Linewidth of Laser Diodes with Nonuniform Phase-Amplitude α-Factor

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Abstract—The linewidth of a laser diode having a phase-amplitude factor α that varies arbitrarily along the path is calculated. For simplicity, an ideal single-mode ring-type laser diode with only one wave circulating is considered. The theory is exact in the limit of large injected currents, provided parameters such as the carrier temperature do not vary and the gain or loss per wavelength is small. It is found that when the electron–hole pairs are injected independently of each other (that is, when the pump fluctuations are spatially uncorrelated shot noises) the linewidth is half the value obtained earlier for the linear regime multiplied by $(1 + \alpha^2)_{av}$ where the round-trip averaging is affected with respect to the reciprocal of the power gain. Specific examples, in particular a sequence of amplifiers and partially reflecting mirrors, are considered.

I. Introduction

A LASER diode consists of a piece of active material of length L located between two reflectors of power reflectivity R. When the power gain per pass equals $1/R$, the device oscillates at a frequency ν_o, but the oscillation is not strictly monochromatic, even for a single-mode oscillator. The field spectrum is usually Lorentzian, at least near ν_o, with a full width at half-power points $\Delta\nu$ [1]. The purpose of this paper is to evaluate $\Delta\nu$ in the saturated regime, in which the injected current density is independent of the laser dynamics and the carrier density fluctuates as a function of time. When the carrier density in the active material varies, the complex permittivity ϵ is perturbed by $\delta\epsilon$. The "phase-amplitude coupling factor" $\alpha \equiv -\mathrm{Re}\,(\delta\epsilon)/\mathrm{Im}\,(\delta\epsilon)$ is on the order of 5 [2] if the oscillation occurs near the maximum of the gain curve (with respect to the sign of α, note that we are using an $\exp(-i2\pi\nu t)$ convention). Lax showed in 1967 on the basis of a simple circuit model [3] that the linewidth $\Delta\nu$ is enhanced by a factor $(1 + \alpha^2)$ when the cavity conductance is independent of frequency. Henry [4] first pointed out the importance of this factor for index-guided laser diodes.

In the theories presently available, the α-factor is assumed to be the same everywhere along the laser length. As a matter of fact, the α-factor may vary because of varying saturation conditions, varying injected current density, or compositional changes (intentional or not) in the active layer. Stepwise variations of α also occur in ring-type resonators consisting of any number of amplifying laser diodes (antireflection-coated laser diodes) and Faraday isolators introduced to help select one propagating wave. The linewidth formula for this particular configuration, given at the end of this paper, could be verified experimentally with existing technology.

Quite generally, we investigate whether longitudinal changes of α may significiantly affect the linewidth. We show that Lax's $(1 + \alpha^2)$ factor must be replaced by $(1 + \alpha^2)_{av}$ where "av" stands for an averaging taken along the laser diode. For ring-type diodes with only one wave circulating, the averaging is to be effected, not with respect to the longitudinal z-coordinate itself, but with respect to the reciprocal of the power gain. For a conventional laser diode with a folded optical axis, the averaging is to be taken with respect to the injected current density. If the spontaneous emission factor $n_{sp} \neq 1$, the linewidth enhancement factor is $[n_{sp}(1 + \alpha^2)]_{av}$. In this expression, n_{sp} as well as α may vary arbitrarily along the laser length. Exactly the same result is applicable when any number of active elements are connected in parallel, as we recently reported [5].

The averaging procedures just quoted are valid only if the electron–hole pairs are injected in the active material independently of each other, in both space and time. In other words, the injected current density is supposed to exhibit spatially uncorrelated shot noise. The shot noise fluctuations of the pump (i.e., of the injected current) are usually deemed to have a negligible effect on the linewidth. This conclusion, however, does not hold when α varies. Indeed, if shot noise is suppressed, e.g., by driving the active elements with electrons from a space-charge-limited thermoionic tube, one should subtract from the above expressions half the spatial variance of α. A concise presentation of these results has been given [6].

Let us now clarify the difference between the saturated regime considered in this paper and the linear regime treated earlier [7]. In the linear regime, the field intensity fluctuations are not supposed to influence the carrier density significantly. The active medium optical parameters are therefore time invariant. The α-factor is irrelevant, and there are no relaxation oscillations since the carrier density is a constant. While for most lasers the linear theory is applicable only below threshold, in laser diodes it can also occur above threshold if the diode is driven by a constant dc voltage and the resistances of the confining layers are small. Indeed, the energy spacing between

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quasi-Fermi levels, and therefore the carrier density, is in that case independent of both time and space.

In the linear regime, the laser output is essentially amplified spontaneous emission, and the optical power fluctuates according to a Rayleigh (exponential) distribution. The linewidth $\Delta\nu$ of the oscillating field is given by the well-known Schawlow–Townes (ST) formula [1] when the cavity has a high quality factor or is homogeneously filled. The ST formula is not applicable to laser diodes, however, because gain and loss do not occur at the same location in space and have large values. A correcting factor that accounts for this effect was first given by Petermann [8] in the linear theory of gain-guided lasers. Petermann's transverse K-factor and a longitudinal K-factor [9] follow from the general three-dimensional formula given in [7], which is also in agreement with Henry's results [4].

The simple linear regime is generally not applicable to laser diodes because, in the first place, the diode is current driven. The voltage across the active layer is not fixed by the power supply, and the carrier density can therefore react to counteract the perturbing effects of spontaneous emission. In the second place, the voltage drop in the confining layers is usually larger than or comparable to the voltage drop across the active layer. A plausible model, then, is that the injected current *density* J at any point z along the laser length is independent of the laser dynamics. This is the regime assumed in the present paper.

