Dielectric-clad monopoles: A most necessary feature of any antenna is that it must be an efficient radiator. Therefore the loss associated with the cladding must be kept low. From radiation-pattern gain and efficiency measurements made on a selection of clad monopoles (as summarised in Fig. 2), the following empirical relationship was found to hold:

$$\eta = \eta_0 (1 - 1{\cdot}75 \tan\delta . \log V) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where η is the overall efficiency, η_0 is the limiting efficiency of the unclad monopole core with no cladding, $\tan\delta$ is the loss tangent and V is volume in cm^3 of the cladding. The lengths and diameters of the various clad monopoles are indicated in Fig. 2.

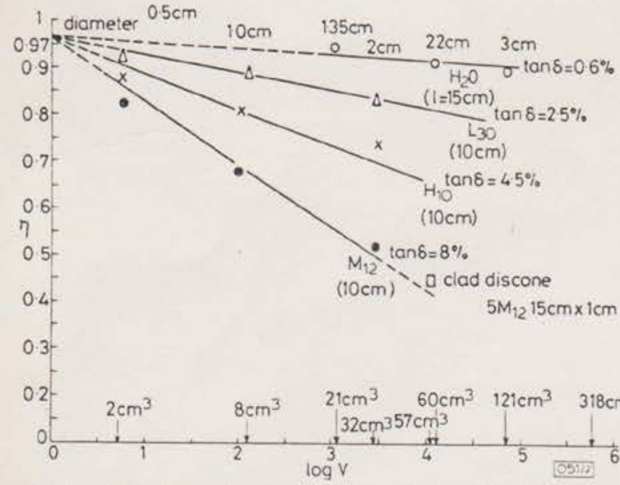

Fig. 2 *Radiating efficiency of M_{12} 'dielectric-clad' discone compared with efficiencies of series of clad monopoles*

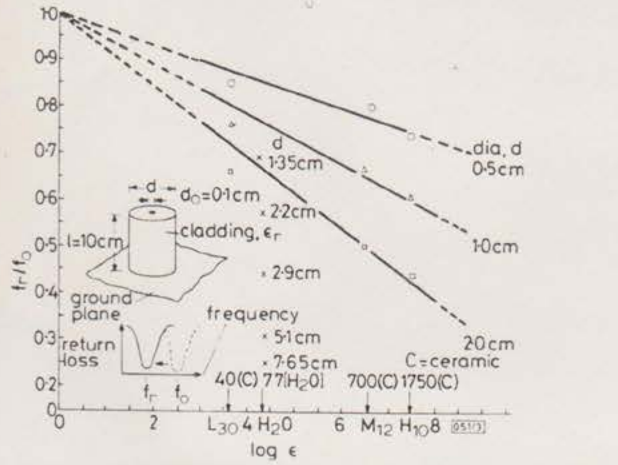

Fig. 3 *Resonance characteristics of series of dielectric-clad monopoles as function of cladding diameter and permittivity*

It can be seen that the radiating efficiency of the clad discone is about 45%, when the effect of mismatch has been allowed for, and is just the extension of the characteristic due to the clad monopole material M_{12} from which the discone was formed.

The materials used in the dielectric cladding should be chosen such that the loss tangent is low for a reasonable cladding volume or diameter to produce the required resonance-frequency reduction while still maintaining a high radiating efficiency. Fig. 3 illustrates the degree of reduction in the resonance frequency f_r that is possible for a fixed monopole length of 10 cm, as a function of the cladding diameter d and permittivity ε_r (f_0 is the resonance frequency of the unclad monopole of the same physical height l and diameter d_0 as the clad-monopole core). Form this data, for both ceramic and water-filled monopoles, the following empirical relationship was found to hold:

$$f_r/f_0 = \frac{\log \varepsilon_r}{k} \left[(d/d_0)^{1/e} - 1 \right] \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where k is a coefficient dependent on the core diameter d_0:

$$k = (0{\cdot}01\, d_0 + 0{\cdot}036)^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Based on these relationships, it was decided that the slant height of the discone should consist of 5 clad rods of M_{12}, each 15 cm long by 1 cm in diameter. It should also be noted that, for the dielectric monopoles to be of use, the resonance characteristic of the clad monopole must contain only one resonance (the first quarter-wave resonance) in the frequency range of interest.

