2.6 Measurements

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I. The Delay Factor

It is usually obtained by the analysis of the standing wave pattern measured by a probe moving along the line short-circuited at one end and excited at the other. The main difficulty is encountered in the space harmonics the magnitude of which is not always small compared with the fundamental, particularly near the π-mode (this problem is encountered in the ladder line); then we observe a beat between the fundamental and first harmonic, with a swift variation at the pitch periodicity and a slow variation with a periodicity Θ (distance between two successive minima).

The delay factor can be obtained from the value of Θ:

$$\tau = \frac{\lambda}{2p} - \frac{\lambda}{2\Theta} \tag{1}$$

In the case of the interdigital line, the fundamental $m = 0$ has zero magnitude at the middle of the line and the space harmonic $m = -1$ is the strongest one in that region except near the low frequency cutoff, where the space harmonic $m = +1$ has nearly the same propagation constant.

The field patterns can be further complicated by free propagation radiated from the ends or from discontinuities on the circuit, and this often results in an error by a factor of two on the delay factor.

In some cases, therefore, other methods are more useful. First, if the structure is limited to n cells, it constitutes a cavity having n resonant frequencies for each mode of propagation; the measurement of these frequencies gives n points of the dispersion curve. Notice that, if the reflecting planes are not such that the geometrical images of the enclosed structures with respect to them duplicate perfectly the infinite structure, each resonant frequency is split into two. These problems are also encountered in mode analysis of magnetron circuits.

Another method uses a beam flowing near the structure. The gain is maximum near the synchronism between the beam and the structure; then the delay factor is obtained from the beam velocity. Another structure of known characteristics could, in some cases, be substituted for the beam. Finally, the delay factor can be measured by the method of the S curve. The delay line is approximately matched to a slotted section. Then, one moves a reflection from cell to cell along the delay line and one follows the position of the minimum in the slotted section.

II. The Coupling Impedance

This important characteristic of a line (defined in Section 2.1.) indicates the strength of the field in the interaction space for a given power flowing through it.

If we construct with the structure a cavity as described previously and if we introduce pieces of dielectric material there will result a shift of the resonance frequencies, to which the coupling impedance can be shown to be proportional; the relation between them is given in Section 2.1, VIII. We shall consider the cases of slabs of infinite dielectric constant or infinite thickness ($\beta e > 2$). Then, simple expressions are obtained:

$$\mathfrak{R}' = -\frac{\Delta f}{f} \cdot \frac{v_{\text{ph}}}{v_{\text{g}}} e^{2\beta a} \times \begin{cases} +1 & (\epsilon \text{ infinite}) \\ \frac{\epsilon+1}{\epsilon-1} & (\text{e infinite}) \end{cases} \tag{2}$$

In the case where the line has a high conductivity, the frequency measurements are very accurate, but systematic errors can result from the finite width of the structure. Experimental results on a ladder line can be seen in Fig. 1 b (circles).

It is better to use a second method. A conductive layer of surface conductivity σ is placed parallel to the structure and the insertion loss is measured. This loss depends upon the coupling impedance by the relation given in Section 2.1, VIII, the author's section in this book previously referred to. In the case of small conductivity, the impedance is given by

$$\mathfrak{R}' = -\operatorname{Log} A \, \frac{\sqrt{\epsilon_0/\mu_0}}{\sigma} \frac{1}{\tau} \frac{e^{2\beta a}}{\beta L} \qquad \left(\sigma \ll \frac{2}{\tau}\sqrt{\frac{\epsilon_0}{\mu_0}}\right) \tag{3}$$

A is the ratio between the output and the input power; L is the length of the attenuation.