We consider ring-type diodes because the basic concepts are most easily explained in that configuration [see Fig. 1(a)]. These diodes, incidentally, have the practical advantage (over conventional laser diodes with a folded optical axis) of a reduced sensitivity to spurious external reflections since the power eventually reflected back (e.g., from the fiber input) may be absorbed at the input end of the oscillator [see Fig. 1(b)]. Fabrication techniques are in progress [10]. The assumption is made that some nonreciprocal materials along the path prevent counterclockwise waves from reaching the threshold of oscillation. We are therefore not concerned with the complex refractive index grating that two counterpropagating waves would create if the (ambipolar) carrier diffusion length L_D were comparable to or smaller than the guided wavelength. The diffusion length L_D, as well as the current spreading length, is on the order of a few micrometers, and therefore much smaller than the diode length $L \approx 500$ μm. It is easy to show that if the current density J injected in a ring-type diode with one wave circulating is z-independent, the local gain varies linearly with a minimum-to-maximum ratio equal to the mirror power reflectivity R [Fig. 1(b)]. From a dynamical point of view, each elementary length of the diode (called dz in the mathematical treatment) possesses independent noise sources, and the field fluctuations are only partly correlated along the length. The laser diode therefore must be considered a multiple-active-element oscillator.

In the saturated regime, the relative power fluctuations are small and have a negligible direct effect on the linewidth, which depends mostly on phase or frequency fluctuations. The fundamental noise sources (to be discussed below) are white, Gaussian, and stationary. They result in an instantaneous frequency deviation $\delta\nu(t)$ which is also Gaussian and stationary by linearity, but not white. In this paper, we evaluate the linewidth $\Delta\nu$ in the limit of large injected currents, assuming that the average parameters (e.g., the carrier temperature) do not vary. In that limit, the field spectrum in Lorentzian near ν_o, and the linewidth $\Delta\nu$ is equal to π times the (single-sided) spectral density of the $\delta\nu(t)$ process at zero baseband (or "Fourier") frequency. This is the low-deviation limit treated in great detail by Rowe [11, eq. (199) and Fig. 4–10(a). Notice that Rowe uses double-sided spectral densities].

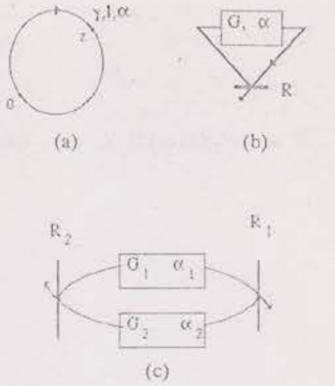

Fig. 1. (a) Schematic of a ring-type laser. The z-coordinate along the path varies from 0 to L, and an arbitrary dependence on z of the phase-amplitude coupling factor α is considered. $\gamma(z)$ is the power gain defined from the origin, and $l(z)$ is the power loss. Only the clockwise wave is considered. (b) Ring-type laser incorporating a lossless amplifying medium and a single partially reflecting mirror. (c) Ring-type laser incorporating two lossless amplifiers and two partially reflecting mirrors.

In a phase-shift keyed optical link, the important quantity is the statistics of the phase jitter: $\Phi(t + T) - \Phi(t)$ where T is the time slot duration. These statistics cannot be obtained from the linewidth $\Delta\nu$ alone when the frequency deviation spectrum is not white. We therefore acknowledge that the linewidth $\Delta\nu$ that we evaluate does not fully characterize the laser diode noise properties. To characterize the laser noise properties fully, one must consider nonzero baseband frequencies, as was done in [12] for a single active element. Detailed results for variable α will be reported elsewhere.

In our opinion, the usual rate equations for photon and carrier numbers are not appropriate to evaluate the linewidth of variable-α laser diodes.

Consider first a single active element with a K-factor larger than unity. According to previous theories based on rate equations, the linewidth would be proportional to $K(1 + \alpha^2)$. The correct expression for a thin-slab model turns out to be, however, $K(1 + \alpha^2)/(1 + \alpha\kappa)^2$ where $K = 1 + \kappa^2$. The latter expression was first given by Lax [3] in 1967 in a different context and rediscovered by us [13], specifically in connection with thin-slab laser diodes. Similar expressions can be obtained for conventional gain-guided lasers. One reason for the (quantitatively very important) discrepancy is that conventional theories require

that α be independent of the transverse coordinates, while $K \neq 1$ values imply that transverse inhomogeneities are present.

In the present paper, the transverse K-factor is assumed to be unity. The conventional rate equations, however, suffer from other, less conspicuous, deficiencies. One of the rate equations describes the time evolution of the number of photons in the cavity, that is, of the electromagnetic energy stored between the diode end facets divided by the photon energy $h\nu$. There exists, however, no general expression for the stored energy in a medium with gain or loss in terms of the complex permittivity and its first derivative with respect to frequency. The expression sometimes employed, proportional to $d[\nu \operatorname{Re}(\epsilon)]/d\nu$, is not accurately quoted from Landau and Lifshitz's book [14], in which it is specified that the permittivity must be real. A simple example (a laser cavity incorporating a matched transmission line) clearly shows that the stored energy may be increased arbitrarily without affecting the laser operation. This ambiguity, noted by Henry [4] in connection with internal losses, was too quickly dismissed.

Because classical textbooks on laser diodes begin with Einstein's relation between spontaneous (A) and stimulated (B) emission coefficients, which depend for their definition on the concept of energy, one gets the feeling that the concept of energy is indispensable. Einstein's relation, however, is unnecessary. Given the carrier density (n, expressed is the number of electrons per cubic meter) and temperature (T), the solution of Schrödinger's equation for a semiconductor (supplemented by the Fermi–Dirac distribution) is capable of giving directly the (positive or negative) semiconductor conductivity σ at some frequency ν. Using Kane's k-p method [15] and some approximations (parabolic bands, k-conservation rule, etc.), one obtains, for example, at $T = 0$ K a minimum value $\sigma = -4\ 10^{-5}\ n^{1/3}$ S/m. This result is derived from first principles without any reference to the concept of energy or spectral density [16]. (The negative conductivity σ is introduced in this paper in the form of a conductance per unit length G_a.) The optical power P_{sp} going into radiation modes can be evaluated from the Nyquist-like current sources associated with $\sigma(\nu)$ and Maxwell's equations. To within the constant $e/h\nu$, this P_{sp} power is the threshold current if we leave aside nonradiative recombinations. Therefore, the need to introduce the concept of energy in laser diode theory nowhere arises, at least in the small-signal approximation.

