

Optimization of the index profile of an optical fiber at N wavelengths

J. Arnaud and A. M. Desage

The paper provides simple expressions for the dopant concentrations that optimize the bandwidth of a fiber at a number of wavelengths. The solution is related to Olshansky's multiple α profiles. Numerical application is made for germania, fluor, and phosphorus doping at wavelengths where the losses are small. The effect of a free parameter on the fiber Δ and its bandwidth is outlined.

1. Introduction

It becomes increasingly important for wavelength multiplexing to design multimode fibers that provide large transmission capacities over a broad spectral range. It is well known that a fiber doped with only one dopant such as germania can be optimized at only one carrier wavelength and that the transmission capacity of the fiber drops fairly quickly with increasing distance from that wavelength.¹ If, however, two dopants are used, such as germanium and fluorine, it may be possible to optimize the fiber, not only at one particular wavelength but also in the neighborhood of that wavelength. An exact procedure for achieving this result has been presented in Ref. 2. We exhibit here the class of analytical solutions that results from the exact equations when we introduce the approximation that the index is a linear function of the dopant concentrations, an approximation used also in Refs. 3 and 4. The class of solutions that we find, related to Olshansky's multiple- α solution, depends on one free parameter. The limitations in the fiber Δ as a function of that free parameter will be outlined. Preliminary results were in Ref. 5.

We shall use the paraxial form of the ray equations. As a result, the optimum profile in the absence of dispersion is found to be a square-law profile [$n(r) \sim r^2$], while the exact result is $n(r) \sim r^{2\kappa}$, with $\kappa = 1 - 6\Delta/5$. For the usual Δ values, $\Delta \approx 1\%$; however, the correction term $6\Delta/5$ is too small to be of practical signifi-

cance. Likewise, for the optimum profile, we find an unlimited bandwidth, while the exact result is of the order of 20 GHz · km for the usual Δ values. There again, random departures from the optimum profile are such, with present fabrication techniques, that the paraxial ray theory appears to be sufficiently accurate at the moment. The method described in this paper is applicable also to the exact ray equations, but the results are less transparent. This is also for pedagogical purposes that we first describe the time-of-flight equalization for helical rays, that is, for rays keeping a constant distance r from the axis. But we later show that the optimum profile obtained is also optimum for all rays (or all modes), and, furthermore, we generalize the result to noncircularly symmetric-index profiles.

We have introduced in our work the so-called linear approximation, according to which the refractive index is at any wavelength a linear function of the dopant concentrations (or rather, less stringently, a linear function of any monotonic functions of the dopant concentrations). For a single dopant this amounts to saying that $dn/d\lambda$ varies linearly with n as the dopant concentration varies. While this approximation is reasonable for small dopant concentration, it is opened to question for a large dopant concentration (≥ 10 mole %?). The existing experimental data are too inaccurate to enable us to evaluate the nonlinear effect. This justifies the present use of the linear approximation (made also by virtually all authors to date).

Within the linear approximation, there is a strong discrepancy between authors for the variation with wavelength of the optimum $\alpha = 2\kappa$, profile exponent for phosphoric oxide doping (see Fleming⁶ vs Shibata⁷). Due to this uncertainty we have based our calculations on the most recent measurements⁷ for that dopant.

A word is in order concerning the arbitrary parameter θ used later in our work. It is a fact, already recognized by Olshansky, that once two dopants have been specified, the optimum profile is not yet uniquely deter-

The authors are with Universite de Limoges, U.E.R. des Sciences, Laboratoire d'Electronique des Microondes, 87060 Limoges Cedex, France.

Received 3 April 1982.

0003-6935/82/234264-07\$01.00/0.

© 1982 Optical Society of America.

mined. The solution of the equations depends mathematically on an arbitrary parameter that we label θ . If $\theta = 0$, we get the solution proposed by Kaminow and Presby,³ and usually (but not always) one gets for $\theta = 0$ the largest Δ values for the fiber. For $\theta = \infty$, the two dopants are in fact counteracting, and usually the fiber Δ is small (yet positive, so that the solution remains physically acceptable). Any intermediate θ value is permissible. The choice of θ thus depends on technological considerations that are not discussed in detail in our paper. It seems that nonzero θ values are of interest.

II. Basic Theory

We shall restrict ourselves to paraxial ray theory. Furthermore, we shall neglect the dispersion of the material on-axis and discuss later the effect of such a dispersion. Let $n(x, y, f)$ denote the refractive index of the fiber material at a point x, y of the cross section and optical frequency f . Next we define a normalized index

$$U(x, y, f) = 1 - n(x, y, f)/n_0 \quad ; \quad n_0 = n(0, 0, f), \quad (1)$$

which we call the potential by analogy with the mechanical problem.

