Reprinted from Optics Letters, Vol. *13*, page 728, September 1988.

Multielement laser-diode linewidth theory

J. Arnaud

Equipe de Microoptoélectronique de Montpellier, Unité Associée au Centre National de la Recherche Scientifique, 392, Université des Sciences et Technique du Languedoc, Place E. Bataillon, F 34060, Montpellier Cêdex, France

Received January 14, 1988; accepted June 21, 1988

A simple formula is presented for the linewidth–power product of multielement laser diodes, under the condition that the active elements are submitted to the same field. The theory is particularly applicable to laser diodes incorporating two thin active layers of different composition. Such diodes can be modeled by two active conductances in parallel with a linear admittance. If the injected current exhibits full shot noise, the laser linewidth is given by the same expression as for a single active element, except that α and α^2 must be replaced by their average values weighted by the injected bias currents, where α denotes the phase-amplitude coupling factor. The same result is applicable to conventional index-guided laser diodes.

A general three-dimensional formula for the product $\Delta\nu P$ of a laser diode in the linear regime has been given,[1] where $\Delta\nu$ denotes the full-width half-power linewidth and P is the total generated power. This theory provides in a single comprehensive formula both the transverse (Petermann) K factor and a longitudinal factor evaluated in Ref. 2 for distributed-feedback lasers. In the linear regime the carrier density and therefore the medium optical parameters are independent of time. The optical power fluctuates according to a Rayleigh distribution.

In reality, single-mode lasers exhibit little power fluctuation above threshold. This is because the diode is current driven and/or the series resistance in the confining layers is large. The carrier density can then partly counteract the perturbating effects of spontaneous emission. This is the saturated regime considered in this Letter.

In the limit of large injected currents and small baseband frequencies, the output optical power fluctuations simply reflect those of the pump, i.e., of the injected current J.[3] The injected current is assumed here to exhibit full shot noise with one-sided spectral density $2eJ$. The hole and electron fluctuations fully correlate through space-charge fields. In the saturated regime the laser linewidth depends on slow frequency fluctuations and negligibly on power fluctuations. For optical communication applications, the frequency range of interest is about 10–1000 MHz. We therefore do not consider flicker noise, which is important mainly below 10 MHz. For simplicity, it is assumed that the quantum efficiency is unity.

A laser diode with a thin active layer and mirror reflectivities close to unity can be modeled as a single active conductance $-G_1$ in parallel with a linear admittance $Y(\nu) \equiv G(\nu) + iB(\nu)$, where ν denotes the optical frequency (see Fig. 1 and ignore for the moment the layer labeled 2). The complete theory for oscillators incorporating a single active element has been given by Lax.[4] Lax has shown that the linewidth–power product is given by the well-known Schawlow–Townes formula multiplied by $\frac{1}{2}[\cos(\theta - \phi)]^{-2}$, where $\tan\theta \equiv \alpha$ is the phase-amplitude coupling factor of the active element[5] and $\tan\phi \equiv G_\nu/B_\nu \equiv \kappa$, where the subscript ν denotes derivatives with respect to the optical frequency ν, evaluated at $\nu = \nu_0$. Arnaud[6] has shown that the quantity $1 + \kappa^2$ corresponds to the factor K introduced by Petermann in the linear theory of gain-guided lasers. Lax's theory is complete because for a single active element the details of the saturation mechanism and the shot-noise contribution can be ignored. By the same token, when only the injected current is varied, such an oscillator cannot be frequency modulated at small frequencies.

The purpose of the present Letter is to extend Lax's theory to laser-diode configurations that can be modeled by any number of active elements submitted to the same field. The theory is more involved than that of single-active-element oscillators because the details of the carrier-rate equations must be accounted for. The linewidth formula has been reported in Ref. 7 for the special case where the α factors are all equal to zero. Because the α factors play an important role in laser diodes, arbitrary α values are now considered. It was pointed out[7] that considerable simplification occurs when the injected current exhibits full shot noise. This conclusion holds also for the case considered here, where the α factors are nonzero.

Our theory differs conceptually from that given in Ref. 8 because the concept of stored energy (or that of "confinement factors") is not used. We have explained[9] why this concept should be avoided in the general situation even though it may prove useful for high-quality factors or homogeneously filled resonators.

For clarity, only laser diodes incorporating two thin active layers of different composition (separated by a high-band-gap layer to prevent carrier diffusion) are considered in detail [Fig. 1(a)]. The electric field E is supposed to be directed along the y axis (TE polarization) and not to vary appreciably within the pair of active layers. The optical loss at the end mirrors is

0146-9592/88/090728-03$2.00/0

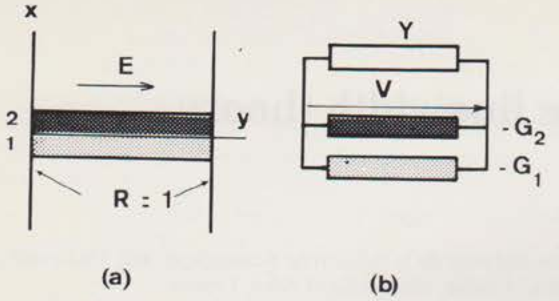

Fig. 1. (a) Laser diode consisting of two thin active layers (labeled 1 and 2) of slightly different band-gap energies and different α factors at the operating frequency. The optical wave propagates essentially along the z axis, perpendicular to the figure. The planes labeled $R = 1$ are perfectly conducting. (b) Electrical circuit model of the laser in (a), which consists of two negative conductances in parallel with a linear admittance $Y(\nu)$. The voltage V across the circuit is proportional to the electrical field E in the active layer of the laser diode.

replaced by a loss uniformly distributed along the path.

