Fig. 2 shows typical electron field-effect mobility data as a function distance from the Si/SiO_2 interface in both as-grown (*a*) and DSPE-regrown SOS (*b*). The precipitous drop in carrier mobility with decreasing distance from the sapphire substrate in as-grown SOS has been reported previously[4] and is consistent with the rise in microtwin density near the Si/Al_2O_3 interface observed in XTEM studies of as-grown SOS.

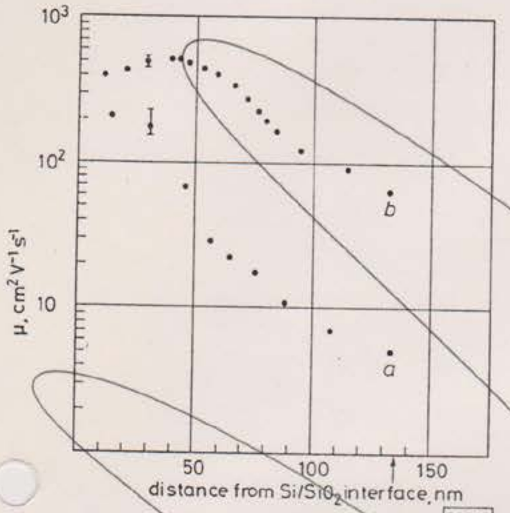

Fig. 2 *Electron mobility against distance from Si/SiO_2 interface*

Arrow indicates position of Si/Al_2O_3 interface
a As-grown SOS
b DSPE-regrown SOS

By contrast, the DSPE-regrown film shows significantly higher electron mobility than as-grown films at all depths. The reduction in mobility near the Si/SiO_2 interface in DSPE-regrown SOS can be attributed to interface scattering and to incomplete depletion of carriers from the near-surface region. Both of these effects are expected to reduce measured mobilities at depths within a Debye length (~ 20 nm) of either the Si/SiO_2 or Si/Al_2O_3 interface. As the film is depleted to greater depths, the electron mobility rises to approximately 500 cm^2/Vs at slightly less than 100 nm from the sapphire substrate; it then drops gradually to about 100 cm^2/Vs within a Debye length of the Si/Al_2O_3 interface. These results are qualitatively in agreement with the ion channelling and XTEM studies of the crystallography of DSPE-regrown SOS cited earlier. However, the mobility data from within about 50 nm of the sapphire substrate requires additional measurements and consideration. These data could be due to a small residual defect concentration, lower dopant activation (and hence larger Debye length), very high Si/Al_2O_3 interface scattering or resolution limits of the measurement technique.

Finally, in addition to significantly improving carrier mobility, the DSPE process also reduced the statistical variation of the mobility at distances less than 100 nm from the sapphire substrate. The error bars in Fig. 2 represent the full range of measured electron mobilities at the depth indicated for 12 randomly selected devices fabricated from both as-grown and DSPE-regrown SOS. This improvement in the uniformity of carrier transport properties in DSPE-regrown SOS is a reflection of the overall reduction in defect density and to the concomitant reduction in the statistical spread in defect concentration.

In conclusion, DSPE-regrown SOS has been shown to have excellent device-grade electron mobility within 100 nm of the Si/Al_2O_3 interface. Enhancement MOSFETs fabricated in SOS films of these dimensions should show well behaved long-channel properties well into the submicrometre range of lateral device dimensions. Further work is in progress to determine the suitability of sub-100 nm-thick DSPE-regrown SOS for laterally scaled-down devices.

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G. A. GARCIA
R. E. REEDY

Naval Ocean Systems Center
San Diego, CA 92152-5000, USA

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NATURAL LINEWIDTH OF SEMICONDUCTOR LASERS

Indexing terms: Lasers and laser applications, Semiconductor lasers

A formula is given for the natural linewidth of high-gain lasers, which is applicable to arbitrary three-dimensional geometries. This formula agrees with Petermann's result for lasers with transversely inhomogeneous gains (*K*-factor) and with previous results for the effect of small mirror reflectivities. It does not agree with the Schawlow-Townes (ST) formula used by most authors in evaluating the $\alpha = \Delta n_r/\Delta n_i$ factor of conventional semiconductor lasers. The difference between the two formulas is significant when the mirror power reflectivity is less than about 0·6. Furthermore, the modified formula gives directly the linewidth of lasers coupled to long external cavities. Saturation effects, however, are neglected in this letter.