Dielectric-clad discone: Fig. 4 shows the return loss of the dielectric-clad discone over the frequency range 200–1300 MHz, compared with an all metal skeletal version of the same physical size as the clad discone. It can be seen that there is a considerable improvement. It was also found that dielectric cladding at the base of the discone produced a proportionally greater effect on the low-frequency response than was obtained by cladding the cone rods, thus indicating that, by preferentially loading the discone to achieve the desired result, it is also possible to reduce loss. Completely encapsulating the discone in a high-permittivity material does not maintain the required broadband behaviour.[3]

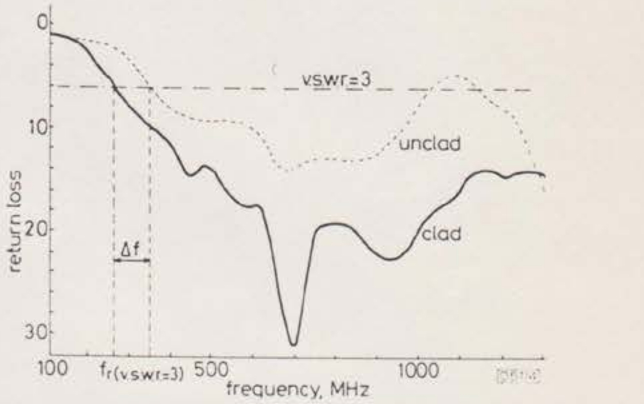

Fig. 4 *Return loss of skeletal discone, with and without cladding*

A recently available material having $\varepsilon_r \simeq 900$ and $\tan\delta \simeq 0{\cdot}2\%$ (as well as excellent frequency- and temperature-stable dielectric properties) will enable the radiating efficiency of a discone clad with this material in the manner described to approach 95%.

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MICROBENDING LOSS OF MULTIMODE SQUARE-LAW FIBRES: A RAY THEORY

Indexing terms: Optical fibres, Optical-waveguide theory

Using a simple ray theory, we show that the steady-state microbending loss of multimode square-law fibres is equal to $8\gamma/\Delta$ dB/m. We have defined $\Delta = \Delta n/n$, and γ is the power-spectral density of the fibre axis curvature in any meridional plane at the natural frequency of ray oscillation, expressed in reciprocal metres. The steady-state irradiance distribution in the fibre core is also given.

It is well known that random bends of multimode optical waveguides cause the amplitude of the rays to increase on the average.[1] Eventually, the rays are lost to the outer parts of the guiding system, as shown in Fig. 1. This effect, called microbending loss, is of great practical importance in fibre optics. In the present letter we derive new simple expressions for the steady-state loss and the irradiance distribution in the fibre core, using a ray theory.

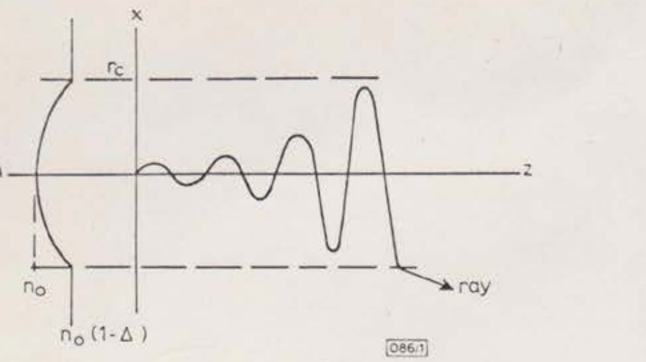

Fig. 1 *Schematic representation of amplitude of ray in multimode fibre*

The rays are sinusoids whose amplitude grows or decays in random fashion under the influence of the curvature $C(z)$ of the fibre axis

Large-capacity optical fibres have index profiles that do not depart very much from square laws. Because the curvature spectrum is usually broad, the fine details of the index profile do not very much influence the microbending loss. A truncated square-law profile

$$n(x,y) = \begin{cases} n_0[1-\Delta(x_1^2+y_1^2)], & x_1^2+y_1^2 \le 1 \\ n_0(1-\Delta), & x_1^2+y_1^2 > 1 \end{cases} \quad . \quad . \quad (1)$$

where we have defined

$$x_1 = x/x_c \qquad y_1 = y/y_c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

is therefore a realistic model for many graded-index multimode fibres.