Let us first consider a circular fiber, where n depends only on the radius r , $U(x, y, f) = U(r, f)$, $r = (x^2 + y^2)^{1/2}$, and give special consideration to helical rays, that is, to rays that remain at a constant distance from the axis. The time of flight $t(r, f)$ of pulses traveling along such rays is given by (see Appendix A)

$$\tau = U_\rho - U_\nu - U_r, \quad (2)$$

where we have made the changes of variable

$$\rho = \log(r^2) \quad ; \quad \nu = \log f \quad (3)$$

for later convenience, and τ is a relative time of flight

$$\tau = t(r, f)/t_0 - 1 \quad ; \quad t_0 = t(0, f). \quad (4)$$

[For helical rays, $\rho = \text{constant}$ along the trajectory so that the average in Eq. (A7) is unnecessary.] Thus U is considered a function of ρ and ν (instead of r and f), and U_ρ, U_ν denote partial differentiation of U with respect to ρ and ν , respectively, that is,

$$U = \frac{\partial U}{\partial \rho} = \frac{1}{2} r \frac{\partial U}{\partial r}, \quad (5a)$$

$$U_\nu = \frac{\partial U}{\partial \nu} = f \frac{\partial U}{\partial f}. \quad (5b)$$

The problem we seek to solve is to find combinations of dopants so that $\tau = 0$ at as many wavelengths as possible. As long as the dopant concentrations remain small, it is reasonable to assume that n is, at any wavelength (or frequency), a linear function of the dopant concentrations. This linear approximation is used in the next paragraph.

III. Linear Approximation

This approximation is expressed by the condition that $U(\rho, \nu)$ has the form

$$U(\rho, \nu) = \sum_{i=1}^N \Delta_i(\nu) u_i(\rho) \quad (6)$$

for N dopants. The u_i ($i = 1, 2, \dots, N$) denotes monotonic functions of the dopant concentrations, so that $u_i(0) = 0$. For example, u may be the difference between the dopant concentration at r in mole % and the concentration on axis. The Δ_i ($i = 1, 2, \dots, N$), on the other hand, is frequency-dependent coefficients. If we introduce the form Eq. (6) into Eq. (2) and specify that τ is equal to zero for all ρ and frequencies $\nu_1, \nu_2, \dots, \nu_N$, we obtain the following system of equations:

$$\sum_{i=1}^N \Delta_i(\nu_1) u_{i\rho} - \Delta_{i\rho}(\nu_1) u_i - \Delta_i(\nu_1) u_{i\nu} = 0;$$

$$\sum_{i=1}^N \Delta_i(\nu_N) u_i - \Delta_{i\nu}(\nu_N) u_i - \Delta_i(\nu_N) u_{i\nu} = 0, \quad (7)$$

where the subscript ν denotes differentiation with respect to ν . This system is conveniently written in matrix form

$$\Delta u_\nu = (\Delta + \Delta_\nu) u, \quad (8)$$

where we have defined the vector $u = (u_1, u_2, \dots, u_N)$, and the matrix

$$\Delta = \begin{bmatrix} \Delta_1(\lambda_1) & \dots & \Delta_N(\lambda_1) \\ \Delta_1(\lambda_N) & \dots & \Delta_N(\lambda_N) \end{bmatrix}. \quad (9)$$

Equivalently, provided Δ is nonsingular, we have

$$\frac{d\mathbf{u}}{d\rho} = \mathbf{M}\mathbf{u} \quad ; \quad \mathbf{M} = \mathbf{1} + \Delta^{-1}\Delta_\nu. \quad (10)$$

The solution of the first-order matrix equation in Eq. (10) is well known. We have⁸

$$\mathbf{u} = \sum_{i=1}^N \mathbf{V}_i e^{\rho \kappa_i} = \sum_{i=1}^N \mathbf{V}_i r^{2\kappa_i}, \quad (11)$$

where $\kappa_i, \mathbf{V}_i, i = 1, 2, \dots, N$ are, respectively, the eigenvalues and eigenvectors of the \mathbf{M} matrix, that is, the solutions of

$$\mathbf{M}\mathbf{V} = \kappa \mathbf{V}. \quad (12)$$

The \mathbf{V} being unnormalized, expansion coefficients are unnecessary. In the second expression in Eq. (11) we have replaced e^ρ by r^2 , since $\rho = \log r^2$. The proof of Eq. (11) is straightforward. As we can see, our solution is much simpler than that given by Olshansky in Ref. 4. The exact comparison is difficult, but the two solutions are probably physically equivalent.

The solution in Eq. (12) is physically meaningful if the eigenvalues κ_i are real positive. Note that the eigenvectors \mathbf{V}_i are defined only to within arbitrary factors. If the eigenvalues κ_i are all real and positive, we have at our disposal $N - 1$ parameters. (A common factor for all the \mathbf{V} is immaterial.) If only some of the κ_i are real and positive, the number of arbitrary parameters is correspondingly reduced. If we consider the case of two dopants only, $N = 2$, one of the eigenvalues is always close to unity because the correction $\Delta^{-1}\Delta_\nu$ to the unit matrix is small in all practical cases. It follows that the second eigenvalue is always real, but it may be negative. We shall discuss later the practical significance of negative eigenvalues. In the next section we

show that the basic result in Eq. (11) is applicable to arbitrary paraxial rays (not just helical rays) and also to some noncircular index profiles.