This somewhat idealized kind of laser diode can be modeled by the electrical circuit shown in Fig. 1(b), which consists of two negative conductances, $-G_1$ and $-G_2$, modeling the two active layers in parallel with a linear admittance, $Y(\nu)$, modeling the external or confining lossy layers, extending to $\pm\infty$ along the x axis in Fig. 1(a). The voltage V across the electrical circuit is proportional to the electrical field E in the active layers of the laser diode.

Let us first consider the steady-state (unperturbed) oscillation. A steady oscillation of the circuit in Fig. 1(b) occurs at frequency ν_0 if

$$G = \textstyle\sum_k G_k, \qquad k = 1, 2, \qquad B(\nu_0) = 0. \tag{1}$$

Let the total admittance be incremented by some small admittance y_T. The resulting complex change $\delta\nu$ of the oscillation frequency is given by $-Y_\nu\delta\nu = y_T$. The admittance change y_T consists of two kinds of terms: noise current sources associated with the positive or negative conductances of the circuit and the reaction of the medium. The complex-current noise source associated with the positive conductance G is denoted by $c + is$, while the complex-current noise source associated with the negative conductance $-G_k$ is denoted by $c_k + is_k$. Each conductance $-G_k$ is perturbed by a small conductance g_k owing to some change in carrier number in that active element, which also involves a change in susceptance equal to $-\alpha_k g_k$ according to the definition of the phase-amplitude coupling factor α. This factor is usually positive and of the order of 5. We can therefore write y_T in the form

$$y_T = (c + is)/|V| + \textstyle\sum_k (c_k + is_k)/|V| + g_k(1 - i\alpha_k). \tag{2}$$

The complex voltage V across the circuit has been replaced by its modulus $|V|$. This is permissible because the noise sources are white and the phase of white processes is arbitrary.

Slow changes of oscillation frequencies must be real; otherwise the amplitude fluctuations would be unbounded. For a single active element, it suffices then to specify that y_T in Eq. (2) equals $-Y_\nu\delta\nu$ and to eliminate g between the real and imaginary parts to obtain the real frequency deviation $\delta\nu$. The result is the Lax formula cited earlier. But if the oscillator incorporates two or more active elements, the number of equations obtained is insufficient and one must consider also the carrier-rate equations.

Under our approximations, the carrier-rate equation asserts that for each element k, the rate of photon generation equals the rate of electron–hole pair injection at any instant of time:

$$J_k/e = P_k/h\nu = G_k|V|^2/2h\nu, \tag{3}$$

where J_k denotes the current injected in the active element k and P_k is the optical power generated by that element. The total power generated (or absorbed, since at small frequencies conservation of energy implies conservation of power) is $P = \sum_k P_k$.

Consider next a small change j_k of the current J_k injected in the active element k. The change in conductance of that element is $g_{Tk} \equiv c_k/|V| + g_k$, according to Eq. (2). If we denote the relative change of $|V|^2$ by ρ, Eq. (3) gives, to the first order,

$$j_k/J_k = \rho - g_{Tk}/G_k. \tag{4}$$

Because $\delta\nu$ is real, the basic relation $-Y_\nu\delta\nu = y_T$ can be written as

$$-G_\nu\delta\nu = g_T = c/|V| + \textstyle\sum_k g_{Tk}, \tag{5a}$$

$$B_\nu\delta\nu = b_T = s/|V| + \textstyle\sum_k (s_k + \alpha_k c_k)/|V| - \alpha_k g_{Tk}. \tag{5b}$$

If g_{Tk} is expressed as a function of j_k and ρ with the help of Eq. (4), Eq. (5a) can be solved for ρ and the result substituted into Eq. (5b). Therefore, $\delta\nu$ can be expressed in terms of the independent white random processes c, s, c_k, s_k, and j_k.

The one-sided spectral density of the current $i(t)$ associated with any (positive or negative) conductance G is (see, e.g., Ref. 3)

$$S_{ii} = 2h\nu|G|, \tag{6a}$$

where the vertical bars indicate absolute value. It is convenient for narrow-band processes to replace $i(t)$ by a slowly varying complex current $I = c + is$, where

$$S_{cc} = S_{ss} = 4h\nu|G|, \qquad S_{cs} = 0. \tag{6b}$$

These formulas express the zero-point fluctuation of the electromagnetic field.

