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OPTICALLY-PUMPED SEMICONDUCTOR SQUEEZED-LIGHT GENERATION

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ABSTRACT

The theory presented shows that light emitted by low-temperature semiconductors under intense optical pumping (with fluctuations at the shot-noise level: SNL) should be amplitude-squeezed down to half the SNL at nonzero frequencies. Amplitude squeezing may be obtained also at zero frequency when spontaneous carrier recombination is significant. It is essential that the optical gain depend on photon emission rate, e.g., as a result of spectral-hole burning. A commuting-number theory that agrees exactly with Quantum Theory for large particle numbers is employed. Comparison with results previously reported for 3-level atom lasers is made.

1 INTRODUCTION

It is desirable that the flow of photons emitted by lasers fluctuate as little as possible, in particular for the measurement of small absorptions. Quantum theories proposed by Golubev [1], Yamamoto [2], Haake [3], Benkert [4] and others, show that complete quieting can be achieved when the pump does not fluctuate. The basic concept is that under ideal conditions each injected excited-state atom or electron-hole pair generates a photon. Accordingly, nonfluctuating pumps entail nonfluctuating photon flows as long as particle storage can be ignored, i.e., at small baseband frequencies. Nonfluctuating electrical pumps consist simply in a battery and a resistor.

Because many semiconductors cannot be electrically pumped, it is important to determine whether squeezed-light may be obtained with optical pumping whose fluctuations are usually at the shot-noise level (SNL). The previous discussion shows that light fluctuations are in that case, at best, at the SNL at zero frequency if spontaneous carrier recombination is negligible. But below-shot-noise operation is possible at nonzero frequencies. A quantum theory by Kolobov and others [5] indicates that this is the case for 3-level-atom lasers. Below-shot-noise light fluctuations may occur also at zero frequencies provided spontaneous carrier recombination is significant.

It is important to examine whether results similar to those reported in [5] hold for semiconductors. The theory presented in [6] that takes spectral-hole-burning (SHB) into account suggests that complete quieting of the light emitted by optically-pumped semiconductors could indeed be obtained. When statistical fluctuations of the optical gain are considered however [7], optimum squeezing turns out to be only half the SNL.

A more complete theory is given in this paper. The single-mode semiconductor laser is supposed to operate near peak gain, i.e., the gain does not depend importantly on frequency, and strong index confinement is assumed. Under such conditions frequency fluctuations do not react back on amplitude fluctuations. Because the laser operates well above threshold, fluctuations are small compared with steady-state values.

For the present situation of large particle (electrons and photons) numbers, the full Quantum formalism is not required. It suffices to employ a commuting-number theory [8]-[9] that generalizes Lax's semiclassical theory [10]. Amplitude noise is the sum of two contributions. a) an intrinsic contribution that occurs with quiet pumps and is due to atomic quantum jumps. b) an extrinsic contribution expressing the system response to pump fluctuations. The two contributions are independent and add up. In the case of semiconductors in which SHB is significant the gain depends on the photon generation rate. Furthermore, statistical fluctuations of the gain, to be discussed later, should in that case be considered.

The intrinsic contribution vanishes at zero baseband frequency as we discussed before. It grows as a function of frequency because of photon storage, approaching the SNL at frequencies of the order of the cold-cavity linewidth. (This simplified picture that ignores carrier storage is valid at high ouput-power levels). The pump-fluctuation contribution is proportional to the amplitude-modulation response of the laser. It decays slowly as a function of frequency when SHB can be neglected, but drops quickly when SHB effects are present. Considering the two contributions

together, the reduced modulation bandwidth entails a dip below the SNL at moderate frequencies. Below SNL operation with a pump fluctuating at the SNL thus requires gain compression.

An equivallent electrical circuit applicable to semiconductor lasers, redrawn in Fig.1 from [6], helps us visualize what is happening. For simplicity, spontaneous carrier recombination is omitted. The schematic involves primarily two resistances: a negative resistance: $-(1+\beta)$, where β is proportional to the SHB effect, relating to the nonlinear emitting element, and a positive unity resistance relating to the linear absorber (perhaps a detector). The relative pump-rate fluctuation $\Delta J/J$ is represented by the current source on the left, while the photon rate fluctuation $\Delta Q/Q$ is represented by the current on the right. The average values of J and Q are equal. It follows from the schematic that $\Delta Q = \Delta J$ at zero frequency irrespectively of SHB since the capacitances can be ignored, in agreement with the discussion given at the beginning of this introduction.

Consider next nonzero frequencies. The capacitance C_c, which is equal to the cold-cavity photon lifetime τ_p, can be neglected at the frequencies presently considered. The negative capacitance C_e expressing carrier storage is inversely proportional to $\partial G/\partial N$, the partial derivative of the optical gain G with respect to carrier number N. At low temperatures $\partial G/\partial N$ vanishes because the states are fully occupied and C_e therefore tends to infinity. This implies that pump fluctuations get short-circuited and are unconsequential at nonzero frequencies. But the laser can oscillate stably only as a result of gain compression, e.g., SHB.