IV. Generalization

We now consider more generally noncircular index profiles

$$U(x, y, f) = 1 - n(x, y, f)/n_0 \quad ; \quad n_0 = n(0, 0, f). \quad (13)$$

The relative time of flight τ defined earlier can be evaluated from the integral [see Eqs. (A7) and (A9)]

$$\tau = \lim_{z \rightarrow \infty} \frac{1}{z} \int_0^z [1/2(x^2 + y^2) - U - U_\nu] dz, \quad (14)$$

$$\tau = 1/2 \left(x \frac{\partial U}{\partial x} + y \frac{\partial U}{\partial y} \right) - U - U_\nu. \quad (15)$$

As in the previous section, let us make the assumption (linear dispersion) that $U(x, y, f)$ has the form

$$U(x, y, f) = \Delta_1(f) u_1(x, y) + \dots + \Delta_N(f) u_N(x, y). \quad (16)$$

In that case, the expression for τ is, from Eqs. (15) and (16),

$$\tau = \sum_{i=1}^N \Delta_i 1/2 \left(x \frac{\partial u_i}{\partial x} + y \frac{\partial u_i}{\partial y} \right) - \Delta_i u_i - \Delta_{i\nu} u_i, \quad (17)$$

where $\nu = \log f$. We seek to satisfy the conditions

$$\tau(f_1) = \dots = \tau(f_N) = 0 \quad (18)$$

for N frequencies f_1, \dots, f_N . If we define the matrix

$$\Delta = \begin{bmatrix} \Delta_1(f_1) & \dots & \Delta_N(f_1) \\ \Delta_1(f_N) & \dots & \Delta_N(f_N) \end{bmatrix} \quad (19)$$

and vector \mathbf{u} by

$$\mathbf{u} = (u_1, \dots, u_N), \quad (20)$$

we can rewrite the conditions in Eq. (18) in the compact form

$$1/2 \left(x \frac{\partial \mathbf{u}}{\partial x} + y \frac{\partial \mathbf{u}}{\partial y} \right) - (\mathbf{1} + \Delta^{-1}\Delta_\nu) \mathbf{u} = \mathbf{0}, \quad (21)$$

where Δ_ν denotes the derivative of the Δ matrix with respect to ν .

Let us call \mathbf{V}_i, κ_i the eigenvectors and eigenvalues, respectively, of the matrix

$$\mathbf{M} = \mathbf{1} + \Delta^{-1}\Delta_\nu, \quad (22)$$

that is,

$$\mathbf{M}\mathbf{V}_i = \kappa_i \mathbf{V}_i. \quad (23)$$

Then, a solution of Eq. (21) can be written

$$\mathbf{u} = \sum_i \mathbf{V}_i h_i^{\kappa_i}, \quad (24)$$

where h_i denotes homogeneous functions of degree 2 in x and y . Indeed, using the Euler theorem on homogeneous functions, we have

$$1/2 \left(x \frac{\partial h_i}{\partial x} + y \frac{\partial h_i}{\partial y} \right) = h_i, \quad (25)$$

and thus, substituting the expression for \mathbf{u} in Eq. (24) in the conditions in Eq. (21), we obtain

$$\sum_i \kappa_i \mathbf{V}_i h_i^{\kappa_i} - \mathbf{M}\mathbf{V}_i h_i^{\kappa_i} = \mathbf{0}. \quad (26)$$

Each term of the sum vanishes because all the \mathbf{V}_i, κ_i are the eigenvectors and eigenvalues, respectively, of \mathbf{M} . Note that the $h_i, i = 1, \dots, N$ need not be all the same. For example,

$$h_1 = (x/x_c)^2 + (y/y_c)^2, \quad (27a)$$

$$h_2 = [(x/x_c)^4 + (y/y_c)^4]^{1/2}, \quad (27b)$$

are acceptable expressions.

Once the dopant concentrations that make τ vanish at the frequencies f_1, \dots, f_N have been found, the time of flight of pulses traveling on a particular ray at any frequency f is given by

$$\tau(\text{ray}, f) = \sum_i [(\kappa_i - 1)\Delta(f) - \Delta_\nu(f)] \cdot \mathbf{V}_i \bar{h}_i^{\kappa_i}. \quad (28)$$

The quantities $\bar{h}_i^{\kappa_i}$ are the z average of the κ_i powers of the homogeneous functions $h_i(x, y)$. They depend on the two constants of motion of the ray in a way not known analytically, in general, unless $N = 1$.

In circularly symmetric fibers, however, we may set $h = (r/r_c)^2$. There are helical rays that correspond to constant values of r , and, for such rays, h is a constant, and thus $h^\kappa = (r/r_c)^{2\kappa}$. If a ray oscillates between r_1 and r_2 it may be that the fine details of the ray trajectory between r_1 and r_2 are not very important for the evaluation of $h^\kappa(r)$, and thus we may perhaps use the trajectory applicable to square-law media.

In Eq. (28), $\Delta(f) = [\Delta_1(f), \dots, \Delta_N(f)]$ is a vector, not a matrix as in Eq. (19).