The carrier rate equation as usually written is also ambiguous. First, it is usually not specified that the current injected in the diode exhibits shot noise fluctuations. These fluctuations, however, are not intrinsic to the laser, but depend on the driving circuit [17] and may or may not exist.

Second, the diffusion coefficients of the Langevin noise term in the carrier rate equation given by various authors differ by a factor of two when the electron–hole inversion is complete and spontaneous emission can be neglected (compare, e.g., [4] and [18]). Because in some of these theories the photon shot noise is included in the amplitude fluctuations, while in others it is not included, the measurable amplitude fluctuations predicted at zero baseband frequency end up being the same. However, other results, and in particular the predicted linewidth of oscillators incorporating more than one active element, are not the same in the two formulations.

Our theory differs conceptually from the conventional rate equation theories. It is based on Nyquist-like noise *currents* (not "photon events") and on the law of conservation of particle *rate*. The concept of photon *number* or of energy is nowhere used. The usual photon rate equation is replaced by an equation for the instantaneous complex resonant frequency. At low baseband frequencies, it suffices to specify that the oscillation frequency is real. This does not imply that the field fluctuations vanish, but only that they are bounded. Previous theories (e.g., [4], [19]) have used also Nyquist-like noise currents as basic noise sources. However, because the time-dependent particle rate conservation law (in which one must include the Nyquist currents) is not written down in these theories, only partially valid results were obtained. Our semiclassical theory agrees exactly with the predictions of quantum mechanics as established by Yamamoto and others [17], even in the case of external electronic feedback, for both the amplitude and phase fluctuations. Yamamoto [17] considerably improved our understanding of laser fluctuations by being careful in distinguishing between the internal field (on which most previous quantum theories had focused, but which cannot easily be measured and is therefore of little interest) and the external field, which is the quantity of interest. Instead of "internal field" and "external field," we find it preferable to speak of the "internal energy" and of the (well-defined and measurable) *dissipated power*, respectively, because the generated power is not necessarily dissipated externally. In our basic theory, we do not distinguish between power dissipated or scattered away, internally or externally. A separate calculation is needed to evaluate that part of the total dissipated (or scattered) power which we chose to consider "useful."

The basic concepts relating to noise sources are presented in Section II. Readers not interested in the mathematical details may go directly from there to Section VI, where the basic formula is given together with the required notation and a number of practical examples are worked out. The laser diode linewidth is expressed in terms of a simple integral along the path for any variations of the phase-amplitude coupling factor $\alpha(z)$, the gain $\gamma(z)$, the loss $l(z)$, and the injected bias current density $J(z)$. This formula is exact under the assumptions stated above. To our knowledge, the general result in (42) is new, and we are unaware of valid alternative procedures that would lead to it. The case where the spontaneous emission factor $n_{sp}(z)$ is not unity and the case where shot noise is suppressed are treated in Sections VII and VIII, respectively. In Section IX, we show that for a conventional laser diode the α-averaging may be effected with respect to the injected bias current density, but additional approximations are required.

Let us now consider the intermediate steps. The model for the ring-type optical cavity is a transmission line of characteristic impedance Z_c, closed on itself; see Section III and the Appendix. This transmission line is loaded by a conductance G, split into a passive conductance G_p and an active conductance G_a. The change of the complex oscillating frequency of the diode due to any small change of G (perturbation formula) is evaluated in Section IV. The carrier rate equation is written down in a simplified form in Section V. We specify that the rate of injected electron–hole pairs equals the photon generation rate. This assumption holds when the driving current is much higher than the threshold current and when the quantum efficiency is close to unity.

The modulation properties of the laser follow from the general formulas established in Section V. Frequency modulation can be obtained from slow injected current changes only when α varies along the diode length and at least two electrodes are used, as other authors have pointed out [20], if we do not consider thermal effects. These modulation properties, however, will not be discussed in detail.

II. Basic Noise Sources

The resonant frequency of a cavity is perturbed by the fluctuations that are associated with any dissipative or active element that it contains (fluctuation–dissipation theorem). Specifically, one associates with any (positive or negative) conductance G a white Gaussian fluctuating current $i(t)$ whose (one-sided) spectral density is [19]

$$S_i = 2h\nu\,|G| \tag{1}$$

where the vertical bars mean "absolute value." For the narrow-band processes of interest to us, it is convenient to express $i(t)$ in the form of a slowly varying complex current $I(t) \equiv c(t) + is(t)$. The spectral densities of these c and s processes are

$$S_c = S_s = 4h\nu\,|G|; \qquad S_{cs} = 0. \tag{2}$$

If G represents a passive (time-independent) positive conductance, the perturbation is just that due to the fluctuating current whose spectral properties are defined in (2). However, if G is negative and represents stimulated emission, the circuit is perturbed not only by the fluctuating current in (2), but also by the reaction of the medium, that is, by the (generally complex) changes of G due to carrier density fluctuations. This is why the carrier rate equation must be considered even at vanishingly small baseband frequencies.

In order to exhibit more clearly the essential features, an ideal laser is considered in which spontaneous recombination in modes other than the oscillating mode is negligible compared to stimulated recombination. Furthermore, the quantum efficiency is supposed to be unity, and the electron–hole population inversion is assumed to be complete. Then the instantaneous rate of electron–hole pair generation equals the rate of photon generation in the mode. It is further assumed that the injected current density J exhibits shot noise; that is, the electron–hole pair arrival times are independent of each other. The hole arrival times are fully correlated with the electron arrival times through space-charge fields. This assumption, commonly made in semiconductor noise theory, holds at least at the small baseband frequencies considered. Thus, the spectral density of the injected current fluctuation j is taken to be

$$S_j = 2e\,|J| \tag{3}$$

where $-e$ denotes the electronic charge. Notice the similarity of S_i and S_j.