In the equivallent circuit the intrinsic noise sources are represented by two independent voltage sources whose spectral densities are independent of SHB [6]. One of these sources (v_e) represents dipole noise while the other (v_a) represents vacuum fluctuations. Since the total resistance of the circuit is equal to the SHB parameter β, the electrical current ($\Delta Q/Q$) representing amplitude noise vanishes in the large-β limit. Complete quieting of the light beam is then expected at $f \neq 0$, as indicated earlier, but statistical fluctuations of the gain are not considered in this equivalent circuit.

Laser oscillator noise sources are described in Section 2. The population inversion factor is evaluated in Section 3 for semiconductors. Amplitude noise of an optically pumped semiconductor laser at T = 0K is derived in Section 4. It is here essential that the optical gain depend on photon rate, perhaps because of SHB. Nonzero temperatures are considered in Section 5. Our conclusion in Section 6 is that semiconductors under intense optical pumping should be able to generate light with fluctuations at approximately half the SNL at moderate baseband frequencies.

Appendix A explains the physical origin of the intrinsic noise sources, and Appendix B the nature of the gain statistical fluctuations. Appendix C gives a classical derivation of 3-level laser amplitude noise.

Let us clarify our notation. Particle rates (number of particles emitted or absorbed per unit time) such as the output photon rate Q(t) consist of an average part $\langle Q \rangle$ and a fluctuation $\Delta Q(t)$ of zero mean. Brackets indicating time-average values are omitted when no confusion may occur. The quantity

$$X \equiv \langle Q \rangle^{-1} S_{\Delta Q}(f) \tag{1}$$

where $S_{\Delta Q}$ denotes the double-sided spectral density of ΔQ and f the baseband frequency, is unity at the SNL. We will denote by q(t) the intrinsic noise source, a white (i.e., flat spectrum) random function of time. The measured noise ΔQ expresses the system response to intrinsic noise sources relating to stimulated emission and absorption, as well as to pump and spontaneous-carrier-recombination fluctuations. For particle numbers, such as the photon number m, we also consider an average part $\langle m\rangle$ and a zero mean fluctuation Δm. In Appendix C statistical fluctuations are denoted by δ to distinguish them from total fluctuation Δ.

A normalized baseband frequency

$$\Omega \equiv f/f_o, \qquad f_o \equiv 1/2\pi\tau_p \tag{2}$$

is defined, where τ_p denotes the cavity photon lifetime and f_o the cold-cavity linewidth.

2 LASER OSCILLATOR NOISE SOURCES

Laser action takes place between two levels separated in energy approximately by hν (where h denotes Planck's constant and ν the oscillation frequency) called respectively "emitting" (upper) level and "absorbing" (lower) level. The rate at which electrons are injected in the emitting level or extracted from the absorbing level is denoted by J, the spontaneous decay rate by S and the net rate of photon emission in the oscillating mode by R.

In the steady-state

$$\langle J\rangle - \langle S\rangle = \langle R\rangle = \langle Q\rangle \tag{3}$$

if the internal optical losses are neglected.

Let us now consider in succession relations applicable to the optical cavity, to the (linear) absorber and to the emitter. The rate dm/dt at which the photon number m increases is the difference between the rate R at which photons enter the cavity and the rate Q at which photons leave the cavity (and eventually get absorbed in the detector)

$$dm/dt = R - Q \tag{4a}$$

On the average, $\langle R\rangle = \langle Q\rangle$ as given in Eq.(3). The fluctuating part of Eq.(4a) reads

$$j\,\Omega\,\Delta m/\tau_p = \Delta R - \Delta Q \tag{4b}$$

if we introduce the normalized frequency Ω defined in Eq.(2) and assume an $\exp(j\,2\pi\,f\,t)$ time dependence.

The absorber of radiation (e.g., a detector) is linear and cold. It may be viewed as a large collection of atoms, essentially all of them in the ground state. Only a comparatively insignificant number of atoms get excited under the influence of incoming light and, eventually, decay spontaneously to the ground state. Accordingly

$$Q = m/\tau_p + q \tag{5a}$$

The first term on the right-hand-side of Eq.(5a) expresses a deterministic linear relation between the absorbed photon rate Q and the photon number m while q(t) is an intrinsic noise term. Separating the average part from the fluctuating part we obtain

$$\langle Q\rangle = \langle m\rangle/\tau_p \tag{5b}$$

and

$$\Delta Q = \Delta m/\tau_p + q \tag{5c}$$

$$S_q = Q \tag{5d}$$

Equation (5d), where "Q" stands for the average value $\langle Q\rangle$, asserts that the intrinsic noise term q(t) fluctuates at the SNL. A detailed discussion of this key point is in Appendix A.