V. Equalization in the Neighborhood of a Frequency

Instead of equalizing the times of flight at two distinct frequencies f_1 and f_2 , we may want to equalize the times of flight at frequency f_0 and in the neighborhood of that frequency. These conditions are expressed by

$$\tau(r, f_0) = 0, \quad \partial^2 \tau / \partial f^2 |_{f=f_0} = 0, \quad (29)$$

$$\partial \tau / \partial f |_{f=f_0} = 0, \quad \partial^N \tau / \partial f^N |_{f=f_0} = 0.$$

The calculations are almost identical to the ones given in the previous sections. However, it is easier here to relate the two-dopant case to the behavior of each dopant taken independently. Under the linear approximation in Eq. (6) we have the same solution as in Eq. (11) if the Δ matrix in Eq. (10) is replaced by

$$\Delta = \begin{bmatrix} \Delta_1 & \dots & \Delta_N \\ \Delta_1^{(1)} & & \Delta_N^{(1)} \\ \Delta_1^{(2)} & & \Delta_N^{(2)} \end{bmatrix}, \quad (30)$$

where the superscript i denotes the i th derivative with respect to $\nu = \log f$, and all quantities are evaluated at $f = f_0$. Δ being a function of f_0 , upper dots will refer to derivatives with respect to f_0 .

Let us give now the explicit expressions for the case of two dopants and equalization of τ and $\partial \tau / \partial \nu$ only. The solution is provided by the system of equations

$$\begin{aligned} du_1/d\rho &= M_{11}u_1 + M_{12}u_2, \\ du_2/d\rho &= M_{21}u_1 + M_{22}u_2, \end{aligned} \quad (31)$$

where $M = 1 + \Delta^{-1}\dot{\Delta}$ or explicitly

$$\begin{aligned} M_{11} &= (\dot{\Delta}_1\dot{\Delta}_2 + \Delta_1\dot{\Delta}_2 - \dot{\Delta}_1\Delta_2 - \dot{\Delta}_1\Delta_2)/K, \\ M_{12} &= (\dot{\Delta}_2^2 - \Delta_2\dot{\Delta}_2)/K, \\ M_{21} &= (\dot{\Delta}_1\dot{\Delta}_1 - \Delta_1^2)/K, \\ M_{22} &= (\Delta_1\dot{\Delta}_2 + \Delta_1\dot{\Delta}_2 - \dot{\Delta}_1\Delta_2 - \dot{\Delta}_1\Delta_2)/K, \\ K &= \Delta_1\dot{\Delta}_2 - \Delta_2\dot{\Delta}_1. \end{aligned} \quad (32)$$

(Here the upper dots are equivalent to the superscripts 1 and the double upper dots to the superscript 2.)

It is instructive to consider also the problem from a different viewpoint. Let us assume first a linear relationship between dopant concentrations

$$u_2 = \eta u_1, \quad (33)$$

where η is, for the time being, a constant.

The fiber Δ (here Δ denotes as usual the relative change in refractive index, a scalar quantity) is then

$$\Delta = \Delta_1 u_1 + \Delta_2 u_2 = (\Delta_1 + \eta \Delta_2) u_1. \quad (34)$$

Since we are effectively dealing now with a single doping parameter (even though two doping materials are involved), we may use the well-known condition for profile optimization at one wavelength and define an optimum κ by

$$\kappa - 1 = \Delta^{-1}\dot{\Delta} = (\Delta_1 + \eta \Delta_2)^{-1}(\dot{\Delta}_1 + \eta \dot{\Delta}_2). \quad (35)$$

In general, the κ value defined in Eq. (35) varies with frequency, but one hopes to be able to make $d\kappa/df = 0$ at some selected frequency f_0 by a proper choice of η , which is so far undefined.

If only dopant 1 (respectively, dopant 2) were used, we would define optimum κ values as

$$\kappa_1 - 1 = \Delta_1^{-1}\dot{\Delta}_1 \quad (\text{dopant 1 only}), \quad (36a)$$

$$\kappa_2 - 1 = \Delta_2^{-1}\dot{\Delta}_2 \quad (\text{dopant 2 only}). \quad (36b)$$

It is convenient to introduce these quantities in Eq. (35). We obtain

$$\kappa' = (\kappa_1 + \eta \kappa_2)/(1 + \eta), \quad (37)$$

where we have set for brevity

$$\eta' = (\Delta_2/\Delta_1)\eta; \quad \kappa' = \kappa - 1. \quad (38)$$

Note that

$$\eta' = (\kappa_2' - \kappa_1')\eta'; \quad \eta' = -(\kappa - \kappa_1)/(\kappa - \kappa_2). \quad (39)$$

The condition $\dot{\kappa} = 0$ is, after rearranging, a second-degree equation for the optimum value of $\eta = u_2/u_1$:

$$\begin{aligned} A\eta'^2 + B\eta' + C &= 0, \quad B = \kappa_1 + \kappa_2 + (\kappa_2 - \kappa_1)^2, \\ A &= \kappa_2, \quad C = \kappa_1, \end{aligned} \quad (40)$$

whose solution is

$$\begin{aligned} \eta &= (\Delta_1/\Delta_2)[-(\kappa_1 + \kappa_2) - (\kappa_2 - \kappa_1)^2 \\ &\pm [(\kappa_1 - \kappa_2)^2 + (\kappa_2 - \kappa_1)^4 + 2(\kappa_1 + \kappa_2)(\kappa_2 - \kappa_1)^2]^{1/2}]/2\kappa_2. \end{aligned} \quad (41)$$

This is the final result. It relates the two optimum concentration ratios that equalize the times of flight of

all rays about some frequency f_0 and the close neighborhood of f_0 . This value of η is expressed in terms of the optimum κ values (κ_1, κ_2) that would be required if the two dopants were used independently and the frequency derivatives of these κ values.