The fluctuations relating to two distinct points in space are uncorrelated. Thus, if $G(z)$ represents a conductance per unit length in the transmission line model and $J(z)$ is the injected current per unit length,

$$S_{ii'} = 2h\nu\,|G(z)|\,\delta(z - z') \tag{4}$$

$$S_{jj'} = 2e\,|J(z)|\,\delta(z - z') \tag{5}$$

where $i \equiv i(z)$, $i' \equiv i(z')$, $j \equiv j(z)$, $j' \equiv j(z')$, and $\delta(\cdot)$ is Dirac δ-function.

III. The Transmission Line Formalism

A ring-type laser can be modeled approximately as a transmission line with distributed gain and loss, closed on itself. Provided that the gain or loss per wavelength is small, as is usually the case, the complex propagation constant $k(z)$ can be written as (see the Appendix)

$$k(z) = k_o(z, \nu) + iZ_c(z)\,G(z)/2 \tag{6}$$

where $k_o(z, \nu)$ is real and $G(z)$ denotes the conductance per unit length of the transmission line. The characteristic impedance Z_c of the transmission line is not affected significantly by G and can therefore be considered real.

Only waves propagating in the positive z-direction are considered. Some nonreciprocal effects are implied that prevent counterpropagating waves from reaching the threshold of oscillation. According to classical transmission line theory, the voltage $V(z)$ is (see the Appendix)

$$V(z) = V(0)\exp\left[i\int_0^z k(z, \nu)\,dz\right] \tag{7}$$

and the resonance condition $V(L) = V(0)$ is, using (6),

$$\int_0^L k_o(z, \nu)\,dz = 2q\pi;$$

$$\int_0^L Z_c(z)\,G(z)\,dz = 0 \tag{8}$$

where q denotes the longitudinal mode number.

The power $S(z)$ propagating along the path at z is

$$S(z) = |V(z)|^2/2Z_c(z), \tag{9}$$

and it evolves with z according to

$$S(z) = S_o\exp\left[-\int_0^z Z_c(z)\,G(z)\,dz\right];$$

$$S_o \equiv S(0) \tag{10a}$$

$$S^{-1}dS/dz = -Z_cG. \tag{10b}$$

Using (9) and (10), we verify that there is no net power gained or lost along the path

$$\int_0^L \tfrac{1}{2}G(z)\,|V(z)|^2\,dz = \int_0^L (Z_cG)(|V|^2/2Z_c)\,dz$$

$$= S(0) - S(L) = 0. \tag{11}$$

Let us now split $G(z)$ into $G_p(z) - G_a(z)$ where $G_p(z)$ represents the passive part, while the active part $G_a(z)$ expresses stimulated emission. Because the electron–hole population inversion is assumed to be complete in the relevant range of energies, no stimulated absorption process takes place. Since no carrier-dependent loss mechanism is present, G_p can be considered a constant. The stimulated process which brings electrons from the split-off band to the top of the heavy-hole valence band is not considered here. It occurs only when the spin-orbit splitting energy is comparable to the bandgap energy.

Let us define a loss factor $l(z)$ and a gain factor $\gamma(z)$ by

$$l(z) = \exp\left[\int_0^z Z_cG_p(z)\,dz\right];$$

$$dl/dz = Z_cG_pl \tag{12a}$$

and

$$\gamma(z) = \exp\left[\int_0^z Z_cG_a(z)\,dz\right];$$

$$d\gamma/dz = Z_cG_a\gamma \tag{12b}$$

respectively. Note that

$$l(0) = \gamma(0) = 1; \qquad l(L) = \gamma(L) \equiv \Omega. \tag{13}$$

With the above notation, (10) reads

$$S(z) = S_o\gamma(z)/l(z). \tag{14}$$

Using (12b), (9), (14), and the fact that $\gamma = l$ at both ends of the integration interval, the total generated power can be written as

$$P_T = \int_0^L \tfrac{1}{2}\left[G_a(z)\,|V(z)|^2\right]dz$$

$$= S_o\int_1^\Omega d\gamma/l(\gamma) = S_o\int_1^\Omega dl\,\gamma/l^2 \tag{15}$$

where we found it convenient to use γ in place of z as an independent variable and we consider the loss "l" as a function of γ. Note that γ and l are nondecreasing functions of z. Therefore, l is a nondecreasing function of γ, and conversely.

In the next section, the effect of a small complex change of G on the complex resonance frequency is considered.

IV. The Perturbation Formula

If the active medium is perturbed, $G(z)$ being incremented by some small complex admittance per unit length $y_T(z) \equiv g_T(z) + ib_T(z)$, the real part of the second relation in (8) must remain equal to zero. We therefore have

$$\int_0^L Z_c(z)\,g_T(z)\,dz = 0. \tag{16}$$

As presently established, this relation holds only for perturbations $y_T(z)$ that vary slowly with z. Consider, however, a small current source I at some z-location. It splits into two currents: $+I/2$ in the forward direction and $-I/2$ in the backward direction. The $+I/2$ current generates a forward-propagating voltage wave $\delta V = Z_cI/2$. Therefore, the complex phase shift introduced by the small current I is equal to $iZ_cy_T/2$ where $y_T = -I/V$. The imaginary part of this expression, upon integration, gives again (16). This alternative derivation shows that (16) can be employed for perturbations that are uncorrelated along the z-axis and therefore do not vary slowly.

Expanding now $k_o(z, \nu)$ to first order in ν, the frequency deviation $\delta\nu$ is obtained from the imaginary part of the first relation in (8) in the form

$$2\pi\delta\nu = (2\tau)^{-1}\int_0^L Z_cb_T(z)\,dz \tag{17}$$

where τ denotes the round-trip time, that is, the integral of $\partial k_o/\partial(2\pi\nu)$ over the round-trip length L. Equations (16) and (17) can be obtained alternatively from the general perturbation formula [7], which involves the adjoint or counterpropagating voltage. But in the present case, a direct approach is physically more appealing. The time dependence of the perturbations need not be shown explicitly at the moment because only the adiabatic or slowly varying regime is considered.

