Guidance of Surface Waves by Multilayer Coatings

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A periodic sequence of layers with alternately high and low refractive indices can guide loosely bound surface waves parallel to the layers. Most of the power flows in free space, and, thus, the losses may be considerably smaller than the bulk losses of the dielectric materials used. Possible applications are briefly discussed.

It is well known that periodic sequences of layers with alternately high and low refractive indices provide high reflectivities (up to 99.9% in the visible) for incident plane waves. It may not be as well recognized that these periodic layers can guide loosely bound surface waves. Our interest in such surface waves is that because most of the power flows in free space, the losses are considerably smaller than the bulk losses of the dielectric materials used.

Loosely bound surface waves can also be guided by single, thin slabs of dielectric. However, such slabs are difficult to support and appear impractical for optical communication. A single layer with a refractive index that is slightly larger than the refractive index of a dielectric substrate may also guide loosely bound waves. In that case, however, more than half the power flows in the layer or the substrate, and the reduction in loss is not significant.

In this paper we will first derive the dispersion equation of a semi-infinite sequence of periodic layers on the basis of the transmission line representation. Simple approximate expressions are subsequently given, applicable to the case of loosely bound waves. In conclusion, we briefly discuss possible applications.

Let us consider the two-dimensional configuration shown in Fig. 1, where an infinite number of periodic dielectric layers occupy the half-space $x < 0$. We are interested in waves that propagate in the z direction and decay exponentially in the x direction. The two eigenstates of polarization, that clearly correspond to electric fields, either parallel $(E_{\parallel})$ or perpendicular $(E_{\perp})$ to the x–z plane, will be considered.

In order to obtain the (imaginary) propagation constant β_x in the x direction, we apply the transverse resonance procedure[1] to the transmission line equivalent of homogeneous layers.[2] In the present case, this procedure amounts to equating to zero the sum of the impedances, observed at the plane $x = 0$ looking toward the periodic layers ($x < 0$) and looking toward free space ($x > 0$). The impedance of the periodic layers is obtained by evaluating the voltage-current transfer matrix for one section (i.e., two layers) and specifying that the impedance is the same at the input and output planes of that section. The normalized form of this impedance is easily found to be

$$Z_{in} = \{A - D \pm [(A + D)^2 - 4]^{1/2}\}/2C, \quad (1a)$$

where

$$A \equiv \cos\beta_1 l_1 \cos\beta_2 l_2 - Z_2^{-1} Z_1 \sin\beta_1 l_1 \sin\beta_2 l_2,$$

$$D \equiv \cos\beta_1 l_1 \cos\beta_2 l_2 - Z_1^{-1} Z_2 \sin\beta_1 l_1 \sin\beta_2 l_2,$$

$$C \equiv j[Z_1^{-1} \sin\beta_1 l_1 \cos\beta_2 l_2 + Z_2^{-1} \sin\beta_2 l_2 \cos\beta_1 l_1], \quad (1b)$$

$$\beta_i = k n_i [1 - (\sin\theta/n_i)^2]^{1/2}, \quad i = 1, 2, \quad (1c)$$

and

$$Z_i = n_i^{-1}[1 - (\sin\theta/n_i)^2]^{1/2}, \quad i = 1, 2 (E_{\parallel}), \quad (1d)$$

or

$$Z_i = n_i^{-1}[1 - (\sin\theta/n_i)^2]^{-1/2}, \quad i = 1, 2 (E_{\perp}). \quad (1e)$$

It can be shown that for waves that decay as $x \to -\infty$, the sign of the square root in Eq. (1a) has to be the same as that of the (real) quantity $A + D$.

In the above equations, $k = 2\pi/\lambda$ denotes the free-space propagation constant; n_1 and n_2 the refractive indices of the first and second layers, respectively; and l_1 and l_2 their thicknesses. The angle θ denotes a generalized angle of incidence on the layers. In our case, θ is a complex angle and $\cos\theta$ is an imaginary quantity that is related to the imaginary propagation constant β_x by the relation
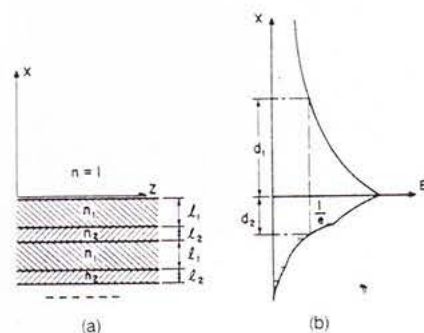
Fig. 1. (a) Besides their reflecting properties periodic layers ($x < 0$) can guide surface waves for suitable values of the refractive indices n_1, n_2 and thickness l_1, l_2. (b) Approximate field distribution in the layers and in free space.

$$\cos\theta = \beta_x/k. \qquad (2)$$

The condition for transverse resonance is given by

$$\cos\theta = -Z_{in}, \quad (E_\parallel), \qquad (3a)$$

$$1/\cos\theta = -Z_{in}, \quad (E_\perp). \qquad (3b)$$

Equations (1), (2), and (3) are transcendental equations for β_x. A simple approximate solution to these equations can be obtained if we assume that the surface wave is loosely bound to the layers; i.e., that the decay per wavelength above the layers is small. In this case, $j\cos\theta = j\beta_x/k$ is a small, positive quantity that tends to zero as k tends to the cutoff propagation constant k_0. Thus, from Eq. (3) it follows that as $k \to k_0$, $Z_{in} \to j0$, ($E_\parallel$), or $Z_{in} \to -j\infty$, ($E_\perp$). At the cutoff, the dielectric layers act either as an electric wall ($E_\parallel$) or as a magnetic wall ($E_\perp$), and plane waves can propagate in the region $x > 0$.