Expressions relating to the emitting element are analogous to Eq.(5). But there are now atoms in both the emitting and the absorbing states. Accordingly, there is a stimulated emission rate R_e and a stimulated absorption rate R_a, the net rate R being the difference between the two. These rates are the sums of deterministic terms, proportional to the photon number m (neglecting unity compared with m, which is of the order of 1000) and intrinsic fluctuations, according to

$$R_e \equiv G_e\,m + r_e \tag{6a}$$

$$R_a \equiv G_a\,m + r_a \tag{6b}$$

$$S_{re} = R_e\,, \qquad S_{ra} = R_a \tag{6c}$$

$$R = R_e - R_a = G\,m + r, \qquad G \equiv G_e - G_a\,, \qquad r \equiv r_e - r_a \tag{6d}$$

Because the $r_e(t)$ and $r_a(t)$ processes are independent, the spectral density of r is

$$S_r = R_e + R_a \equiv (2\,n_p - 1)\,R \tag{6e}$$

where we have introduced for later convenience the population inversion factor

$$n_p \equiv R_e/(R_e - R_a) \tag{6f}$$

and R, R_e and R_a are here understood as average values. The population-inversion factor (or spontaneous-emission factor) n_p in Eq.(6f) is unity when the population inversion is complete, i.e., when $R_a = 0$. Detailed expressions for n_p are given in the next section.

3 POPULATION INVERSION AND SHB FACTORS

In the present section all the quantities considered are understood as average values, i.e., noise is not considered.

Consider first 2-level atoms. If there are on the average n_e atoms in the emitting state and n_a atoms in the absorbing state in the optical cavity, the stimulated emission and absorption rates are proportional to these numbers, respectively, that is

$$R_e = A\, n_e\, m, \quad R_a = A\, n_a\, m \tag{7}$$

where A is a constant (the same constant A applies to stimulated emission and to stimulated absorption). Equation (6f) thus reads

$$n_p = n_e/(n_e - n_a) \tag{8}$$

In semiconductors the situation is more complicated. Each level pair corresponding to the same electronic momentum (one level in the conduction band and one in the valence band) is analogous to a 2-level atom. But as a result of Pauli's exclusion principle that prevents two electrons from occupying the same state (assuming that the spin degeneracy has been lifted), the collection of these "atoms" exhibits a large spread in energy spacings. Only one per-cent of them, approximately, has the energy spacing appropriate for interaction with the optical field at frequency ν, namely $h\nu$ to within approximately $h/2\pi\tau_i$, where τ_i denotes the intraband scattering time, of the order of 0.1 ps. If we denote by n_o the number of interacting states and by N the total carrier number, we have $n_o << N$. We are presently concerned with these n_o interacting states. The complication alluded to earlier stems from the fact that both levels of a pair may be unoccupied or both occupied, a situation that does not normally occur with atoms. These level pairs are optically inactive but the latter kind contributes to the total electron number, and therefore to the carrier dynamics.

Let f_c and f_v denote the probability of occupancy of the emitting and absorbing states, respectively. The average values of n_e and n_a are respectively

$$n_e = n_o f_c, \quad n_a = n_o f_v \tag{9}$$

Assuming that the occupation of the emitting and absorbing levels are independent processes

$$R_e = A\, n_o\, f_c\, (1 - f_v)\, m, \quad R_a = A\, n_o\, f_v\, (1 - f_c)\, m \tag{10a}$$

$$R = R_e - R_a \equiv G\, m \tag{10b}$$

$$G = A\, n_o\, (f_c - f_v) \tag{10c}$$

The general expression in Eq.(6f) for the population inversion factor reads now

$$n_p = R_e/(R_e - R_a) = f_c(1 - f_v)/(f_c - f_v) \tag{11a}$$

From now on we consider for simplicity symmetrical bands. Then $f_v = 1 - f_c$ and Eq.(11a) reads

$$n_p = f_c^2 / (2 f_c - 1) \tag{11b}$$

Net gain requires that f_c be larger than 1/2, and thus n_p is positive.

At low and moderate optical powers the carriers are in thermal equilibrium within their respective bands and the probability f_c is given by Fermi's statistics. It increases monotonically as a function of the total carrier number N, and eventually reaches the value unity.

At high power density, the thermal equilibrium condition no longer holds. f_c then depends not only on N but also on the rate at which electrons and holes recombine (SHB). This recombination rate is approximately equal to the photon generation rate R if one neglects spontaneous recombination of interacting carriers. The recombination rate per state is therefore R/n_o.