The perturbing admittance y_T is the sum of three terms: 1) The first term is the fluctuating current $i_a = c_a + is_a$ associated with the active conductance G_a, divided by the voltage V. 2) The second term is the fluctuating current $i_p = c_p + is_p$ associated with the passive conductance G_p, divided by the voltage V. We find it convenient to replace V by $|V|$ in these expressions. This is permissible for white processes. 3) The third term is the change $g(1 - i\alpha)$ of the medium admittance where $\alpha \equiv -b/g$ denotes the phase-amplitude coupling factor. α is usually positive and may vary along the laser length. Thus,

$$y_T = (c_a + is_a)/|V| + (c_p + is_p)/|V| + g(1 - i\alpha). \tag{18}$$

Separating the real and imaginary parts in (18), we have

$$g_T = (c_a + c_p)/|V| + g;$$

$$b_T = (s_a + s_p)/|V| - \alpha g. \tag{19}$$

Using (19), (17) can be written in the form

$$2\pi\delta\nu = (2\tau)^{-1}\int_0^L Z_c[(s_a + \alpha c_s + s_p)/|V| + \alpha g_a]\,dz \tag{20a}$$

where $g_a \equiv -c_a/|V| - g$ represents the total change of G_a. Equation (20a) is conveniently written as an integral over the gain factor γ using (12b):

$$2\pi\delta\nu = (2\tau)^{-1}\int_1^\Omega d\gamma \cdot \gamma^{-1}[(s_a + \alpha c_a + s_p)/G_a|V| + \alpha g_a/G_a]. \tag{20b}$$

To evaluate g_a, we need the relative fluctuation $\rho(z)$ of the propagating power $S(z)$ defined in (9):

$$\rho(z) = \delta S(z)/S(z). \tag{21}$$

If we take the logarithmic variation of (10), we obtain

$$\rho(z) = \rho(0) - \int_0^z Z_c g_T\,dz; \quad d\rho/dz = -Z_c g_T \tag{22}$$

where g_T denotes, as before, the total conductance change. With the help of (12b), the second expression in (22) is written as

$$\gamma\, d\rho/d\gamma = -g_T/G_a \tag{23}$$

or, using the first relation in (19) and the definition of g_a following (20a),

$$\gamma\, d\rho/d\gamma = g_a/G_a - c_p/G_a|V|. \tag{24}$$

The remaining step to obtain g_a requires that the carrier rate equation be written down.

V. The Carrier Rate Equation

It is assumed that the rate of injected electron–hole pairs equals the rate of photon generation in the oscillating mode at any time. We therefore have, equating the generation rates,

$$J(z)/e = G_a(z)\,|V(z)|^2/2h\nu. \tag{25}$$

If we increment $J(z)$ by $j(z)$, we have from (25), since g_a denotes the variation of G_a,

$$j/J = g_a/G_a + \rho. \tag{26}$$

According to (24) and (26), the relative voltage intensity fluctuation ρ obeys the differential equation

$$d(\gamma\rho)/d\gamma = j/J - c_p/G_a|V| \equiv j_n, \tag{27}$$

whose solution, in order that $\rho(\Omega) = \rho(1)$, is

$$\gamma\rho(\gamma) = \int_1^\gamma j_n\,d\gamma + (\Omega - 1)^{-1}\int_1^\Omega j_n\,d\gamma. \tag{28}$$

Let us summarize what has been achieved so far. Equation (20b) gives the frequency deviation $\delta\nu$. Into that equation enters g_a, which is expressed in terms of ρ and j in (26), and ρ itself is expressed in terms of j and c_p in (27) and (28). We have therefore achieved our aim of expressing $\delta\nu$ as a sum of the five uncorrelated processes: c_a, s_a, c_p, s_p, and j. In the present extended structure model, these currents are functions of z, and the sum actually involves integrals over z (or γ). When an integral is performed over γ instead of z, we must use the following transformation of the Dirac δ-function:

$$\delta(z) = (d\gamma/dz)\,\delta(\gamma). \tag{29}$$

The usual absolute value in this transformation is unnecessary since $d\gamma/dz \geq 0$.

Let us recall now that if $\delta\nu = ax + by + \cdots$ is the weighted sum of independent processes $x(t)$, $y(t)$, $\cdots$ of spectral densities S_x, S_y, $\cdots$, respectively, the spectral density of $\delta\nu$ is

$$S_{\delta\nu} = |a|^2 S_x + |b|^2 S_y + \cdots. \tag{30}$$

We are therefore in position to evaluate $s_{\delta\nu}$.

Because the algebra is involved, it is convenient to define the normalized processes

$$p \equiv (S_o/2h\nu)^{1/2} c_p/G_a|V|;$$

$$S_{pp'} = \gamma\, dl/d\gamma\, \delta(\gamma - \gamma') \tag{31a}$$

$$a \equiv (S_o/2h\nu)^{1/2} c_a/G_a|V|;$$

$$S_{aa'} = l(\gamma)\,\delta(\gamma - \gamma') \tag{31b}$$

$$m \equiv (S_o/2h\nu)^{1/2} j/J;$$

$$S_{mm'} = l(\gamma)\,\delta(\gamma - \gamma'). \tag{31c}$$

Relations similar to (31a) and (31b) can be applied to s_a and s_p. The expressions for the spectral densities in (31) follow from the basic equations (4) and (2), (12), and the transformation rules in (29) and (30).

