu m$ is shown in Fig. 1. The calculated optimum profile is shown in Fig. 2, where $h(x, y)$, in the present case, is $(r/a)^2$. It

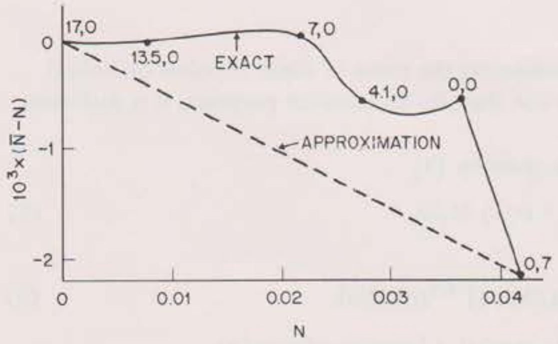

Figure 1 Plain line: Variation of the relative group index $\bar{N}$ as a function of the relative phase index $N \equiv 1 - n^2/n_0^2$ for various materials at $\lambda_0 = 1.06\ \mu m$. The dots correspond to measured points [2] with the concentrations of germanium and boron oxide in mole percent shown near the dots. The dashed line corresponds to the linear approximation.

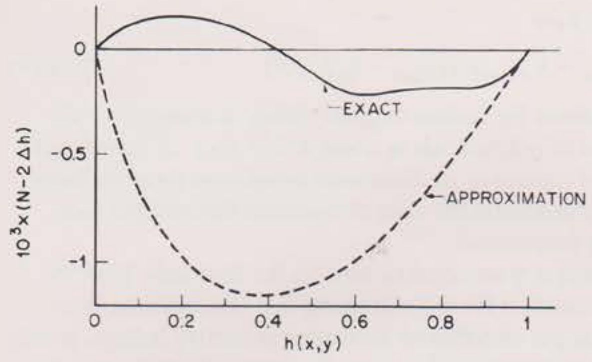

Figure 2 Plain line: Deviation of the optimum profile from a square-law profile, for the class of materials in Fig. 1. For the special case of circularly-symmetric fibres, set $h(x, y) \equiv (r/a)^2$, where a denotes the core radius. The dashed line follows from the linear approximation.

differs considerably from the profile obtained by assuming that $\bar{N}$ is proportional to N. The approximate (power-law) profile is shown by a dashed line in Fig. 2.

3.2 Deviations from optimum profiles

The profile $r(k)_{opt}$ that equalizes the times of flight for all helical rays and quasi-monochromatic sources is given in Equation 11. Let us call d the deviation of $(r/k)(\partial k/\partial r)$ from its optimum value $(r_{opt}/k)/(\partial r_{opt}/\partial k)$. It readily follows from Equation 9 that the time of flight per unit length deviates from a constant by $-d/2$. Let, for example, the refractive index profile $n(r)$ deviate from the optimum profile $n_{opt}(r)$ by

$$n(r) - n(r)_{opt} = n_0\,\delta\,\sin(2\pi N_0 r/a) \tag{14}$$

where $\delta \ll 1$ and N_0 represents the number of periods along the radius. After rearranging we find that the maximum variation of t is approximately

$$\Delta t = \pi N_0 \delta. \tag{15}$$

Thus Δt varies linearly with the number of periods N_0 if the amplitude δ is a constant. For large values of N_0, however, wave optics effects, that are not accounted for by the result in Equation 15, become significant.

3.3 Two (or more) quasi-monochromatic sources

Let us suppose that we want to transmit signals at two different optical frequencies, say ω_1 and ω_2, with $\omega_1 - \omega_2 \ll \omega_1 + \omega_2$. Because of their frequency separation, these two signals can be combined or separated, at least in principle, with the help of interference filters or gratings. We seek to design a fibre that has optimum transmission capacity at these two frequencies. The relative frequency separation being small, it is sufficient to ensure that the condition Equation 10 is satisfied in the region of the central frequency. In addition to the condition $t = t_0$, we require that $\partial t/\partial\omega = dt_0/d\omega$. When these conditions are fulfilled, pulses at different carrier frequencies do not broaden greatly, but they may arrive at different times.

In order to satisfy the conditions set up above, we need materials whose optical properties depend on at least two parameters. for example the concentrations of two dopants. Let these two parameters be denoted as u, and v. The wavenumber k is considered a function of u, v and ω, while u and v are unknown functions of the radius r.