Let $x = \bar{x}(z)$, $y = \bar{y}(z)$ describe the distorted fibre axis. We define the curvature laws

$$C_x(z) = d^2\bar{x}/dz^2 \qquad C_y = d^2\bar{y}/dz^2 \quad . \quad . \quad . \quad . \quad (3)$$

and assume that $C_x(z)$, $C_y(z)$ are stationary Gaussian random processes of zero mean and microscopic correlation. The spectral power density of these processes is

$$\gamma_{x,y} = 2\int_0^\infty \langle C_{x,y}(z)C_{x,y}(z+\zeta)\rangle \cos(\Omega_{x,y}\zeta)d\zeta \quad . \quad (4)$$

where $\langle\ \rangle$ denotes an average over a large number of similar fibres (ensemble average). Alternatively, γ_x, γ_y can be defined for a single fibre by spatial averagings, since $C_x(z)$, $C_y(z)$ are stationary. $\Omega_x = x_c/\sqrt{(2\Delta)}$, $\Omega_y = y_c/\sqrt{(2\Delta)}$ denote the angular frequencies of ray oscillation.

The probability density P that a ray has position x, y and slope $\dot{x}$, $\dot{y}$ at z and time t is the solution of the Fokker-Planck equation,

$$\partial P/\partial z + v_g^{-1}(e_x, e_y)\partial P/\partial t = (\gamma_x/8\Delta)(\partial^2 P/\partial x_1^2 + \partial^2 P/\partial x_2^2)$$
$$+ (\gamma_y/8\Delta)(\partial^2 P/\partial y_1^2 + \partial^2 P/\partial y_2^2) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where x_1, y_1 have been defined in eqn. 2 and

$$x_2 = \dot{x}/\sqrt{(2\Delta)},\ y_2 = \dot{y}/\sqrt{(2\Delta)},\ \dot{x} = dx/dz,\ \dot{y} = dy/dz,$$

$$e_x = x_1^2 + x_2^2,\ e_y = y_1^2 + y_2^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The excitation conditions are assumed to be such that P depends on x_1, x_2 only through the sum e_x, and on y_1, y_2 only through the sum e_y. The group velocity v_g differs from a constant if the fibre material suffers from inhomogeneous dispersion. An explicit expression for v_g wa given in Reference 2. The right-hand side of eqn. 5 expresses a diffusion in the phase space x_1, x_2, y_1, y_2 caused by microbending. To obtain this result, we used Unger's expression in Reference 1 for the ray position $x(z)$, $y(z)$ and Siegert's results on first-passage probability.[3] Some of the details are given in the Appendix.

We neglect the optical losses in the core and the transmission of slightly leaky rays. Thus a ray is lost to the cladding when $e = e_x + e_y$ reaches unity. The boundary condition to impose on P is therefore $P = 0$ when $e = 1$.

In the following, we discuss the continuous-wave operation, with $\partial P/\partial t = 0$. We assume that the fibre profile and the fibre axis deformations are circularly symmetric, $x_c = y_c = r_c$, $\gamma_x = \gamma_y = \gamma$. We also assume that the excitation is circularly symmetric, that is, P depends on e_x, e_y only through the sum $e = e_x + e_y$. The steady-state solutions $P_m(z, x_1, x_2, y_1, y_2)$ of eqn. 5 are easily found. We have

$$P_m = e^{-\frac{1}{2}} J_1(v_m e^{\frac{1}{2}}) \exp(-v_m^2 \gamma z/8\Delta) \quad . \quad . \quad . \quad . \quad (7)$$

where

$$e = x_1^2 + x_2^2 + y_1^2 + y_2^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

J_1 denotes the Bessel function of order 1, and v_m the mth zero of J_1, e.g. $v_1 = 3{\cdot}8317$. The lowest steady-state loss is therefore, for $m = 1$ in eqn. 7,

$$\text{loss} = 7{\cdot}969\,\gamma/\Delta \quad \text{dB/m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This result is about 25% higher than Marcuse's approximate result in Reference 4, obtained with a modal theory.*

The steady-state irradiance in the fibre core is obtained by integrating P_m in eqn. 7 with respect to x_2 and y_2. The result of this integration is

$$I(r) = J_0(v_m r/r_c) - J_0(v_m) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The variation of the irradiance in the fibre core given in eqn. 10 is plotted as a function of the normalised radius r/r_c in Fig. 2. For comparison, the irradiance in straight fibres excited by Lambertian sources is shown as a dotted line. Note that, in square-law fibres, the far-field irradiance patterns are similar to the near-field patterns.