Let us consider first the first parenthesis in the integrand of (20b). For that term, we have

$$\delta\nu^{(1)} = (4\pi\tau)^{-1}\int_1^\Omega d\gamma\,\gamma^{-1}(s_a + \alpha c_a + s_p)/G_a|V| \tag{32}$$

and therefore, using the mathematical results in (30), (31a) and (31b),

$$S_{\delta\nu(1)} = (4\pi\tau)^{-2}(2h\nu/S_o)\int_1^\Omega d\gamma \cdot [(1 + \alpha^2)\,l(\gamma) + \gamma\, dl/d\gamma]/\gamma^2. \tag{33}$$

The second term in (20b) can be written in terms of the normalized processes as

$$\delta\nu^{(2)} = (4\pi\tau)^{-1}(2h\nu/S_o)^{1/2}\int_1^\Omega d\gamma\,(\alpha/\gamma)\,f(\gamma) \tag{34a}$$

where

$$f(\gamma) \equiv m - \gamma^{-1}\int_1^\gamma d\gamma_1\,(m - p) - \gamma^{-1}(\Omega - 1)^{-1}\int_1^\Omega d\gamma_1\,(m - p) \tag{34b}$$

and we have used (26), (27), and (31). Using again the mathematical result in (30), we obtain

$$S_{\delta\nu(2)} = (4\pi\tau)^{-2}(2h\nu/S_o)\int_1^\Omega d\gamma\int_1^\Omega d\gamma' \cdot (\alpha\alpha'/\gamma\gamma')\,S_{ff'}. \tag{35}$$

Let us evaluate $S_{ff'}$. Notice that $f(\gamma)$ in (34b) consists of two terms linear in p and three terms linear in m. Consider first the terms in p. They lead to $2 \times 2 = 4$ terms for $S_{ff'}$, namely,

$$S_{ff'} = (\gamma\gamma')^{-1}\Big\{\int_1^{\min(\gamma,\gamma')} d\gamma\,\gamma(dl/d\gamma) + (\Omega - 1)^{-1}\int_1^\gamma d\gamma\,\gamma(dl/d\gamma) + (\Omega - 1)^{-1}\int_1^{\gamma'} d\gamma\,\gamma(dl/d\gamma) + (\Omega - 1)^{-2}\int_1^\Omega d\gamma\,\gamma(dl/d\gamma)\Big\}. \tag{36}$$

Consider next the terms linear in m. There are three such terms that we label: 1, 2, and 3, leading to nine terms in the solution. The terms 22′, 23′, 32′, and 33′ are identical to the ones obtained above except that the integrand is "l" instead of "$\gamma(dl/d\gamma)$." When these terms are added to those in (36), the integrals can be evaluated in closed form since

$$\gamma(dl/d\gamma) + l = d(l\gamma)/d\gamma, \tag{37}$$

and we obtain, after rearranging,

$$(\Omega\gamma' l' + \gamma l)/\gamma\gamma'(\Omega - 1) \quad \text{if } \gamma > \gamma' \tag{38a}$$

$$(\Omega\gamma l + \gamma' l')/\gamma\gamma'(\Omega - 1) \quad \text{if } \gamma < \gamma'. \tag{38b}$$

The terms 12′, 13′, 21′, and 31′ exactly cancel out the terms in (38). The only remaining term is therefore 11′, and we have

$$S_{ff'} = l(\gamma)\,\delta(\gamma - \gamma'). \tag{39}$$

Substituting the result in (39) into (35), we obtain

$$S_{\delta\nu(2)} = (4\pi\tau)^{-2}(2h\nu/S_o)\int_1^\Omega d\gamma\,\alpha^2 l(\gamma)/\gamma^2. \tag{40}$$

Adding now the results in (33) and (40) and rearranging, we obtain

$$S_{\delta\nu} = S_{\delta\nu(1)} + S_{\delta\nu(2)} = (4\pi\tau)^{-2}(4h\nu/S_o)\int_1^\Omega d\gamma(1 + \alpha^2)\,l/\gamma^2 \tag{41}$$

where we have used the fact that $d\gamma(\gamma dl/d\gamma - l)/\gamma^2 = d(l/\gamma)$ gives a zero contribution since l/γ assumes the same value, namely unity, at both ends of the integration interval.

The spectral density of the instantaneous frequency deviation $\delta\nu$ is expressed in (41) as an integral over a round trip in the ring-type diode. The two functions $\alpha(\gamma)$ and $l(\gamma)$ enter, but the injected current density J does not enter explicitly. As is well known [11], the laser linewidth $\Delta\nu$ is equal to π times the spectral density $S_{\delta\nu}$ of the frequency deviation process. The resulting expression for $\Delta\nu$ is expressed in a practical form and applied to specific examples in the next section.

VI. The Linewidth Formula

If we introduce the total generated or dissipated power P_T given in (15), the laser (full width at half power) linewidth $\Delta\nu$ is given by

$$\Delta \equiv 2\pi\Delta\nu P_T/h\nu = (\tfrac{1}{2})\,\tau^{-2}\int_1^\Omega d\gamma/l\int_1^\Omega d\gamma(1 + \alpha^2)\,l/\gamma^2 \tag{42}$$

where $h\nu$ denotes the photon energy, and $\tau = L/v_g$ is the round-trip time where L is the round-trip length and v_g is the group velocity. If v_g is a function of z, τ is given by a simple integral.

On the right-hand side of (42) enters the product of two integrals over a round trip in the resonator. $\gamma(z) \geq 1$ denotes the power gain defined from the origin of the z-coordinates, with $\gamma(0) = 1$. $l(z) \geq 1$ is the power loss similarly defined, with $l(0) = 1$. After a round trip, $\gamma(L) = l(L) \equiv \Omega$ since the total power gain equals the total power loss in the absence of perturbations. $\gamma(z)$ and $l(z)$ are nondecreasing functions of z, and therefore, l is a nondecreasing function of γ. On the right-hand side of (42), the phase-amplitude coupling factor α, as well as l, is considered a function of γ. The laser linewidth is therefore easily evaluated from a simple integration if one knows the three functions $\gamma(z)$, $l(z)$, and $\alpha(z)$, which can be reexpressed as two functions, $l(\gamma)$ and $\alpha(\gamma)$, by eliminating z. The expression in (42) is independent of the choice of origin, even though this is not obvious by inspection.