If we set $\rho \equiv \log r$, and denote partial differentiations with respect to ρ, ω, u or v by subscripts (e.g., $k_\rho \equiv \partial k/\partial\rho \equiv r\partial k/\partial r$), Equation 10 is

$$k_u u_\rho + k_v v_\rho = k(k_\omega^2/k_{0\omega}^2 - 1) \equiv F(u, v, \omega) \tag{16a}$$

and differentiation with respect to ω gives

$$k_{\omega u} u_\rho + k_{\omega v} v_\rho = F_\omega. \tag{16b}$$

The solution of Equation 16 is straightforward. We have

$$\mathrm{d}v/\mathrm{d}u = v_p/u_p = -(k_{\omega u} - k_u F_\omega/F)/(k_{\omega v} - k_v F_\omega/F). \tag{17}$$

Once the Sellmeier-law coefficients have been measured for various sampled values of u and v (see Appendix), we can express k, k_ω and $k_{\omega\omega}$ in the form of polynomials in u and v. The r.h.s. of Equation 17 is then a known function of u and v at some central frequency ω. Numerical integration gives the curve in the u, v plane that satisfies Equation 16. This curve defines the class of materials that ensures that optimum profiles remain optimum at neighbouring frequencies.

Let us assume for convenience that the parameters u, v are equal to zero on the fibre axis. Near the axis, Equation 17 turns out to be a second degree equation for u/v. Assuming that this equation has real solutions, we can proceed away from the axis in the direction of decreasing refractive indices. It may happen that at some point along the path in the u, v plane the index stops decreasing. In that case a limit value for Δ is reached.

We have verified that the condition in Equation 17 can be satisfied for a mixture of phosphor oxide, germanium oxide and silica at the frequency where $\mathrm{d}\tau_0/\mathrm{d}\omega = 0$. Related results have been obtained by Kaminow and Presby (in preparation). Their results, however, are based on the rather restrictive assumption that the refractive index is proportional to dopant concentration at any wavelength.

4. General rays. Non circularly symmetric profiles

In the previous section we have restricted ourselves to helical rays. The question may be raised whether it is sufficient to equalize helical rays. This as we shall show is approximately the case. The formalism used in the present section can handle non-circularly symmetric profiles without further complication. We shall therefore treat the general case.

The equations that describe the motion of optical pulses in inhomogeneous media (the space-time Hamilton equations) are given in [5]. In that reference, it is shown (Equations 2 and 4) that the time of flight t of a pulse along a ray, and the axial wave number (or propagation constant) k_z are given, respectively, by*

$$k_z t = \frac{1}{2}\int_0^L (\partial K/\partial\omega)\,\mathrm{d}z \tag{18}$$

and

$$k_z^2 = L^{-1}\int_0^L [K + X(\partial K/\partial X) + Y(\partial K/\partial Y)]\,\mathrm{d}z. \tag{19}$$

In Equations 18 and 19, L denotes the length of the fibre and we have defined

$$X \equiv x^2; \quad Y \equiv y^2; \quad K \equiv k^2. \tag{20}$$

In these equations, K and its derivatives are functions of X, Y and ω. X and Y in turn, are known functions of z once a ray trajectory $X = X(z)$, $Y = Y(z)$ has been specified. Thus, the integrands in Equations 18 and 19 are functions of z. It follows that any linear combination of $k_z t$ and k_z^2 can be expressed as an integral over z.

Let us set

$$ak_z^2 + bk_z t + c = 0 \tag{21}$$

and define the three constants a, b, c from the condition that the variation of t when k_z varies be minimal. When k_z is smaller than the cladding wavenumber $k_c \equiv (\omega/c)n_c$, where n_c is the cladding index, the optical power for the ray considered leaks out. We may therefore restrict ourselves to rays that have $k_z > k_c$. On the other hand, the maximum value of k_z is the wavenumber on axis k_0. Thus, the permissible variation of k_z is

$$k_c < k_z < k_0. \tag{22}$$

*Marcatili's result [4] is essentially the ratio of Equations 18, and 19, the integration being performed over r instead of z. To make this transformation, note that $\mathrm{d}r/\mathrm{d}z = k_r/k_z$, where k_r and k_z denote respectively the radial and axial components of the wave vector.