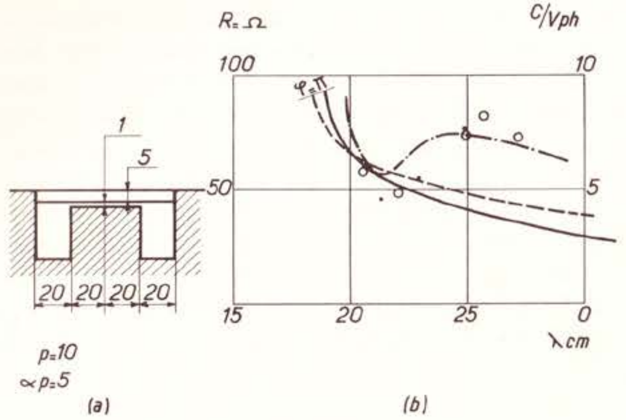

FIG. 1. Measurement of the coupling impedance. (a) Sketch of the line (all dimensions in millimeters); (b) ——— theoretical dispersion curve; (— — —) experimental dispersion curve (—·—) theoretical coupling impedance; (o) experimental points obtained with the dielectric; (·) experimental points obtained with the attenuator. ($\mathfrak{R}' \simeq 0.003R$.) All dimensions are in millimeters.

Experimental results on a ladder line are indicated in Fig. 1 b (black points). Finally, one can here again measure the coupling impedance with an electron beam. It will be simpler and more accurate to use an O-type beam. One knows that the gain is zero for a shift from synchronism which is related to the coupling impedance (*1*) (J. Kompfner dip condition).

In a similar way, it is possible to use another circuit instead of the beam and to measure the Kompfner condition (for two forward waves or for two backward waves). All these methods, except the latter two, measure the total field and not the field of the space harmonic considered for interaction. E. J. Nalos (*2*) has refined the experimental procedure and obtained the magnitude of the space harmonics.

III. Matching of the Input and Output Circuits

We want now to test the transition between a coaxial line or a waveguide and a delay structure. If we suppose that the delay structure is perfectly matched at the output, we can measure the input coaxial line or waveguide VSWR corresponding to the mismatch of the input circuit.

It must be noticed that in lossless reciprocal systems the VSWR measured on the delay structure when the coaxial line is matched is the same as the VSWR previously defined.

The interdigital line is in principle easy to match to a coaxial line because the input admittance at the free end of the first finger is equal to $c\gamma'$, where γ' is the capacity per unit length between adjacent fingers, and is real and constant. If $c\gamma'$ is also the characteristic conductance of the coaxial line, matching is easy. Different kinds of tapered sections can be used also: on the larger walls of a rectangular waveguide working with the H_{01} mode, one can place fingers of increasing length, and load the waveguide capacitively in a progressive manner. When the length of the fingers approaches the size of the small sides of the guide, the H_{01} mode is transformed into interdigital line propagation. In the case of the ladder line, however, the input impedance, defined for instance between the middle of the first bar and ground, is not real nor constant but if the dispersion is mainly low, good matching can be achieved.

IV. Attenuation of the Structure

The attenuation of the structure is usually made of a metal deposit on the bars of the structure (the metal is for instance kovar + kanthal deposited by the "shoop" process); its permability depends upon the dc magnetic field and so does the attenuation. The insertion loss is usually of the order of 10 to 20 db. The simplest method to measure it would be to short-circuit the line after the attenuation and to measure the VSWR at the input. Then, we would have, if the attenuation in power is A,

$$\text{VSWR} = \frac{1 + A}{1 - A} \tag{4}$$

This supposes perfect matching between the slotted line, with which the VSWR is measured, and the structure; as a matter of fact this is not the case, but if we move a short circuit after the attenuation we can obtain A from the variations of the VSWR at the input.

List of Symbols

a	distance in the Oy direction
f	frequency
e	dielectric thickness
ϵ_0	vacuum permittivity

μ_0	vacuum permeability
λ	free space wavelength
v_{ph}	phase velocity
β	delayed propagation constant
p	pitch
v_{g}	group velocity
m, n	integers
θ	beating wavelength
$\mathfrak{R}'$	mean coupling impedance
ϵ	relative dielectric constant
L	length of line
A	power attenuation
τ	delay factor
σ	surface conductivity

References

1. D. A. WATKINS AND A. E. SIEGMAN, The helix impedance measurements using an electron beam, *J. Appl. Phys.* **27**, p. 917 (1953).
2. E. J. NALOS, Measurements of circuit impedance of periodically loaded structures by frequency perturbation. *Proc. I.R.E.* (*Inst. Radio Engrs.*) **42**, p. 1508 (1954).