Within the relaxation-time approximation, the probability f_c obeys an equation of the form

$$df_c/dt = (f_{co} - f_c)/\tau_i - R/n_o \tag{12a}$$

where f_{co} is the thermal equilibrium value of f_c and τ_i is the intraband relaxation time. Because the intraband scattering time is of the order of 0.1 ps, the adiabatic approximation that amounts to neglecting df_c/dt is applicable.

At low temperatures f_{co} is unity and Eq.(12a) thus reads

$$f_c = 1 - \tau_i\, R/n_o \tag{12b}$$

The optical gain is from Eq.(10c) and Eq.(12b)

$$G(R) = A\, n_o\, (2f_c - 1) = A\, n_o - 2\, A\, \tau_i\, R \tag{13}$$

The optical gain G no longer depends on N because the state is fully occupied at low optical power, but it depends on R because of SHB.

It follows from Eq.(13) that the spectral-hole-burning parameter

$$\beta \equiv - (R/G)\, dG/dR = 2\, A\, \tau_i\, R/G \tag{14a}$$

can be written after rearranging using Eq.(13)

$$\beta = 2\,(1 - f_c)/(2f_c - 1) \tag{14b}$$

This SHB parameter goes to infinity as f_c approaches 1/2.

We are now in position to treat semiconductor laser noise at T = 0K.

4 COLD SEMICONDUCTOR

As we discussed earlier, at low temperatures and for low optical fields, the relevant states in the conduction band of pumped semiconductors are filled (occupation probability $f_c = f_{co} = 1$) and the relevant states in the valence band are empty (occupation probability $f_v = f_{vo} = 0$). Accordingly, the optical gain is independent of the carrier number N as long as N exceeds some critical value. (See Fig. 2 without the spectral hole). Oscillation stability is ensured by the decrease of f_c and increase of $f_v = 1 - f_c$ as the optical power increases, which entail a decreasing gain. This is the SHB effect represented in Fig.2. Because the intraband scattering times are very small, less than 1 ps even at low temperatures, this effect can be considered instantaneous as far as the dynamics is concerned.

Considering that G depends on R according to Eq.(13), the steady-state oscillation condition in Eq.(3) is explicitly

$$G(\dot{R}) = 1/\tau_p \tag{15a}$$

and

$$J - N/\tau_s = R \tag{15b}$$

where τ_s denotes the spontaneous recombination lifetime. Equation (15a) can be solved for R. The value of N then follows from Eq.(15b) if the pump rate J is known.

Let us now go back to the noise problem and restore the intrinsic noise term. Equation (6d) reads

$$R = G(R)\ m + r \tag{16}$$

with G(R) given in Eq.(13), and the spectral density of r in Eq.(6e) and Eq.(11b).

We here implicitly assume that the optical gain G is a deterministic function of R. It should not be forgotten, however, that f_c represents the probability that a state is occupied. The state occupency fluctuates as a function of time with variance $f_c\ (1 - f_c)$, a well-known result. For the time being, however, we proceed with Eq.(16) as it stands.

To first order, Eq.(16) reads

$$(1 + \beta)\ \Delta R = \Delta m/\tau_p + r \tag{17}$$

if we introduce the SHB parameter β according to Eq.(14a) and the steady-state conditions are used.

Eliminating Δm from Eqs.(4b), (5c) and (17) we obtain the output photon rate fluctuation

$$\Delta Q = \{r + [j\ \Omega(1 + \beta) - 1]\ q\} / [j\ \Omega(1 + \beta) + \beta] \tag{18}$$

which is here expressed as a weighted sum of the intrinsic noise terms r(t) and q(t). Note that the pump fluctuations, if any, do not enterin Eq.(18). This special situation holds only at T = 0 K and $\Omega \neq 0$.

Because the noise sources q (vacuum fluctuation) and r (dipole noise) are independent and have the spectral densities given in Eq.(5d) and Eqs.(6e) and (11b), respectively, the normalized spectral density of ΔQ reads from Eq.(18)

$$X \equiv \langle Q\rangle^{-1}\ S_{\Delta Q}(f) = 1 + (2\ n_p/\beta^2 - 1) / [1 + (1 + 1/\beta)^2\ \Omega^2] \tag{19}$$

The population inversion factor n_p is expressed in term of the occupation probability f_c in Eq.(11b). The SHB parameter β is expressed in terms of f_c in Eq.(14b). Thus the noise spectrum X(f) depends only here on the f_c value.

When the frequency tends to zero, the value $X = 2\ n_p/\beta^2$ is approached (but X jumps to unity at $\Omega = 0$). It follows from Eq.(19) that light is amplitude-squeezed (i.e., $X < 1$) at small (but nonzero) frequencies when

$$\beta^2 > 2\ n_p \tag{20}$$

i.e., $f_c < 0.7$.