For $\alpha = 0$, (42) gives half the result applicable to the linear regime. Notice that the second integral in (42) can then be derived from the first one by changing l to $1/l$ and γ to $1/\gamma$, as well as the sign, as is appropriate for adjoint (counterpropagating) fields [7]. The result in (42) for any $\alpha(z)$ function is half the result applicable to the linear regime multiplied by $(1 + \alpha^2)_{\text{av}}$, where the average is taken, not with respect to the z-coordinate itself, but with respect to $ld(1/\gamma)$. Let us emphasize that this simple result holds only when the injected bias current densities exhibit spatially uncorrelated shot noise. Otherwise, we would end up with numerous complicated integrals that could not be solved in closed form. The result in (42) applies in the limit of large injected currents when the rate of electron–hole pair generation equals the rate of photon generation in the oscillating mode, and the pairs' arrival times are independent of each other in both space and time.

Let us apply (42) to a number of interesting special cases. Consider first the case where the loss equals the gain everywhere along the path: $l = \gamma$, and α is a constant. A straightforward integration gives

$$\Delta = \tau^{-2} (\ln \Omega)^2 (1 + \alpha^2)/2. \tag{43}$$

This is the standard ST formula [1] multiplied by the phase-amplitude coupling term $(1 + \alpha^2)/2$. This result is also applicable to the case where the generated power, instead of being dissipated along the ring, is continuously radiated away, perhaps because of curvature loss.

The case where the ring laser is lossless except for a loss localized near the origin of the z-axis is of greater practical interest. It is immaterial from the point of view of laser operation whether the power is actually dissipated on the path or radiated away through a partially reflecting mirror of power reflectivity $R = 1/\Omega$ and eventually dissipated in some detector or at infinity. The configuration involving a reflecting mirror is shown in Fig. 1(b). The expression in (42) is, in that case,

$$\Delta = \tau^{-2}[(1 - R)^2/R]\,[(1 + \alpha^2)_{av}/2] \tag{44}$$

where

$$(\alpha^2)_{av} \equiv (1 - R)^{-1} \int_R^1 \alpha^2 \, d(1/\gamma). \tag{45}$$

In other words, in a ring-type laser which is lossless except for the coupling loss, the average of α^2 should be taken with respect to the gain reciprocal $1/\gamma$.

To give a specific example, let us consider the plausible law $\alpha = 6/\gamma$, implying that α varies from 6 to $6R$. We obtain from (44) and (45)

$$2\pi\Delta\nu P\tau^2/h\nu = (1 - R)^2 \,[1 + 12(1 + R + R^2)]/2R. \tag{46}$$

As we discussed in the introduction, the α-factor may vary either because of changes in material composition along the path or, for homogeneous layers, because of changing saturation conditions. The latter effect is more important for ring-type diodes than for conventional diodes. In ring-type diodes, the ratio of maximum to minimum local gain is as large as $1/R = 3$ if $R = 0.33$, for example. Just *after* the partially reflecting mirror, the local gain is the largest because the field intensity is the weakest. The α-factor is large at that location (perhaps $\alpha = 6$) because we are on the "red" side of the gain versus frequency curve, according to the Kramers–Kronig relations. The opposite is true just *before* the partially reflecting mirror, α assumes there a reduced value and may even be negative.

Let us consider a ring-type laser incorporating in succession along the path an amplifier of power gain G_1, constant α-factor denoted by α_1, a partially reflecting mirror of power reflectivity R_1, a second amplifier with parameters G_2 and α_2, and finally, a second partially reflecting mirror of power reflectivity R_2, as shown in Fig. 1(c). Application of (42) gives, for the linewidth of that configuration,

$$\Delta = \tau^{-2}(g_1/r_1 + g_2/r_2)\,[(1 + \alpha_1^2)\, r_1 g_1 + (1 + \alpha_2^2)\, r_2 g_2]/2 \tag{47a}$$

$$g_k \equiv \sqrt{G_k} - 1/\sqrt{G_k}; \qquad r_k \equiv \sqrt{R_k}; \qquad k = 1, 2. \tag{47b}$$

The case of a single amplifier and a single mirror is given by (47) with the terms labeled by "2" deleted.

VII. Incomplete Population Inversion

The previous theory is applicable at low temperatures. At elevated temperatures (e.g., room temperature), the active medium exhibits not only stimulated emission, expressed by the negative conductance G_a, but also stimulated absorption, expressed by the positive conductance G_b. In that case, it suffices in the previous expressions to replace G_a by the net gain $G_a - G_b$. The spectral density of the noise current associated with $G_a - G_b$ is, however, proportional to $G_a + G_b$. If we define as usual a spontaneous emission factor

$$n_{sp} \equiv G_a/(G_a - G_b) \tag{48}$$

which is on the order of two at room temperaure, we find that the linewidth enhancement factor is

$$[n_{sp}(1 + \alpha^2)]_{av} \tag{49}$$

where the averaging is defined as before.

VIII. Influence of Shot Noise

Yamamoto and Machida [17] have shown both theoretically and experimentally that the amplitude fluctuations of a laser output can be reduced below shot noise, ideally to zero, when the pump (injected current) fluctuations are suppressed. They asserted that the laser linewidth remains unaffected. While their conclusion would be correct if the α-factor were a constant, it does not hold when α varies. In our formulation, suppressing shot noise amounts to suppressing the term j in (27). All of the subsequent equations can be solved exactly, provided the laser suffers from no internal loss. The resulting linewidth enhancement factor is obtained by subtracting from the term in (49) half the spatial variance of α:

$$\text{var}(\alpha) \equiv (\alpha^2)_{av} - (\alpha_{av})^2. \tag{50}$$

This effect should be accessible to experiment.

IX. Conventional Laser Diodes

The previous results have been derived for a ring-type laser diode with only one wave propagating. In a conventional laser diode, forward- and backward-propagating waves create a standing-wave pattern. If the diffusion length is much larger than the wavelength, it is plausible that the previous results could be applicable. It would then suffice to apply the formula in (42) to a round trip in the resonator. In the absence of internal losses, the linewidth enhancement factor is then found to be given again by (49), but the averaging is accomplished from one end facet to the other, with the injected current density as a weighting factor. Note that this is exactly the result obtained when any number of active elements are connected in parallel [5].