It is well known that transverse inhomogeneities in gain enhance the natural linewidth of high-gain lasers, by a factor $K \gg 1$ (see References 1 and 2). By 'natural linewidth' we understand the nonzero linewidth which results from spontaneous emission in the active medium. The theories in References 1 and 2, however, are essentially one-dimensional. I would like to report a formula, derived from first principles, which generalises previous results to arbitrary three-dimensional geometries, and make them more precise. Furthermore, that theory shows that a *K*-like factor exists also in the longitudinal direction. The fact that the ST formula is not applicable when the mirror reflectivity is not close to unity has been recognised earlier[3–5]. While this fact can sometimes be overlooked[6,7] for conventional lasers (error $\sim 10\%$), it is important to take it into account in the case of reduced reflectivity lasers. The departure of this longitudinal *K*-like factor from unity is not as large as in transverse directions, but it is nevertheless significant when the mirror power reflectivities are much smaller than unity. When $R = 0{\cdot}01$, for example, the laser linewidth Δf is 4·6 times the value calculated from the ST formula. The relationship between this longitudinal factor and the *K*-factor has, to my knowledge, never been pointed out. In the present letter, the transverse effect is let aside and only the longitudinal effect is discussed in detail. For the sake of simplicity, saturation is neglected. Therefore, formulas assuming full saturation as the ones given, for example, by Henry[6] should be multiplied by a factor of 2. Aside from this factor, my result agrees with the Schawlow–Townes formula when the mirror power reflectivity R is close to unity (low-gain lasers), but the discrepancy goes to infinity as $R \rightarrow 0$.

The laser field is viewed as amplified spontaneous emission. Spontaneous emission is modelled by electrical currents $J(r)$ randomly oriented and δ-correlated in space. The averaged quantity $\langle J \,.\, J^* \rangle$ is proportional to hf, where h denotes

Planck's constant and f the optical frequency, and to the negative imaginary part of the medium permittivity ε. This representation is discussed in detail, for example, in Reference 8. The rest of the calculation is based on linear electromagnetism.

Let us consider a cavity with perfectly conducting walls which encloses both the laser and the detector of radiation. The medium permittivity ε and permeability μ are in general complex scalar functions of space $\boldsymbol{r} = x, y, z$ and frequency f. The imaginary part of ε is split into $\varepsilon_1 - \varepsilon_2$, where ε_1 represents stimulated absorption (including absorption by the detector) and ε_2 represents stimulated emission. I consider well above-threshold conditions in which the resonating field E, H is almost in a single spatial mode. Under those conditions, the product of laser (full half-power) linewidth Δf and dissipated power P ($P = 2P_0$ is the laser output power if the internal losses are neglected and P_0 is the power per facet in a symmetrical configuration) is given by

$$\Delta f P = 4(hf/2\pi) \left| \int \sigma |E|^2 \, dV \Big/ \int (\varepsilon' E^2 - \mu' H^2) \, dV \right|^2 \quad (1)$$

where $\sigma = 2\pi f \varepsilon_1$ represents the medium conductivity. I have defined $\varepsilon' = \partial(f\varepsilon)/\partial f$, $\mu' = \partial(f\mu)/\partial f$. These quantities would respectively be equal to ε and μ if the medium were nondispersive, but dispersion is important in applications. For any vector E, $|E|^2 = E \, . \, E^*$, where the star indicates complex conjugation and the dot a scalar product, and $E^2 = E \, . \, E$. The integrals in eqn. 1 are over the cavity volume (which includes as one recalls the detector), and $dV = dx \, dy \, dz$.