From Eq. (41) we can write an equation for the optimum κ values, which coincide, of course, with the eigenvalues of the M matrix in Eq. (10).

We have for the optimum 2, dopant κ values:

$$\begin{aligned} \kappa &= [(\kappa_2 - \kappa_1 + \kappa_2^2 - \kappa_1^2) \\ &\pm [(\kappa_1 - \kappa_2)^2 \\ &+ 2(\kappa_1 + \kappa_2)(\kappa_1 - \kappa_2)^2 + (\kappa_1 - \kappa_2)^4]^{1/2}]/2(\kappa_2 - \kappa_1). \end{aligned} \quad (42)$$

The present formulation is useful because sometimes the data are given in terms of $\kappa_1(f)$ and $\kappa_2(f)$, measurements being made on fibers doped with a single dopant. Also, we can see clearly from Eq. (42) a number of interesting features. For example, if $\kappa_1 = \kappa_2$, that is, at the frequency at which the two $\kappa(f)$ single-dopant curves cross, there is only one solution for κ , namely, $\kappa = \kappa_1 = \kappa_2$. The dopant concentration ratio is either

$$u_2/u_1 = -\Delta_1/\Delta_2 \rightarrow U = 0, \quad (43)$$

which is of no practical interest, or

$$u_2/u_1 = -(\Delta_1/\Delta_2)(\kappa_1/\kappa_2). \quad (44)$$

The latter equation can be written

$$\kappa_1 u_1 \Delta_1 + \kappa_2 u_2 \Delta_2 = 0. \quad (45)$$

To understand better the physical significance of this equation, let us assume that u_1, u_2 are the dopant concentrations at $r = r_c$, the core radius. Then $u_1 \Delta_1$ is the total change of relative index due to dopant 1, and, similarly, $u_2 \Delta_2$ is the total change of relative index due to dopant 2. We see that the ratio of these concentration changes is weighted by the respective slopes κ_1 and κ_2 , a result that is intuitively understandable.

VI. Calculated Results

To calculate optimum dopant concentrations we have used the results of Fleming's measurement on doped silica samples.^{6,7}

The unshaded areas in Fig. 1 give realizable dopant combinations that optimize the fiber bandwidth at two wavelengths, λ_1 (horizontal scale) and λ_2 (vertical scale), with real positive exponents κ_1, κ_2 . As we can see, acceptable solutions are rather scarce. Figures 2 and 3 give curves in the dopant concentration plane that are physically acceptable. They are labeled by a parameter θ , which can be chosen arbitrarily from the point of view of the theory. The value of the relative index difference Δ in the fiber cross section, however, depends on θ (see Fig. 4), and, in general, θ will be chosen to maximize Δ (here $\theta_{\text{opt}} \approx -5$).

Figure 5 shows a more conventional result: the variation of the optimum exponents κ for fluor doping alone and phosphoric oxide doping alone. Note that not all the curves published in the literature agree with the ones presented, calculated from Ref. 6.

Figure 6 shows the pulse broadening Δt (bandwidth $\approx 1/\Delta t$) for one and two dopants. This figure gives only

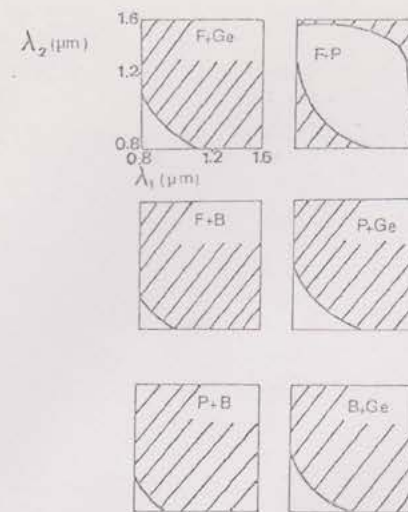


Fig. 1. Unshaded areas give the combination of wavelengths (λ_1, λ_2) at which the profile can be optimized. The combination of dopants considered is indicated on the corresponding figures. Data from Ref. 6. From Ref. 7 and P + Ge all wavelengths are in fact permissible.