For a typical vertical cavity laser diode we estimate [11] that

$$\beta \approx \tau_i^2\ P_{out} \tag{21}$$

where the intraband scattering time τ_i is expressed in ps and the total output power P_{out} is expressed in mW. In that case amplitude squeezing would occur for $\tau_i > 1$ ps at a power of 1.6 mW.

Note that as β goes to infinity, so does n_p, but n_p/β^2 nevertheless vanishes. In that limit X(f) is the reciprocal of $1+(f_c/f)^2$. X thus tends to zero at low nonzero frequencies. The variation of X as a function of f for $\beta = 0$ and $\beta = 5$ is shown in Fig. 3 as dotted lines.

The statistical fluctuations of the optical gain, not considered so far, introduce a floor at half the SNL level (see Appendix B).

5 WARM SEMICONDUCTOR

At nonzero temperatures the optical gain G(N, R) depends on both the carrier number N and the photon rate R, and Eq.(17) generalizes to

$$(1 + \beta)\ \Delta R = P\ \Delta N/\tau_p + \Delta m/\tau_p + r \tag{22}$$

where the P-parameter

$$P \equiv g\ m / N \tag{23a}$$

is proportional to the optical power, and

$$g \equiv (N/G)\ \partial G/\partial N, \qquad \beta \equiv -(R/G)\ \partial G/\partial R \tag{23b}$$

The differential gain g and P vanish at T = 0K as discussed in the previous section. At room temperature g is of the order of 4 and P is of the order of 0.01 [11].

We now need the carrier rate equation

$$dN/dt = J - S - R \Rightarrow \qquad j\Omega\ \Delta N/\tau_p = \Delta J - \Delta S - \Delta R \tag{24a}$$

$$S_{\Delta J} = J \tag{24b}$$

because the pump J is supposed to fluctuate at the SNL.

The radiative spontaneous recombination rate is of the form

$$S = N/\tau_s + s \Rightarrow \qquad \Delta S = \sigma\ \Delta N/\tau_p + s, \tag{25a}$$

$$S_s = S, \qquad \sigma \equiv \tau_p/\tau_s \tag{25b}$$

The method of calculation of X proceeds as in the previous section. After remarkable simplifications we obtain

$$X \equiv 1 + N_o/D_o \tag{26a}$$

$$N_o = (\Omega^2 + \sigma^2)\ (2\ n_p - \beta^2) + 2\ P^2\ S/Q - 2\ P\ \sigma\ \beta \tag{26b}$$

$$D_o = \Omega^2 (P + \beta)^2 + [\Omega^2 (1 + \beta) - P]^2 + 2 P \sigma [\beta + \Omega^2 (1+ \beta)] + \sigma^2 [\beta^2 + \Omega^2 (1 + \beta)^2] \tag{26c}$$

which is the analytical form corresponding to the schematic in [6]. It coincides with Eq.(19) in the limit $g = S = 0$. The expression for n_p in Eq.(11b) valid at $T = 0K$ must be increment by approximately $g/2$ at room temperature [11].

The variation of X in Eq.(26) is shown in Fig.(3) as a function of $\Omega \equiv f/f_o$ as plain lines when spontaneous recombination is neglected ($S = 0$) with $g = 1$, $m/N = 0.6$, and two values of β, namely 0 and 5.

The influence of a relative spontaneous recombination rate $S/Q = 0.25$ (i.e., 20% of the created electron-hole pairs recombine spontaneously) is illustrated in Fig.3 as dashed lines (same parameters as above). These curves (and others not shown) indicate that spontaneous carrier recombination (SCR) does not deteriorate much squeezing as long as β remains large compared with unity. Squeezing in fact now occurs also at zero frequency. The reason is that SCR tends to clamp the carrier number, while the intrinsic fluctuations that it introduces play only a minor role. Here again statistical gain fluctuations entails a level floor at approximately half the SNL.

6 CONCLUSION

We have shown theoretically that amplitude noise from strongly optically-pumped cold semiconductor can be squeezed down to approximately half the shot noise level, as a result of spectral-hole burning. Statistical fluctuations of the optical gain generate terms that are of the same order as spectral-hole-burning terms. Statistical fluctuations should also be accounted for in semiconductor linewidth calculations at high optical power. Moderate spontaneous-carrier-recombination far from being detrimental to amplitude noise enables squeezing to occur at zero frequency. These results, if confirmed experimentally, would be of practical importance for the measurements of small attenuations and sensors, because many semiconductors cannot be electrically pumped.

Optically pumped semiconductor can be given a geometry appropriate to low-loss whispering-gallery modes of resonance. Experiments on whispering-gallery modes have been reported in the microwave [12] and optical [13] ranges. In order to verify that light fluctuations are squeezed high-quantum-efficiency detectors that collect essentially all the emitted light are required. Alternatively, a dual-detector system (Hanburry-Brown and Twiss type experiment) can be employed.