One important feature of this simple formula is that the linewidth predicted for a given power is independent of the distance between the laser and matched detectors of radiation, as one expects on physical grounds. This is because travelling-wave fields neither contribute to the integral in the denominator of eqn. 1, nor, trivially, to the numerator.

I have checked the validity of eqn. 1 for lumped circuits by comparing it to direct calculations analogous in spirit to the one made in Reference 9 for a simple LCR parallel circuit. Note that for the high-gain lasers presently considered the 'cold cavity' obtained by suppressing the gain has a very low Q-factor, and therefore the so-called 'cold-cavity linewidth' that enters into the Schawlow–Townes (ST) formula needs to be precisely defined before any comparison can be made. In this letter the cold-cavity linewidth is not introduced.

It is easy to show that eqn. 1 agrees with Petermann's result[1] if the laser structure is uniform in the longitudinal direction and spatial changes of the modified permittivity ε' can be neglected ($\mu' \doteq \mu_0$ is the free-space permeability). This approximation is reasonable for most semiconductor lasers.

Let us now consider an active homogeneous medium between two partially reflecting plane mirrors of power reflectivity R located at $z = 0$ and $z = -L$, respectively.

The formula used by Henry[6] can be derived either from $\Delta f =$ average spontaneous-emission rate in the mode/2π × number of photons in the mode, or from the ST formula (with the cold-cavity linewidth defined as twice the imaginary part of the complex resonant frequency). This classical result reads (eqn. 26 of Reference 6 with the internal losses neglected: $g = \alpha_m = L^{-1} \ln(1/R)$, $\alpha = 0$, $P = 2P_0$, $n_{sp} = 1$, and multiplied by 2 for the reason explained earlier)

$$\Delta f P = (hf/2\pi)(v_g/L)^2 (\ln 1/R)^2 \quad (2)$$

My eqn. 1 leads to a different result, namely

$$\Delta f P = (hf/2\pi)(v_g/L)^2 (1 - R)^2/R \quad (3)$$

Clearly, eqns. 2 and 3 agree when $R \to 1$, but the result in eqn. 3 is 1·12 times the result in eqn. 2 when $R = 0{\cdot}3$, and 4·6 times when $R = 0{\cdot}01$, corresponding to a single-pass gain of 20 dB. The discrepancy between eqns. 2 and 3 has been noted before.[3–5] The interest of my formulation is that it unites into a simple comprehensive expression Petermann's and earlier results. Furthermore, it enables us to treat complicated geometries.

To show with little mathematics that indeed eqn. 3 follows from eqn. 1 for the configuration considered, let us use a transmission-line analogue of the laser described above: a uniform active transmission line of real characteristic conductance G_c terminated at $z = 0$ and $z = -L$ by identical conductances G, representing the partially transmitting mirrors of power reflectivity R followed by perfect absorbers of radiation (I have checked that more realistic models to essentially the same results). We have therefore

$$r = \sqrt{R} = (1 - G/G_c)/(1 + G/G_c) \quad (4)$$

with all the quantities being real.

Let the active transmission line have an impedance per unit length $-R - iL2\pi f$ and admittance per unit length $-G - iC2\pi f$, where R and G are positive quantities such that $G/R = C/L = G_c^2$. An exp $(-i2\pi ft)$ time dependence is used throughout. Let $V(z)$ and $I(z)$ denote the voltage and current along the line and G the conductances loading the line at $z = 0$ and $z = -L$. One should use in eqn. 1 the correspondences: $E, H \to V, I$; $\sigma \to G \, \delta(z) + G \, \delta(z + L)$, $\varepsilon' \to C$, $\mu' \to L$ (note that ε and μ without primes would correspond to complex capacitances and inductances involving losses or gains).

The general eqn. 1 leads then to the following expression:

$$\Delta f P = 4(hf/2\pi) G^2 \{ |V(-L)|^2 + |V(0)|^2 \}^2 \left| \int (CV^2 - LI^2) \, dz \right|^{-2} \quad (5)$$

the integral being taken from $-L$ to 0.