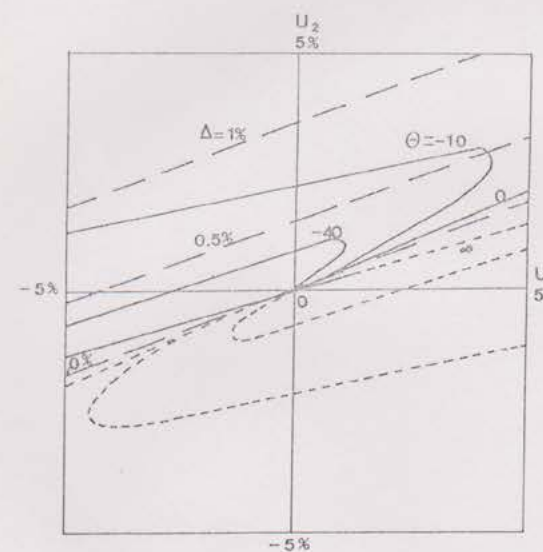


Fig. 2. Heavy lines give the permissible relationship between dopant concentrations (u_1 , phosphoric oxide; u_2 , fluor, in mole % referred to the values on-axis). The labels on the curves (0, -10, -40) refer to the arbitrary parameter θ . The dotted curves are unphysical branches. The dashed straight lines give the $U = 1 - n(r)/n_0$ values. The maximum U value is the fiber Δ (e.g., 1%). The dopant considered here are fluor and phosphoric oxide, and the two wavelengths are $\lambda_1 = 0.8$ and $\lambda_2 = 1.3$ μm .

an estimate of the fiber bandwidth, as only helical rays were taken into account for the sake of simplicity.

The case where one of the two exponents κ is negative is intriguing. Although n diverges near-axis ($r \rightarrow 0$), nevertheless the solution may be of some practical interest if we avoid exciting the central region of the fiber core. Figure 7 gives the permissible dopant concentration curves, Fig. 8 the normalized index profile, and Fig. 9 the fiber bandwidth ($\approx 1/\Delta t$) for that negative κ case. As we can see, such solutions are not as unac-

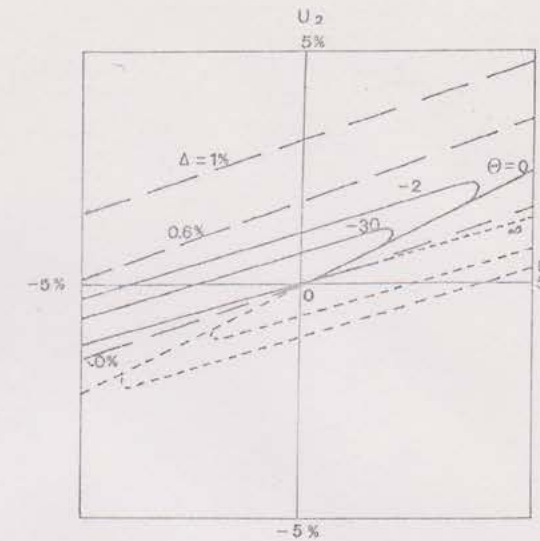


Fig. 3. Same as Fig. 2 for $\lambda_1 = 0.8$ μm and $\lambda_2 = 1.55$ μm .

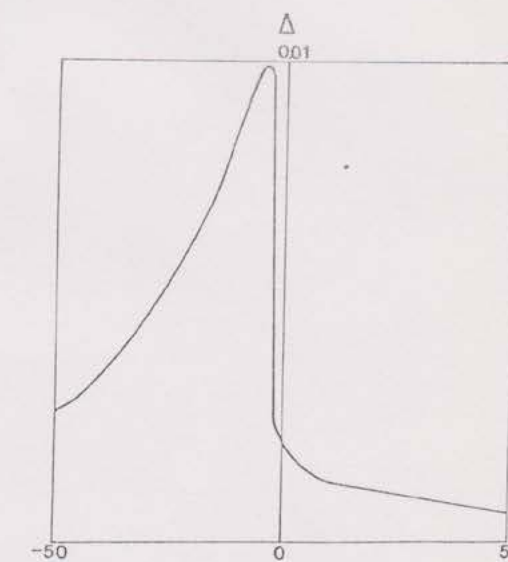


Fig. 4. Maximum values of Δ as a function of the arbitrary parameter θ for a combination of optimum wavelengths: $\lambda_1 = 0.8$ μm and $\lambda_2 = 1.3$ μm with phosphoric oxide and fluor.

ceptable as one may feel at first. If this is the case, the shaded areas in Fig. 1 become permissible.

VII. Conclusion

We have discussed in much more detail than was done previously the optimization at N wavelengths of the bandwidth of a multimode optical fiber excited by a quasi-monochromatic but spatially incoherent source, using N dopants. Although our result seems to be basically the same as the Olshansky ' $N - \alpha$ ' profile, our formulation is considerably simpler because of the use of matrix algebra. We show that these results follow from a more general formulation given earlier by one of us by introducing the approximation that the index is a linear function of the dopant concentration. We have outlined the effect of the free parameter θ on the overall

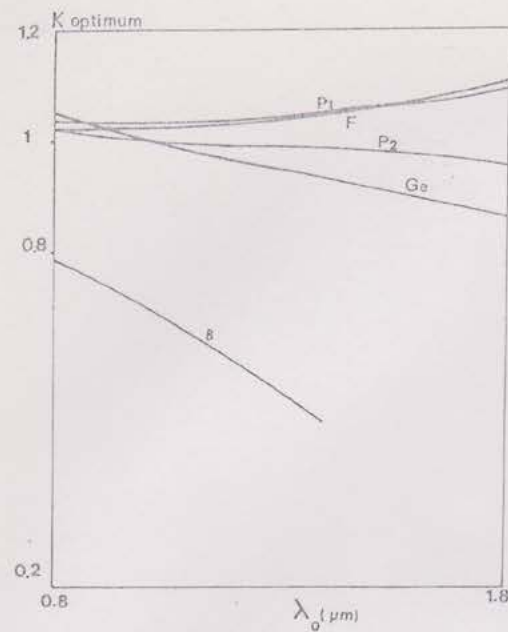


Fig. 5. Optimum profile exponent K_{opt} as a function of wavelength for fluor doping alone (F) phosphoric oxide alone (P_1 , Ref. 6; P_2 , Ref. 7), germanium doping alone (Ge) and boron doping alone (B).