APPENDIX A: BASIC NOISE SOURCE.

Well-above threshold 2-level-atoms are considered in this Appendix, with fluctuations small compared with the steady-state values.

Theories that ignore the noncommuting character of the operators representing light may be erroneous. The phasor model (: mewhat related to normally-ordered Quantum theories, such as the one employed in [5]) is accurate only in special circumstances, as we hope to clarify in a future paper. But the classical theory employed in this paper (that ressembles symmetrically-ordered Quantum theories) appears to be exact

as long as the particle number is large. These two commuting-number theories rest on vastly different concepts: the phasor theory attributes laser noise to spontaneous emission, while the classical theory in [8], [9] attributes laser noise to quantum jumps [14] resulting from stimulated emission.

The classical theory of laser noise is based on a unique intuitive principle: atoms submitted to a constant-amplitude classical field are independent of each other. Emitters and absorbers are treated in a completely symmetrical manner, as was done by Lax [10] (however the nonlinearities are not adequately handled in Lax's paper). The fact that the emitter properties are usually more complicated than those of the detector is of no fundamental significance. With this symmetrical view spontaneous emission is clearly not relevant to laser noise since spontaneous absorption never occurs.

Let us thus consider atoms in the ground-state with nonoverlapping wavefunctions submitted to a classical field of prescribed amplitude. We are asserting that the atoms in that special (and perhaps unrealizable) situation, are independent of each other because they cannot "communicate", so-to-speak, with one another through induced field fluctuations. To wit, the instants at which electrons jump from the ground state to the upper state as a result of stimulated absorption (resp. emission) are independent. This assertion is reminiscent of Glauber's observation [15] that classically prescribed electrical currents radiate light in the coherent state. In fact, our formalism leads readily to Glauber's observation.

The current induced by the electron motion thus fluctuates at the shot-noise level. For the one-photon processes considered in the main text, this implies that the (absorbed or emitted) photon rate fluctuates at the SNL, i.e., the spectral density of the intrinsic fluctuations denoted r(t) or q(t) in the main text are equal to the average rates:

$$S_q = \langle Q \rangle \tag{A1}$$

More generally, complex intrinsic rates $q \equiv q' + i q''$ can be defined. Applying the above underlined principle to detuned atoms one finds that q' and q'' are independent and fluctuate at the SNL. For k-photon processes [16], using again the same argument, one finds that the spectral densities of q' and q'' are multiplied by k

$$S_{q'} = S_{q''} = k \langle Q \rangle \tag{A1}$$

Two-photon absorption ($k = 2$) is relevant to the discussion in the main text because this mechanism may occur in some lasers and has effects similar to spectral-hole-burning. The detailed discussion however is not given here.

APPENDIX B: STATISTICAL GAIN FLUCTUATIONS

Consider first the conduction band and denote by f_c the probability that a particular interacting state is occupied by an electron. If x is a random variable equal to 1 with probability f_c and 0 with probability $1 - f_c$ obviously the mean of x is equal f_c and the variance of x is $f_c (1 - f_c)$. Because we are interested in the limit in which f_c is approaching 1/2 from above, this value is considered from now on.

It is quite plausible that the instants of jump of x from 1 to 0 or from 0 to 1 are independent, i.e., Poisson's distributed. Let the average rate of this underlying point process be denoted by λ. The spectrum of the telegraphic process x(t) is Lorentzian [17] with a low frequency limit equal to $1/4\lambda$. This result is obtained by rescaling Eq.(9.28) of [17], taking the Fourier transform, and setting the frequency equal to zero. In the present context, "low frequency" refers to frequencies much smaller than the reciprocal of the intraband scattering time, i.e., below approximately 100 GHz.

Assuming symmetrical conduction and valence bands, the occupation of the valence band state is a process y(t) independent of x(t) but with the same statistics. The spectral density of $z(t) \equiv x(t) - y(t)$ is therefore equal to $1/2\lambda$.

Because we are interested here in statistical fluctuations only, the total carrier number N and the recombination rate R are considered constant, and the probability f_c is independent of time. The contribution of a single level pair to the optical gain is of the form: A z(t), with A a constant. Assuming that the occupations of the n_o interacting states are independent of one another, the spectral-density of the optical gain is n_o times the single level-pair contribution, i.e.,

$$S_{\delta G} = n_o A^2 / 2\lambda \qquad \text{(B1)}$$

where δ is used to denote statistical fluctuations.