Recall now the classical transmission-line formulas

$$V(z) = Z + r/Z$$
$$I(z)/G_c = Z - r/Z \quad (6)$$

where

$$r = \sqrt{R} \qquad Z = \exp(ikz) \qquad k = (C2\pi f - iG)/G_c \quad (7)$$

The resonance condition is $Z(L) r = \pm 1$ and the group velocity $v_g = G_c/C$.

Introducing eqn. 6 in eqn. 5 leads to eqn. 3, if we make use of eqn. 4 and of the above relations for $Z(L)$ and v_g. It is easy to restore in eqn. 3 the n_{sp} factor and internal losses. The ST result in eqn. 2 would follow from eqn. 5 if it were permissible to replace in the denominator of eqn. 5 V^2 by $|V^2|$ and I^2 by $-|I^2|$. This, however, is not the case in general.

Another configuration of interest is an optical fibre of length L' with power reflection R' at the right end, and a lossless laser of length $L \ll L'$ at the left end. The laser provides on reflection a power gain $\simeq 1/R'$ and a noise spectral density $hf(1/R' - 1)$. For that configuration eqn. 1 leads to a linewidth Δf given by

$$\Delta f P = (1/4)(hf/2\pi)(v_g'/L')^2 (1 - R')^2/R' \quad (8)$$

where v_g' is the group velocity in the fibre and P the output power. This configuration is that of a laser coupled to a long optical fibre. However, instead of considering first the laser diode and subsequently introducing a delayed feedback, as is usually done, we choose to consider the fibre itself as a laser, and the laser diode as an active load on that fibre. Direct application of the ST formula to that configuration would lead to quite incorrect results (because R' is small). The reciprocal dependence of Δf on R' when $R' \ll 1$ given in eqn. 8 is, on the contrary, correct (see, for example, Reference 10).

In conclusion, a general formula for the natural linewidth of high-gain spatially inhomogeneous lasers such as semiconductor lasers, far above threshold, has been reported. For lasers that have large transverse inhomogeneities, Petermann's K-factor is recovered (with some correction terms which we will discuss elsewhere). For conventional lasers, we can account for mirror reflectivities significantly smaller than unity. This effect is particularly conspicuous when the laser is coupled to a long fibre terminated by a small reflection or when the mirror reflectivity is artificially reduced by coatings.

The generalisation of our basic eqn. 1 to arbitrary bi-anisotropic media has been obtained. It will be reported elsewhere. I acknowledge with thanks support from the CNET, France.

J. ARNAUD

Equipe de Microoptoélectronique de Montpellier
Unitée Associée au CNRS, USTL
Pl. E. Bataillon
34060, Montpellier Cedex, France

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USE OF UNIDIRECTIONAL DATA FLOW IN BIT-LEVEL SYSTOLIC ARRAY CHIPS

Indexing terms: Signal processing, Systolic arrays

We show how the architecture of two recently reported bit-level systolic array circuits—a single-bit coefficient correlator and a multibit convolver—may be modified to incorporate unidirectional data flow. This feature has some important advantages in terms of chip cascadability, fault tolerance and possible wafer-scale integration.

Introduction: In a number of recent publications[1,2] we have demonstrated how the systolic concept, applied at the bit level, can be used in the design of high-performance VLSI chips for digital signal processing. Two bit-level systolic array chips have now been successfully produced as a result of a collaborative research programme between RSRE and the GEC Hirst Research Centre. These perform:

(*a*) correlation of a bit-parallel input data stream with single-bit coefficients[3]

(*b*) convolution of a bit-serial input data stream with multibit coefficients.[4] *

The correlator chip is now in production at Marconi Electronic Devices Ltd. (Lincoln). Both chips have been designed so that they can be cascaded to increase the number of coefficient stages and, also in the case of the correlator, the number of bits in the coefficient words.