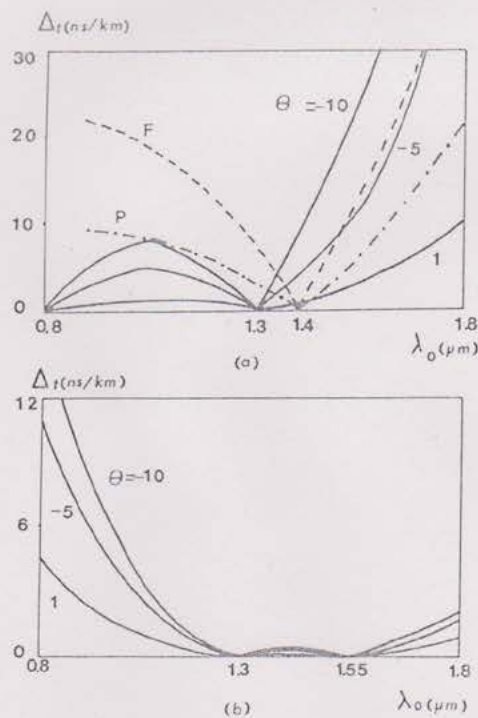


Fig. 6. (a) Pulse broadening as a function of wavelength for fluor alone and optimum wavelength at $1.4 \mu\text{m}$ (dashed curve), phosphoric oxide alone and optimum wavelength at $1.4 \mu\text{m}$ (dash-dot curve), and a combination of these two dopants for optimum bandwidth ($\sim 1/\Delta t$) at 0.8 and $1.3 \mu\text{m}$ and various values of the parameter θ (plain lines). (b) Same as in (a) for P + Ge from Ref. 7.

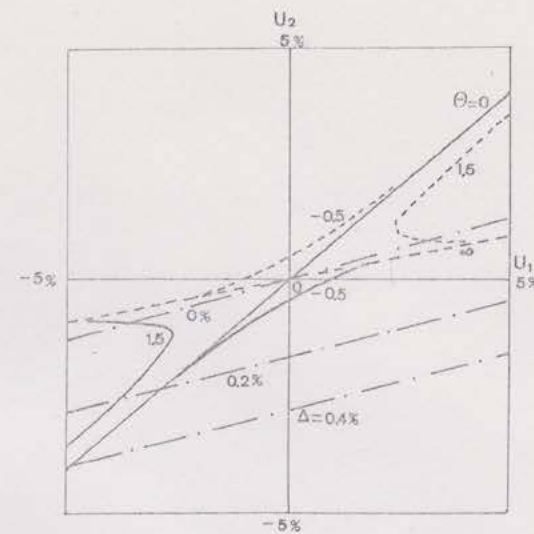


Fig. 7. Curve in the dopant concentration plane (u_1 , B_2O_3 ; u_2 , GeO_2 in mole % referred to values on-axis) for the case where one of the two κ exponents is negative. Other labels as in Fig. 2. The two wavelengths are $\lambda_1 = 0.8 \mu\text{m}$ and $\lambda_2 = 1.6 \mu\text{m}$.

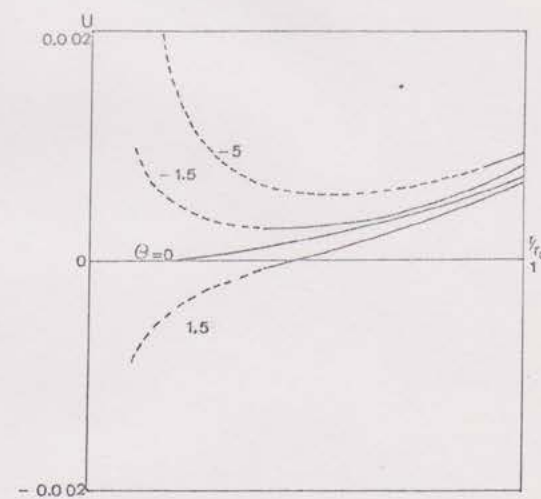


Fig. 8. Normalized index U as a function of normalized radius r/r_c for the case where one of the two κ exponents is negative. If $\theta \neq 0$ the profile is not realizable near-axis. Dashed curves are unphysical, because U must increase monotonically as a function of r (upper curves), and u_1, u_2 do not exceed $\pm 5\%$ (lower curves). We consider the two dopants boron and germanium and the wavelengths $\lambda_1 = 0.8 \mu\text{m}$ and $\lambda_2 = 1.6 \mu\text{m}$.

Δ of the fiber and on its bandwidth at frequencies other than the ones for which it has been optimized.

However, for the practical use of the method, it must be recognized that much uncertainty remains in the dispersion of the materials used, and the calculated profiles should be used with some caution. In that respect, the interferometric technique reported in Ref. 9 should be of great help.