It is not obvious that the ring-type result is applicable to conventional laser diodes because the noise sources are correlated (in fact, identical) at two points that are distinct along the round-trip path, but correspond to the same point in physical space. Note, however, that the current density injected into the laser diode at some z-location splits into two parts: one supplies power to the forward wave, while the other part supplies power to the backward wave. It is well known in electronics that when a current is split into two parts, the shot noise fluctuations of these two parts are uncorrelated.

Consider next the Nyquist-like noise currents. They can be expanded in a Fourier series over some interval of nonzero length ΔL, with $\lambda << \Delta L << L$. Because these noise sources are uncorrelated and approximately stationary over ΔL, the Fourier coefficient of order m is uncorrelated with the Fourier coefficient of order $-m$ [11]. The term of order m excites only the forward-propagating wave, while the term of order $-m$ excites only the backward-propagating wave. It therefore appears that the noise sources are effectively uncorrelated and that the ring-type result is applicable to conventional laser diodes. The range of validity of the approximations made in this section needs to be clarified.

X. Conclusion

A general theory for the linewidth of ring-type laser diodes has been given that accounts for the basic noise sources ($1/f$ noise and thermal fluctuations have not been considered). It is shown that the shot noise contribution is on the same order as the spontaneous emission contribution and is essential to obtain a simple result. The simplification is related to the fact that only when the pump (injected current) exhibits full shot noise does the output field evolve toward a coherent state in the limit of large cavity lengths [17].

The basic concepts and results in this paper appear to be applicable to conventional laser diodes, provided longitudinal hole burning is washed out by diffusion. Whether this is actually the case at high output powers (involving short diffusion lengths) remains open to question.

We have assumed in this paper that the series resistance of the confining layers is large, so that the injected bias current density is independent of the diode dynamics. The opposite assumption of negligible series resistance has been treated in earlier papers. But the real situation is probably somewhere in between and remains to be treated.

Finally, it should be noted that only one electromagnetic mode has been considered. If the power in the side modes remains small compared to that in the main mode, the side modes can be treated in the linear approximation, and their power is Rayleigh (exponentially) distributed. The total side-mode power is therefore Rayleigh distributed. The current needed to generate the side-mode power is subtracted from the constant injected current and has the same effect on the main mode as a nonuniform modulating current density, if the modes are too far apart in frequency to interact through carrier modulation. We therefore concur with Adams's finding [21] that the effect of low-power side modes on the main-mode linewidth is negligible when α is a constant. The effect may be significant, however, if α is nonuniform, as we discussed in this paper.

It is shown in [23], [24] that electronic feedback may squeeze the amplitude fluctuations below shot noise. For arbitrary multiple-active elements, the fluctuations can be expressed simply in terms of the optical circuit scattering matrix [25].

Appendix
The Transmission Line Model

The theory of propagation along nonuniform transmission lines can be found in electrical engineering textbooks. However, it is convenient to derive here the essential formulas. We also briefly discuss the applicability of this formalism to buried heterojunctions.

Consider first a periodic filter with series impedance $Z\,dz$ and parallel admittance $Y\,dz$. The term dz is introduced for later convenience. Ohm's law and Kirchhoff's law read

$$V_{k+1} - V_k = -Z\,dz\,I_k \tag{A1a}$$

$$I_{k+1} - I_k = -Y\,dz\,V_{k+1} \tag{A1b}$$

where $k = \cdots, -1, 0, 1, 2, \cdots$ labels the cells. V is the voltage between the two conductors, and I is the current flowing in one conductor. The sign convention is easily understood from the above equations. In the limit where dz is small (see, for example, [22, eq. (5)–(7)]), these equations become

$$dV/dz = -ZI \tag{A2a}$$

$$dI/dz = -YV. \tag{A2b}$$

These differential equations admit the following solutions:

$$V(z) = V(0) \exp\left[-\int (YZ)^{1/2}\, dz\right] \tag{A3a}$$

$$I(z) = (Y/Z)^{1/2}\, V(0) \exp\left[-\int (YZ)^{1/2}\, dz\right] \tag{A3b}$$

where the integrals are from 0 to z. The z-variation of the characteristic admittance $(Y/Z)^{1/2}$ is supposed to be slow, so that its variation can be neglected in differentiating the right-hand side of (A3b) with respect to z.

The transmission line formalism is applicable to an active layer (with gain or loss) bounded by two perfect conductors, the electrical field being perpendicular to the conductors, provided the ambipolar diffusion length is much larger than the transverse dimensions (thickness and width) of the active material. The layer parameters (e.g., thickness), however, may vary arbitrarily with the longitudinal coordinate z as long as the changes per wavelength are small.

For such an optical waveguide, the series impedance is a pure reactance X, while the parallel admittance possesses a nonzero real part G in addition to the susceptance B. For a system with net loss, G is positive, while G is negative for a system with net gain.

$$Z = iX; \qquad Y = G + iB. \tag{A4}$$

We now define the propagation constant k and the characteristic impedance Z_c according to

$$Z_c = (Z/Y)^{1/2} \approx (X/B)^{1/2} \qquad \text{positive real} \tag{A5a}$$

$$k = i(YZ)^{1/2} \approx k_o + iZ_cG/2;$$

$$k_o \equiv (XB)^{1/2} \qquad \text{positive real.} \tag{A5b}$$

The approximate forms in (A5) have been obtained under the assumption that $G << B$; that is, the gain (or loss) per wavelength is small, but it may accumulate to large values over the full laser length $L >> \lambda$. In (A5b), appropriate signs for the square roots have been selected. Equation (A5) is the result in (6) of the main text, while (A3a), together with (A5), is (7) of the main text.

Because there are no good conductors at optical frequencies and because epitaxy on metals is difficult, active layers are, in the present technology, imbedded into lower-index high-bandgap semiconductors [18]. The transmission line formalism remains approximately applicable to both TE and TM polarizations with minor modifications (introduction of confinement factors [18]), provided the wavefront of the guided mode remains approximately perpendicular to the z-axis. Otherwise, Petermann's K-factor [8] must be introduced.

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