It can be shown from Eq. (1) that at the cutoff (i.e., when $\sin\theta = 1$), both $\beta_1 l_1$ and $\beta_2 l_2$ must be odd multiples of $\pi/2$. Thus, if λ_0 is the cutoff wavelength, l_1 and l_2 are given from Eq. (1c) by

$$l_i = (\lambda_0/4)(n_i^2 - 1)^{-1/2}, \quad i = 1, 2. \qquad (4)$$

Also, Eq. (1) shows that for a solution to exist we must have $Z_1 < Z_2$, ($E_\parallel$), or $Z_1 > Z_2$, ($E_\perp$). With Eqs. (1d) and (1e), it follows that n_1 and n_2 must satisfy the condition

$$n_1^{-2}(n_1^2 - 1)^{1/2} < n_2^{-2}(n_2^2 - 1)^{1/2}, \quad (E_\parallel), \qquad (5a)$$

or

$$n_1 < n_2, \quad (E_\perp). \qquad (5b)$$

From these conditions, the regions in the n_1, n_2 plane that allow the existence of loosely bound surface waves are presented in Fig. 2. The figure indicates that, depending upon the values of the refractive indices, the periodic layers can support waves with both $E_\parallel$ and $E_\perp$, waves with $E_\parallel$ alone, waves with $E_\perp$ alone, or no wave.

Near the cutoff wavelength λ_0, we can set

$$\lambda = \lambda_0 + \Delta\lambda. \qquad (6)$$

and we have $j\cos\theta \ll 1$ and $\sin\theta \simeq 1$. Thus,

$$Z_i \simeq n_i^{-2}(n_i^2 - 1)^{1/2}, \quad i = 1, 2, \quad (E_\parallel), \qquad (7a)$$

or

$$Z_i \simeq (n_i^2 - 1)^{-1/2}, \quad i = 1, 2, \quad (E_\perp). \qquad (7b)$$

In this case, the approximate solution for $j\beta_x$ is easily found to be

$$j\beta_x \simeq \lambda_0^{-1}\pi^2 (Z_1^{-1} - Z_2^{-1})^{-1}(-\Delta\lambda/\lambda_0), \quad (E_\parallel), \qquad (8a)$$

or

$$j\beta_x \simeq \lambda_0^{-1}\pi^2 (Z_1 - Z_2)^{-1}(-\Delta\lambda/\lambda_0), \quad (E_\perp). \qquad (8b)$$

For both polarizations, the layers can support surface waves only for $\Delta\lambda < 0$; i.e., for wavelengths shorter than the cutoff wavelength.

The distance d_1 above the layers, where the field is reduced by a factor $e = 2.718\ldots$, is given by

$$d_1 = (j\beta_x)^{-1}. \qquad (9)$$

This distance d_1 is clearly a sensitive function of the operating wavelength, since it is proportional to $-\lambda_0/\Delta\lambda$.

The distance d_2 inside the dielectric layers, where the field amplitude is reduced by a factor e, can be calculated from the decay constant per pair of layers

$$\alpha = \text{real part of } \{\cosh^{-1}[(A + D)/2]\}. \qquad (10)$$

Neglecting terms of the order of $(-\Delta\lambda/\lambda_0)^2$, we get from Eq. (10) and Eq. (1)

$$d_2 \simeq (l_1 + l_2)\{\cosh^{-1}[(Z_1/Z_2 + Z_2/Z_1)/2]\}^{-1}, \qquad (11)$$

where l_1 and l_2 are given in Eq. (4), and Z_1 and Z_2 are given in Eq. (7a) or (7b). Unlike d_1, the distance d_2 is almost independent of frequency in the domain of interest.

Note that the above equations are based upon the assumption of infinitely many layers. A rough estimate of the number of layers that are requi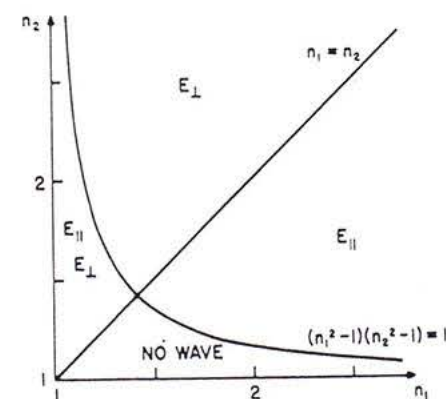red in practice is obtained by assuming that the total thickness is equal to twice the distance d_2. From this rule the approximate number of pairs of layers is found to be

$$N \simeq 2d_2/(l_1 + l_2). \qquad (12)$$

In order to illustrate these results let us work out an example.