The spectral density of the full shot-noise j_k processes is

$$S_{j_k} = 2e|J_k|, \tag{7}$$

where e denotes the absolute value of the electronic charge.

Next, one makes use of the well-known fact that the full-width half-power linewidth $\Delta\nu$ of an oscillator frequency modulated by white noise is equal to $\pi S_{\delta\nu}$, where $S_{\delta\nu}$ is the one-sided spectral density of the frequency deviation process $\delta\nu(t)$.

Our final result can be written in the form

$$\Delta \equiv 2\pi\Delta\nu P/h\nu = (1/2)(4\pi G/B_\nu)^2[1 + (\alpha^2)_{\mathrm{av}}]/(1 + \alpha_{\mathrm{av}}\kappa)^2, \tag{8a}$$

where $P = G|V|^2/2$ is the total power, and

$$\kappa \equiv G_\nu/B_\nu \tag{8b}$$

is negative for the configuration in Fig. 1 but usually small compared with unity. The averaging for any quantity a is defined as

$$a_{\mathrm{av}} \equiv (\textstyle\sum a_k J_k)/(\sum J_k). \tag{8c}$$

The result in Eqs. (8) is the Lax formula quoted earlier, except that α and α^2 are here replaced by their averaged values, weighted by the injected currents. Let us emphasize that Eqs. (8) hold only when the shot-noise spectral densities are $2e|J_k|$ (full shot noise). If shot noise were suppressed, for example, by driving the active elements by space-charge-limited thermoionic tubes, the factor $1 + (\alpha^2)_{\mathrm{av}}$ would be reduced by half the variance of α (if $\kappa = 0$). In a multielement oscillator it is therefore not permissible to ignore shot noise, even at vanishingly small baseband frequencies.

A useful generalization is the case when the electron–hole inversion is incomplete. In this case an active element should be represented by a negative conductance $-G_a$ expressing stimulated emission and a positive conductance G_b expressing stimulated absorption. In all the previous equations, one must then replace G_k by the difference $G_{ak} - G_{bk}$. The spectral density of the noise current relating to G_k, however, is then given by $2\, h\nu(G_{ak} + G_{bk})$, with a plus. The resulting expression is

$$\Delta \equiv 2\pi\Delta\nu P/h\nu$$

$$= (1/2)(4\pi G/B_\nu)^2[n_{\mathrm{sp}}(1 + \alpha^2)]_{\mathrm{av}}/(1 + \alpha_{\mathrm{av}}\kappa)^2, \tag{9a}$$

where

$$(n_{\mathrm{sp}})_k \equiv G_{ak}/(G_{ak} - G_{bk}) \tag{9b}$$

denotes the so-called spontaneous emission factor, which is of the order of 2 at room temperature but may assume different values, as well as α, for the different active elements.

The application of the theory illustrated in Fig. 1 concerns a hypothetical laser incorporating two thin active layers of different composition. Conventional index-guided laser diodes should also be considered as multielement active devices, this time in the longitudinal direction. But because the mirror power reflectivity R is usually not close to unity, the relative field modulus fluctuation ρ varies importantly along the propagation z axis and the theory reported here, which basically assumes that ρ is a constant, does not seem to be applicable. Nevertheless, rather involved calculations (solving the stochastic differential equation for ρ) show that an equation similar to Eqs. (9) with $\kappa = 0$ holds when the driving current density exhibits full shot noise. Numerous complicated terms cancel out in that case. The simplification perhaps occurs because, when full shot noise is present, the optical field tends to be in a coherent state in the limit of large injected currents and large resonator lengths.[3]

The present theory can be further generalized to the case for which the current $c + is$ has a deterministic part (injection locking[10]) and when the injected-current fluctuations are correlated with the other noise sources because of external electronic feedback.

References

1. J. Arnaud, Opt. Quantum Electron. **18**, 335 (1986); see Eqs. (A-11) and (13).
2. J. Wang, N. Schunk, and K. Petermann, Electron. Lett. **23**, 715 (1987).
3. Y. Yamamoto, S. Machida, and O. Nilsson, Phys. Rev. A **35**, 4025 (1986).
4. M. Lax, Phys. Rev. **160**, 290 (1967); see Eqs. (6-6) and (3-24).
5. L. D. Westbrook and M. J. Adams, IEE Proc. J. Optoelectron. **134**, 209 (1987).
6. J. Arnaud, Electron. Lett. **23**, 450 (1987).
7. J. Arnaud, Electron. Lett. **24**, 116 (1988). The spectral densities used in this reference are different from, but equivalent to, those used in the present Letter when $\alpha = 0$, which was the case considered. The interpretation for shot noise must be reconsidered along the lines of the present Letter.
8. R. Lang and A. Yariv, IEEE J. Quantum Electron. **QE-22** (to be published).
9. J. Arnaud, Electron. Lett. **24**, 302 (1988).
10. I. Petitbon, P. Gallion, G. Debarge, and C. Chabran, IEEE J. Quantum Electron. **24**, 148 (1988).