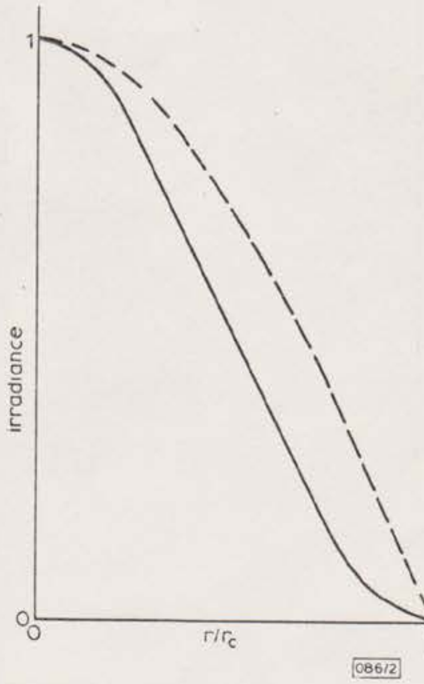

Fig. 2

Plain line shows steady-state irradiance in the fibre core for Gaussian deformations of the fibre axis. Dotted line shows irradiance for an undeformed fibre excited by a Lambertian source

In conclusion, using a simple ray theory, we have obtained new accurate expressions for the steady-state loss and the irradiance of circularly symmetric fibres with random bends. For profiles that may not be amenable to analytic solutions, ray techniques are more economical than modal techniques. For such cases, the present theory may be used as a guide line.

Appendix: Let us first consider the square-law medium in eqn. 1 without the cladding. Let $x(z)$ denote a ray trajectory in the xz-plane and let $C(z)$ denote the curvature of the fibre axis. The subscripts x are omitted in this Appendix for brevity.

The complex ray amplitude $X(z)$ defined by

$$X(z) = \Omega x(z) + i\dot{x}(z),\ \dot{x} = dx/dz \quad . \quad . \quad . \quad . \quad . \quad (11a)$$

is given by[1]

$$X(z) = X(0)\exp(-i\Omega z) + i\int_0^z C(\zeta)\exp[i\Omega(\zeta - z)]d\zeta \qquad (11b)$$

for any curvature law $C(z)$, provided

$$e = |X|^2/2\Delta = x_1^2 + x_2^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

does not exceed unity. If $C(z)$ is a stationary Gaussian process of zero mean and microscopic correlation, the conditional probability that a ray has amplitude e at z, given that $e(0) = e_0$, is found from eqn. 11 with the help of elementary transformations:

$$P(e_1 z|e_0) = (2/\bar{z})\exp[-2(e+e_0)/\bar{z}]I_0(4\sqrt{(e e_0)}/\bar{z}) \quad . \quad . \quad (13)$$

where

$$\bar{z} = \gamma z/\Delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

γ is the power-spectral density of $C(z)$ at Ω and I_0 is the modified Bessel function of order zero. It follows from eqn. 13 that

$$\langle e\rangle = e_0 + \bar{z}/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$\langle e^2\rangle - \langle e\rangle^2 = e_0\bar{z} + (\bar{z}/2)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where $\langle\ \rangle$ denotes an ensemble average.

We now wish to account for the fact that some rays do reach the cladding, that is, e does exceed unity for some of the fibre samples. These samples do not contribute to the average transmitted power. Let us recall Siegert's results on first-passage probability:[3] consider a random process $e(z)$ and let $P(e, z|e_0)$ denote the probability density for e at z, given that $e(0) = e_0$. If the first and second moments $A(e_0)$ and $B(e_0)$, defined, respectively, by

$$A(e_0) = \lim_{\Delta z \to 0} \frac{1}{\Delta z}\int_0^\infty (e - e_0)P(e, \Delta z|e_0)de \quad . \quad . \quad (17)$$

and

$$B(e_0) = \lim_{\Delta z \to 0} \frac{1}{\Delta z}\int_0^\infty (e - e_0)^2 P(e_1 \Delta z|e_0)de \quad . \quad . \quad (18)$$

exist, and all higher moments tend to zero faster than Δz when $\Delta z \to 0$, the probability $f(z|e_0)$ that $e(z)$ does not exceed unity between 0 and z is the solution of the Fokker-Planck equation

$$\frac{\partial f}{\partial z} = A(e_0)\frac{\partial f}{\partial e_0} + \tfrac{1}{2}B(e_0)\frac{\partial^2 f}{\partial e_0^2} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$f(0|e_0) = 1 \qquad f(z|1) = 0$$