It remains to relate the average rate λ of the underlying Poisson's process to the relaxation time τ_i introduced in the main text. Equation (12a) with $df_c/dt \approx 0$ defines τ_i as the ratio of small changes $\delta\langle x\rangle$ of the mean state occupation and small changes $\delta\rho \equiv \delta R/n_o$ of the electron removal rate (resulting in the present context from stimulated carrier recombination). Going back to the telegraphic process, suppose that x is forced to drop from 1 to 0 once every $T \equiv 1/\delta\rho$ seconds. The average duration of a x = 1 state is $1/\lambda$. One of these x=1 states is cut by half on the average over the T-period. It follows that the mean occupation drops by $\delta\langle x\rangle = 1/(2\lambda T)$. We thus end up with the relation

$$2\lambda = 1/\tau_i \qquad \text{(B2)}$$

which is related to the fluctuation-regression theorem.

Thus G in Eq.(16) is not a deterministic function of R. It suffers a fluctuation δG that amounts to adding up a term δG m to the intrinsic fluctuation r(t). Equivalently one should add to $2n_p$ the term

$$m^2 S_{\delta G} / R = n_o A^2 \tau_i m^2 / R = (1 - f_c)/(2 f_c - 1)^2 \qquad \text{(B4)}$$

where we have used the expression for G(R) in Eq.(13), and the steady-state oscillation condition: R = G m. Let us recall that Eq.(B4) is valid when f_c is near 1/2. According to Eq.(19) of the main text, as f_c is approaching the limiting value of 1/2, the photon noise relative to shot noise, X, also approaches 1/2, as is the case for 3-level atoms [5] (see Appendix C).

APPENDIX C: 3-LEVEL ATOMS

The authors in [5] considered the independent injection of 3-level atoms in the emitting state into a single-mode optical cavity. The absorbing state is supposed to decay to the ground state by spontaneous emission at an average rate $\langle K\rangle = n_a/\tau_a$. the other assumptions and notations are as in the main text. Such 3-level atom lasers appear to be similar to semiconductor lasers with spectral-hole burning. The third level can indeed be likened to the semiconductor noninteracting states, lumped into a single level. Amplitude noise turns out to be rather similar.

Because the result in Eq.(5.13) of [5] is obtained from a difficult Quantum theory that employs normally-ordered operators, it is useful to show that the result can be recovered from the classical noise theory.

The system is not closed and equations must be written for both n_e and n_a according to

$$dn_e/dt = J - R, \qquad dn_a/dt = R - K, \qquad dm/dt = R - Q \qquad \text{(C1)}$$

where the pump fluctuation ΔJ is at the SNL, and the rates R, K and Q are given by

$$R = A(n_e - n_a)\, m + r, \quad K = n_a/\tau_a + k, \quad Q = m/\tau_p + q \qquad \text{(C2)}$$

where A is a constant.

The spectral densities of the (independent) noise sources ΔJ, k, q and r are respectively

$$S_{\Delta J} = S_k = S_q = Q \qquad \text{(C3a)}$$

$$S_r = (1 + 2 A \tau_a \tau_p Q)\, Q \qquad \text{(C3b)}$$

where the steady-state conditions have been used to evaluate the population-inversion factor n_p from Eq.(6f). Let us recall that $h\nu \langle Q\rangle$ represents the output optical power. At large power the two working-level populations tend to equalize and the population-inversion factor goes to infinity. Nevertheless the contribution of r vanishes in that limit. A similar observation was made in the main text for semiconductors.

Simple algebra gives the photon noise from the above equations. In the large power limit we obtain

$$X \equiv Q^{-1} S_{\Delta Q} = 1 - (1/2)\, \{(1+\Omega^2)[1 + (\tau_p/2\tau_a)^2/\Omega^2\}^{-1} \qquad \text{(C1)}$$

an expression that coincides with Eq.(5.13) of [5] but is written here in a simpler form. The minimum value of X, namely 0.5, is comparable to the one that one can achieve with semiconductors.

REFERENCES

1 Y. M. Golubev and I. V. Sokolov, Sov. Phys. JETP **60**, 234 (1984).

2 Y. Yamamoto, ed. *Coherence, amplification and quantum effects in semiconductor lasers*, J. Wiley and Sons, New-York, (1991).

3 F. Haake, S.M. Tan and D.F. Walls, Phys. Rev. A **40**, 7121 (1989).

4 J. Bergou, L. Davidovitch, M. Orszag, C. Benkert, M. Hillery and M.O. Scully, Phys. Rev. A **40**, 5073 (1989)

5 M.I. Kolobov, L. Davidovich, E. Giacobino and C. Fabre, Phys. Rev. A **47**, 1431 (1993).

6 Y. Vidal, S. Gaillard and J. Arnaud, Proceedings of "the 4ème journées Nationales Microélectroniques" p.195, La Grande Motte, 21 Oct. 1992, and submitted for publication.