The success of these projects has resulted in a continued investigation of these architectures, and modifications to the original convolver array have been published. These increase the computational efficiency to 100%.[1,2] More recently we have given some consideration to other factors which may influence the design of future bit-level systolic array chips. These include ease of cascadability, fault tolerance and potential wafer-scale integration.

* We should point out that, while for convenience and consistency with other work we have referred to the circuits in this letter as a correlator and convolver, respectively, one can interchange the function of a given device simply by reversing the order of its coefficients

Recent work by Kung and Lam[5] have demonstrated the advantages of having unidirectional data flow in fault-tolerant processor arrays. Most fault tolerance schemes which can be applied to linear arrays involve the routing of information around faulty cells. This in turn can introduce significant transmission delays between cells if long lengths of wire are involved. As a result, the clock rate must be reduced and the system performance is degraded. This problem can be avoided in an array with unidirectional data flow by inserting latches in all data streams which are rerouted around a faulty cell. This does not alter the required data interactions, since the relative delay between all data paths is zero. The degradation in throughput rate is avoided since the bypass propagation can now occupy one entire clock cycle.[5] System latency is, of course, increased slightly. The same technique cannot be applied to arrays with contraflowing data streams, because the relative delay between paths would be nonzero and hence the data interactions would be corrupted. Similar considerations are also important when cascading chips in either a hybrid package or on a printed circuit board. In a unidirectional system latches can be used as a means of buffering to prevent off-chip or chip-to-chip delays from degrading system performance.

Previously published architectures for the correlator and convolver have all involved contraflowing data streams. It is the aim of this letter to present modified versions of both architectures which have unidirectional data flow. In particular, we will show how very subtle changes to the previous bit-level circuits can produce such important changes in overall data flow.

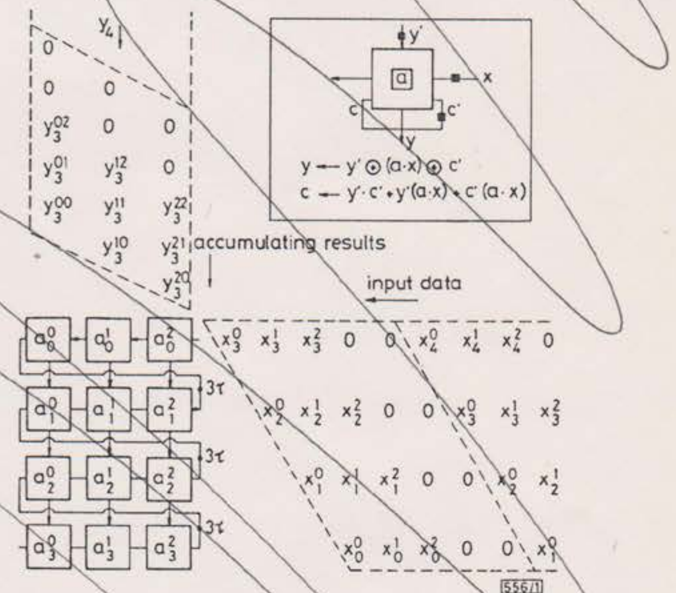

Fig. 1 *Convolver architecture with unidirectional data flow*
Partial products which involve the multiplication of a bit a_i^p with x_{j-i}^q are represented by y_j^{pq}

Unidirectional convolver structure: The modified convolver is illustrated in Fig. 1 and consists of an array of cells whose logic function is indicated. Delay elements are represented by solid squares. This circuit has static coefficients rather than moving ones[1] because both the basic cell and clocking scheme required are simpler. It therefore bears close similarities to the array proposed by Urquhart and Wood.[2] The main difference is in the timing and physical movement of data bits within the array. The function of the array is to compute the output sequence

$$Y_j = \sum_{i}^{N-1} a_i \, . \, x_{j-i} \qquad j = 0, 1, 2 \ldots$$

where $a_i (i = 0, 1, 2, \ldots, N - 1)$ represents a set of fixed n-bit coefficients and $x_i (i = 0, 1, 2, \ldots)$ represents a sequence of input data (signal) values. Fig. 1 depicts a simple example