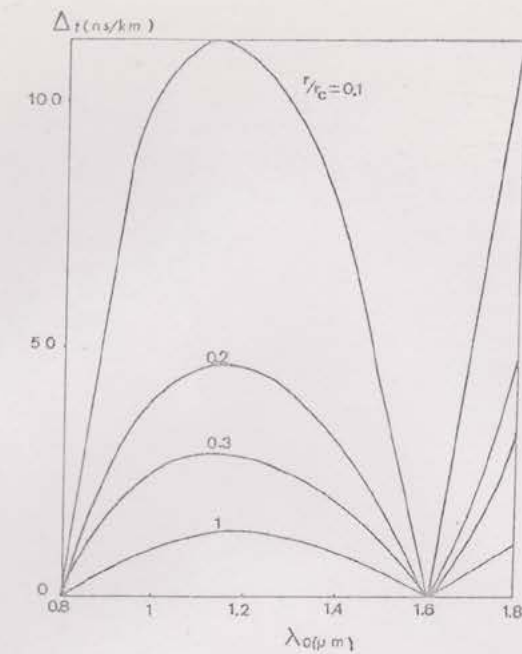


Fig. 9. Relative time of flight for helical rays of radius r as a function of wavelength for the case where one of the two exponents is negative (see Fig. 8) and $\theta = 0.5$. In contrast to the normal case, Δt increases as the ray radius decreases. The dopants are boron and germanium.

Appendix A. Expression for the Flight Time

The time of flight along a ray of trajectory $x = x(z)$, $y = y(z)$ is obtained by integration:

$$t(z) = \int_0^z u^{-1} ds, \quad (\text{A1})$$

where

$$ds = (dx^2 + dy^2 + dz^2)^{1/2} \approx dz [1 + \frac{1}{2}(x^2 + y^2)] \quad (\text{A2})$$

is the elementary arc length, u is the local group velocity,

$$c/u(x,y) = \frac{\partial}{\partial f}(fn) = n + n_r = n_0[(1 + n_{0r}/n_0)(1 - U) - U_r] \quad (\text{A3})$$

from the definition $U = 1 - n/n_0$ and $v = \log f$. Let us define a relative time of flight

$$\tau = t/t_0 - 1. \quad (\text{A4})$$

$$t_0 = z/u_0 = (z/c) \frac{d}{df}(fn_0) = (zn_0/c)(1 + n_{0r}/n_0) \quad (\text{A5})$$

denotes the time of flight of rays on-axis.

$$\tau + 1 = \lim_{z \rightarrow z} \frac{1}{z} \int_0^z [1 + \frac{1}{2}(x^2 + y^2)] \times [1 - U - U_r/(1 + n_{0r}/n_0)] dz. \quad (\text{A6})$$

Usually $n_{0r}/n_0 \ll 1$, and thus it will be omitted in the following. Neglecting terms of order Δ^2 ($U < \Delta$), we obtain

$$\tau = \overline{\frac{1}{2}(x^2 + y^2)} - U - U_r, \quad (\text{A7})$$

where the upper bar denotes the integral and limit in

Eq. (A6). It is convenient to rewrite the first term in Eq. (A7) in a different form.

From the equations of ray motion

$$\ddot{x} = -\frac{\partial U}{\partial x}; \quad \ddot{y} = -\frac{\partial U}{\partial y}, \quad (\text{A8})$$

it follows that

$$\frac{1}{4} \frac{d^2}{dz^2}(x^2 + y^2) = \frac{1}{2}(x^2 + y^2) - \frac{1}{2} \left(x \frac{\partial U}{\partial x} + y \frac{\partial U}{\partial y} \right). \quad (\text{A9})$$

Let us apply that result by considering helical rays $r^2 = x^2 + y^2 = \text{constant}$ in circularly symmetric, but otherwise arbitrary, profiles. We obtain

$$\frac{1}{2}(x^2 + y^2) = \frac{1}{2} \left(x \frac{\partial U}{\partial x} + y \frac{\partial U}{\partial y} \right) = dU/d\rho \quad \rho = \log(r^2). \quad (\text{A10})$$

If we use the expression in Eq. (A10) in Eq. (A7), we obtain the expression for τ used in the main text [Eq. (2)].

References

1. R. Olshansky and D. B. Keck, Appl. Opt. 15, 483 (1976).
2. J. Arnaud, Electron. Lett. 12, 654 (1976).
3. I. P. Kaminow and H. M. Presby, Appl. Opt. 16, 108 (1977).
4. R. Olshansky, Appl. Opt. 18, 683 (1979).
5. J. Arnaud and A. M. Desage, Electron. Lett. 17, 339 (1981).
6. J. W. Fleming, Electron. Lett. 14, 326 (1978).
7. N. Shibata and T. Eda, Trans. Inst. Electron. Commun. Eng. Jpn. E65, 166 (1982).
8. L. Pontriagin, A Course in Ordinary Differential Equations (Hindustan, India, 1967).
9. A. Barthelemy, P. Facq, C. Froehly, and J. Arnaud, Electron. Lett. 18, 247 (1982).