7 J. Arnaud, "European Semiconductor Laser Workshop", Sept. 9-11 (1993), Rigi-Kaltbad, Switzerland (unpublished talk).

8 J. Arnaud and M. Estéban, Proc. IEE, Part J, **137**, 55 (1990).

9 J. Arnaud, Phys. Rev. A , **48**, 2235 (1993).

10 M. Lax, Phys. Rev.,**160** (2), 290 (1967).

11 M. Estéban and J. Arnaud, Proc. IEE, Part J, **138**, 79 (1991).

12 C. Vedrenne and J. Arnaud, Proc IEE, Part H, **129**, 183 (1982).

13 V.B. Braginsky, M.L. Gorodetsky and V.S. Ilchenko, Phys. Lett. A, **137**, 393 (1989).

14 R.J. Cook, in: "Progress in Optics XXVIII", E. Wolf ed., North-Holland, Amsterdam, 1990. C. Cohen Tannoudji, B. Zambon and E. Arimondo, J. Opt. Soc. Am. B, **10**, 2107 (1993).

15 R.J. Glauber, in: "Quantum Optics and Electronics", p.132, Gordon and Breach, 1964.

16 P.N. Butcher and D. Cotter, "The Elements of Nonlinear Optics", Cambridge University Press, Cambridge (1990).

17 A. Papoulis, "Probabilities, Random Variables, and Stochastic Processes", McGraw Hill, New-York, (1965).

CAPTIONS

Figure 1

Electrical model for semiconductor laser noise. The detector is represented by a 1 ohms resistance in series with a noise source v_a representing vacuum fluctuations. The active element is represented by a resistance of - (1 + β) ohms, where β is the spectral-hole-burning parameter, in series with a noise source v_e representing dipole noise whose spectral density is independent of β. At low temperatures g = 0. The capacitance C_e is infinite and may be replaced by a short circuit at any nonzero frequency. The capacitance C_c, equal to the photon lifetime, is negligible in comparison. Amplitude noise vanishes in the limit $\beta \to \infty$, irrespectively of the pump fluctuations.

Figure 2

Schematic picture of optical gain as a function of frequency at low temperature. The dip in the gain curve at the oscillation frequency ν_o represents the spectral-hole-burning effect (ν_g is the band gap frequency). In the limit considered the optical gain depends on the emitted photon rate R but not on carrier number N, as the curve labeled N + dN indicates.

Figure 3

Variation of amplitude noise normalized to the shot-noise-level, X as a function of f/f_o where f_o denotes the cold cavity linewidth. Two values of the spectral-hole-burning parameter β (proportional to the square of the intraband scattering time and to optical power) are considered: 0 and 5.

Dotted lines apply to cold semiconductors. Plain lines apply to room temperature operation, with g = 1 and a photon-to-electron number ratio m/N = 0.6, spontaneous recombination being neglected. Dashed lines: same as plain lines but with spontaneous recombination: S/Q = 0.25. Statistical gain fluctuations contribute a floor at half the SNL, not shown in the figure.

Figure 1

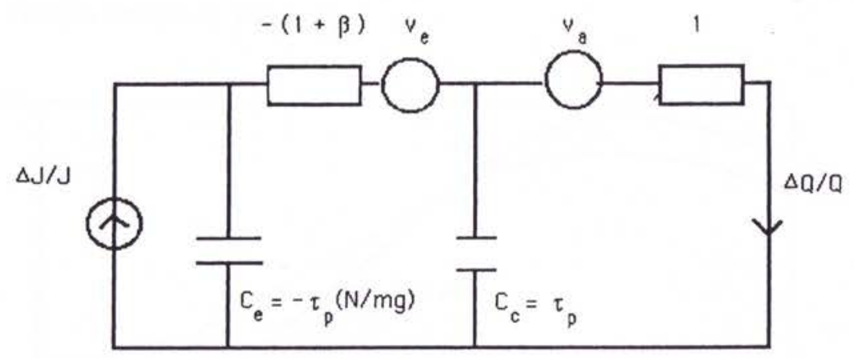

Figure 2

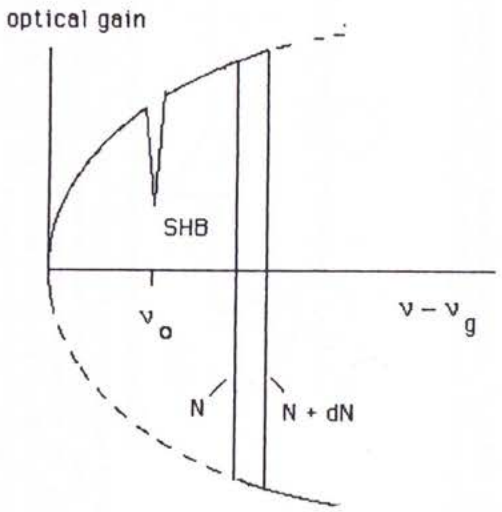

Figure 3