Taking into account the constraint that t must be equal to the time of flight t_0 of pulses along the fibre axis when $k_z = k_0$, it is not difficult to show that the total change of t in Equation 21 is a minimum (and therefore the impulse response width is a minimum) if $t = t_0$ when $k_z = k_c$. This condition defines the values of a, b, c in Equation 21 to within a common factor. Equation 21 becomes

$$k_z^2 - (k_0 + k_c)(t/t_0)k_z + k_0 k_c = 0. \tag{23}$$

Using Equations 18 and 19, we find that this relation is satisfied if

$$\int_0^L \{\tfrac{1}{2}(k_0 + k_c)(L/t_0)(\partial K/\partial\omega) - [K + X(\partial K/\partial X) + Y(\partial K/\partial Y)] - k_0 k_c\}\,\mathrm{d}z = 0. \tag{24}$$

This is the case, in particular, if the integrand in Equation 24 is equal to zero. If we introduce the relative phase index N and the relative dispersion index $\bar{N}$ defined in Equation 13, we find after rearranging that the integrand in Equation 24 vanishes when

$$X(\partial N/\partial X) + Y(\partial N/\partial Y) = (1 + \chi)\bar{N} - N \tag{25a}$$

where

$$\chi \equiv k_c/k_0 = n_c/n_0. \tag{25b}$$

In general, $\bar{N}$ as well as N, may be a complicated function of X and Y. To simplify the solution of Equation 25 we consider materials that can be specified by a single parameter, say d. This is the case, for example, when the dopant concentrations $d_1, d_2, \ldots$ have the form $d_1 = d_1(d)$, $d_2 = d_2(d) \ldots$ where the $d_i(d)$ are arbitrary functions of the parameter d. We assume that when $\bar{N}$ is plotted against N for various values of the parameter d, a unique value of $\bar{N}$ corresponds to a given value of N, that is, $\bar{N}$ can be considered a function of N. Because $\bar{N}$ is a function of N alone, Equation 25 is a partial differential equation for $N(X, Y)$ whose solutions provide the optimum profiles for quasi-monochromatic sources. They are obtained by setting $N = N(h)$ where $h = h(x, y)$ is a homogeneous function of degree 2 in x and y (or of degree 1 in X and Y),* as in Equation 31 below.

Substituting this form for N in Equation 25 and using the Euler theorem* we obtain

$$\frac{\mathrm{d}N}{\mathrm{d}h}h = (1 + \chi)\bar{N} - N \tag{26}$$

which can be integrated in the form

$$h(x, y) = \exp\left\{\int_{2\Delta}^{N} [(1 + \chi)\bar{N} - N]^{-1}\,\mathrm{d}N\right\}. \tag{27}$$

The lower limit of integration is taken to be $N = 2\Delta$, where 2Δ denotes the value assumed by N at the cladding.

Thus, we have $h(x, y) = 1$ when $N = 2\Delta$. On the other hand, we have $h(x, y) = 0$ for $N = 0$ because the integral diverges as N (and $\bar{N}$) approaches 0.

In the special case where $\bar{N}$ is proportional to N, an assumption made implicitly in [5, 6],

$$\bar{N} = D_\kappa N \tag{28}$$

where D_κ is a constant. It is not difficult to show that stating the proportionality of $\bar{N}$ to N is equivalent to stating the linearity of $\mathrm{d}(n^2)/\mathrm{d}\lambda_0$ as a function of n^2, or approximately, for very weakly guiding fibres, the linearity of $\mathrm{d}n/\mathrm{d}\lambda_0$ as a function of n. Thus, we obtain from Equation 27

$$N(x, y) = 2\Delta h(x, y)^\kappa \tag{29}$$

where the exponent κ is related to D_κ by [5]

*A function $f(x, y)$ is said to be homogeneous of degree κ in x and y if $f(\lambda x, \lambda y) = \lambda^\kappa f(x, y)$ for any λ. The Euler theorem on homogeneous functions says that $(x\partial f/\partial x + y\partial f/\partial y)/f = \kappa$. This theorem follows from differentiation of the defining equation with respect to λ and setting $\lambda = 1$.

$$D_\kappa/(1+\kappa) = 1/(1+\chi). \tag{30}$$

When the linear approximation of Equation 28 holds, the optimum relative index profile is a homogeneous function of degree 2κ in x and y, where κ is defined in Equation 30 and $\chi = n_c/n_0$ for minimum impulse response width. The degree κ is unity when $D_\kappa = 1$ (no material dispersion) and $\chi = 1$ (very weakly guiding fibres, $n_c \approx n_0$).