Let the cutoff wavelength be 0.6328 μm (He–Ne line), $n_1 = 1.4$, and $n_2 = 1.6$. Figure 2 indicates that the layers can support surface waves with the electric field perpendicular to the x–z plane. The above equations give $l_1 = 0.162$ μm, $l_2 = 0.127$ μm, $d_1 \simeq 89/(-\Delta\lambda)$ μm, where $\Delta\lambda$ is the difference in Angstroms between the operating and the cutoff wavelengths, $d_2 \simeq 1.2$ μm, and $N \simeq 8$. If $\Delta\lambda = -5$ Å, $d_1 \simeq 18$ μm. In this case, the losses are roughly ten times smaller (in dB) than the bulk losses in the dielectrics, since about 90% of the power is propagating in air and only 10% is propagating in the dielectric.

If the order of the layers were reversed (i.e., if we had $n_1 = 1.6$ and $n_2 = 1.4$), the structure would support waves with the electric field parallel to the x–z plane. However, the number of pairs of layers that would be required in that case would be prohibitively large, about eighty.

So far we have considered only plane layers. Periodic coatings could be deposited on curved as well as on plane substrates (e.g., on the internal surface of hollow quartz rods for long-distance optical communications).[3] A small curvature of the substrate does not significantly modify the layer impedance. The field, however, decays faster (in free space) if the surface is concave, and more slowly (or even radiate) if the surface is convex. (The general field solution is then described by Airy functions.) Therefore, by shaping the surface of the substrate, it may be possible to control the propagation of these surface waves without having to change the layer thicknesses. A variety of integrated optics configurations is therefore conceivable. For example, Fig. 3 shows two methods of making a positive lens that can be used to confine optical beams transversely. It should be noted, however, that loosely bound waves are more prone to radiation than tightly bound waves, and hence one is restricted to bends with small curvatures.[4] It should also be noted that with the techniques presently available in optics, significant losses occur through surface irregularities. Accurate refractive index periodicities can be permanently induced by a standing ultraviolet-wave pattern in polymethylmethacrylate rather than by evaporation techniques,[5] as shown in Fig. 4. If the uv wavelength used is of the order of one-half that of the optical wavelength to be guided by the structure, the standing-wave pattern obtained has the proper period. (The angle of incidence of the uv wave needs, of course, to be taken into consideration.)

This paper was presented at the OSA Topical Meeting on Integrated Optics, Guided Waves; Materials and Devices, Las Vegas, Nevada, 7–10 February 1972.

Fig. 2. Diagram showing under what conditions loosely bound surface waves with the electric field in the x, z plane ($E_\parallel$) or along the y axis ($E_\perp$) can propagate. (The intersection point is at $n_1 = n_2 = \sqrt{2}$.)
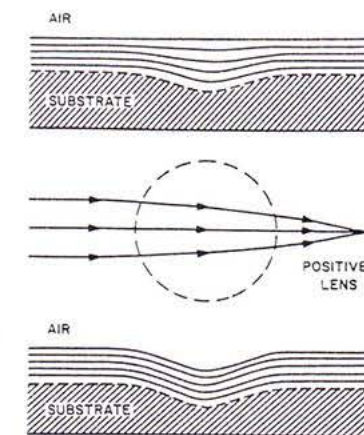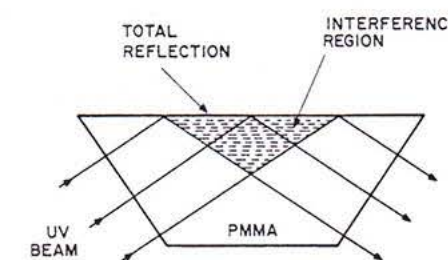

Fig. 3. Optical beams can be confined transversely or focused through slow changes in layer thickness or deformation of the surface.

$$n = 1.5, \quad \Delta n = 0.005$$

$$N \simeq 300 \text{ pairs, } E_\perp$$

$$N \simeq 3000 \text{ pairs, } E_\parallel$$

Fig. 4. Method of fabrication using a Littrow prism (suggested by I. P. Kaminow and H. P. Weber). Typical numbers of layers are given for $E_\perp$ and $E_\parallel$.

References

1. W. L. Weeks, *Electromagnetic Theory for Engineering Applications* (Wiley, New York, 1964), pp. 246–259.
2. R. B. Adler, L. J. Chu, and R. M. Fano, *Electromagnetic Energy Transmission and Radiation* (Wiley, New York, 1965), pp. 354–362.
3. In the configurations considered by R. P. Larsen and A. A. Oliner, Microwave Symp. G-MTT, 17 (May 1967) and by E. A. J. Marcatili, Patent 3,583,786, multiple layers are effectively used in the reflection mode rather than in the surface wave mode, considered in the present paper.
4. E. A. J. Marcatili, Bell Syst. Tech. J. 48, 2103 (1969) and J. A. Arnaud, Bell Syst. Tech. J. 53 (Sept. 1974).
5. This was suggested to us by I. P. Kaminow and H. P. Weber.