For the conditional probability in eqn. 13, it follows from eqns. 15 and 16 that $A(e_0) = \gamma/2$, $B(e_0) = \gamma e_0$. The higher moments tend to zero faster than Δz when $\Delta z \to 0$. Thus the average ray transmission $f(\bar{z}|e_0)$ is the solution of

$$\frac{\partial f}{\partial \bar{z}} = \frac{1}{2}\frac{\partial}{\partial e_0}\left(e_0\frac{\partial f}{\partial e_0}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$f(0|e_0) = 1,\ f(\bar{z}|1) = 0,\ \bar{z} = \gamma z/\Delta$$

It is not difficult to show that the operator on the r.h.s. of eqn. 20 is the first term of the r.h.s. of eqn. 5 in polar coordinates. The l.h.s. of eqn. 5 corresponds to the surviving terms of the space-time Liouville equation given in Reference 5. The full details will be given elsewhere.

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* For a 2-dimensional fibre, there is exact agreement between our and Marcuse's results

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DECISION-DIRECTED REFERENCE CARRIER GENERATION IN AN AUTOCORRELATION RECEIVER

Indexing terms: Demodulation, Error statistics, Phase-shift keying

A receiver for an asynchronous spread-spectrum system is described with special regard to initial synchronisation and error probability in the steady state for a 2-p.s.k. system. The results of simulations and measurements for an analogue and a digital version of the receiver are presented.

The concept of modulation means combining an information carrying signal with a carrier. To separate this information from the carrier, the receiver must know something about the carrier function. In the ideal case, the receiver uses an exact phase-synchronous replica of the carrier function. There are problems of phase synchronisation if the autocorrelation function (a.c.f.), which is frequently used as phase indicator, is flat almost everywhere, as the a.c.f. of a broadband carrier.

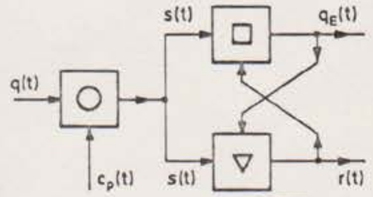

Fig. 1 *Modulation (○), demodulation (□) and remodulation (▽)*

The method described here benerates the replica $r(t)$ of the carrier function directly from the received signal by a decision-directed process. The system shown in Fig. 1 has only one stable state, i.e. $r(t)$ is a periodically repeated function, if the equation

$$[q(t)\circ c_p(t)]\,\square\,([q(t)\circ c_p(t)]\,\nabla\, q_E(t)) = q_E(t) \quad . \quad . \quad (1)$$

has one and only one solution

$$q_E(t) = q(t)$$

The operators are defined as follows:

(*a*) $q(t)\circ c_p(t) = s(t)$; modulation of a carrier $c_p(t)$ by information $q(t)$. $c_p(t)$ is the periodically repeated carrier function $c(t)$.

(*b*) $s(t)\,\square\, c_p(t) = q(t)$; demodulation of the signal $s(t)$ by a carrier $c_p(t)$ gives the information $q(t)$.

(*c*) $s(t)\,\nabla\, q(t) = c_p(t)$; 'remodulation' (carrier regeneration) of the signal $s(t)$ by the information $q(t)$ gives the carrier $c_p(t)$.

For the special case of a 2-p.s.k. spread-spectrum system with information $q(t)$ a binary signal with bit length T and amplitude ± 1 and $c(t)$ a binary signal of m subpulses, two stable states exist if the operators are defined as follows (the subpulse length is T/m; m is called the spreading factor):

○ : multiplication.

□ : multiplication followed by a short-time integration threshold decision and sample-and-hold operation (correlation receiver).