In general we integrate Equation 27 numerically. The function $h(x, y)$ can be, for example

$$h(x,y) = (x/a)^2 + (y/b)^2 \tag{31}$$

or

$$h(x,y) = (r/a)^2. \tag{32}$$

For circularly symmetric profiles, the condition of Equation 27 (which reduces the quasi-monochromatic r.m.s. impulse response width to a value as small as σ(ns/km) $\approx 150\,\Delta^2$) coincides with Equation 12 (which equalizes exactly helical rays) if we make the approximations $\chi \approx 1$, $\bar{N}^2 \approx 0$. These approximations are valid for weakly guiding fibres, $\Delta \ll 1$.

5. Conclusion

Index profiles that reduce the quasi-monochromatic r.m.s. impulse width of multimode fibres to a value as small as $150\Delta^2$ ns/km can be found from a simple integration. No assumption has been made concerning the dispersion of the material. This result reduces to a previous result of Marcatili for the special case of circularly symmetric profiles. We have outlined a procedure to find dopant materials such that optimum profiles remain optimum at neighbouring frequencies. We find that the transmission capacity of multimode fibres excited with LED sources can be as large as 150 Mbit/s over 10 km lengths if the central frequency can be selected. This is at least ten times better than that achieved at the time of writing. How much effort will be needed to approach the theoretical limit is a question difficult to answer at the moment.

Acknowledgement

The author expresses his thanks to E. A. J. Marcatili, I. Kaminow and H. Presby for informing him of their results, prior to publication.

Appendix A. Evaluation of the relative dispersion index $\bar{N}$

A relative phase index N and a relative group index $\bar{N}$ were introduced in the main text

$$N \equiv 1 - k^2/k_0^2 = 1 - n^2/n_0^2 \tag{A1a}$$

$$\bar{N} \equiv 1 - (\partial k^2/\partial\omega)/(\partial k_0^2/\partial\omega) \tag{A1b}$$

where $k = (\omega/c)n$ denotes the wavenumber, ω the angular frequency, and k_0 and n_0 the values of k and n respectively, on the axis.

Let the variations of the refractive index n with the free-space wavelength λ_0 be defined by a three-term Sellmeier law

$$n^2 - 1 = \sum_{\gamma=1}^{3} A_\gamma(1-\pi_\gamma)^{-1}; \qquad \pi_\gamma \equiv (l_\gamma/\lambda_0)^2 \tag{A2}$$

According to (Equation A2) each material is characterized by six coefficients, namely, A_1, A_2, A_3, l_1, l_2, l_3.

Let a dispersion index (not to be confused with the group index) be defined by

$$\bar{n} = n\sqrt{v/u} \tag{A3a}$$

where

$$v/u = (\omega/k)(dk/d\omega) \tag{A3b}$$

denote the ratio of phase to group velocity. In terms of the Sellmeier coefficients, we have

$$\bar{n}^2 - 1 = \sum_{\gamma=1}^{3} A_\gamma(1-\pi_\gamma)^{-2} \tag{A4}$$

It is not difficult to show that $\bar{N}$ is related to $\bar{n}$ by the simple relation

$$\bar{N} = 1 - \bar{n}^2/\bar{n}_0^2 \tag{A5}$$

where $\bar{n}_0$ is the value of $\bar{n}$ on axis.

References

1. M. HORIGUCHI and H. OSANAI, *Elect. Lett.* **12**, (1976) 310.
2. J. W. FLEMING, *Fall Meeting of the American Ceramic Society*, Pocono Manor (Penn.), (October, 1975).
3. S. KAWAKAMI and J. NISHIZAWA, *IEEE Trans.* **MTT-16** (1968) 814–18.
4. E. A. J. MARCATILI, (in preparation).
5. J. A. ARNAUD, *Bell Syst. Tech. J.* **54** (1975) 1179–1205.
6. R. OLSHANSKY and D. B. KECK, *Appl. Opt.* **15** (1976) 483–91.