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Gaussian Light Beams with General Astigmatism

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This paper considers the propagation and diffraction of coherent light beams through nonorthogonal optical systems such as sequences of astigmatic lenses oriented at oblique angles to each other. The fundamental (gaussian) mode has elliptical light spots in each beam cross section and ellipsoidal (or hyperboloidal) wavefronts near the axis. It is found that the orientation of the light spot differs from that of the wavefront, and changes continuously by as much as π radians as the beam propagates through free space. A theory of these general astigmatic beams is given and simple experimental observations are described. The coupling factor between two such beams is also given.

Introduction

This paper considers the passage of gaussian beams of light through nonorthogonal optical systems.[1] A sequence of two or more astigmatic lenses with oblique orientations is an example of such a system. Nonorthogonal optical configurations are encountered in special optical cavities,[2] helical gas lenses,[3,4] and in systems where optical beams are refocused and redirected in two dimensions by spherical mirrors. In nonorthogonal arrangements, the conventional laws[5] which govern the propagation and diffraction of light beams are no longer applicable. It is the purpose of this paper to discuss the laws for light beams produced by such arrangements.

To demonstrate the failure of the conventional laws of beam propagation in these optical systems, let us consider a stigmatic fundamental Gaussian beam, i.e., a beam with a circular light spot and a spherical wave front in every beam cross section. The beam radius or spot size w of this beam is the same for the two transverse rectangular coordinates (x,y), i.e., $w_x = w_y$, and the corresponding radii of curvature of the wavefronts R are also the same $(R_x = R_y)$. The propagation laws for this beam are well known.[5] Assume that this beam passes through a thin astigmatic lens with different focal lengths f in the x and y coordinate $(f_x \neq f_y)$. The resulting beam is "astigmatic," with elliptical light spots $(w_x \neq w_y)$ except in a few isolated cross sections, and with ellipsoidal (or hyperboloidal) wavefronts (with $R_x \neq R_y$). But in each cross section the ellipses of constant intensity and the ellipses of constant phase have the same orientation, and this orientation remains constant while the beam propagates in free space. It is known that the conventional propagation laws can also be applied to this beam since the propagation can be considered independently for the x and the y coordinate. Now let this beam of simple astigmatism pass through another astigmatic lens, oriented in the x–y plane at an angle with respect to the first lens. The beam emerging from this lens is of general astigmatism with constant intensity ellipses and constant phase ellipses (or hyperbolas) oriented at an oblique angle with respect to each other. Just after the second lens, the axis of the phase ellipse is rotated with respect to the phase ellipse of the incoming beam while the intensity ellipse is unaffected. We shall find that, even in free space, the orientation of the ellipses of the outcoming beam changes along the path of propagation. We will restrict the discussion to the propagation of fundamental modes along the axis of lossless, nonaberrated optical systems.

The transformation of astigmatic ray bundles by nonorthogonal optical systems has generally been considered in connection with the transformation of skew rays in optical instruments.[1] The stability condition for periodic nonorthogonal systems has been recently given by Kahn,[6] on the basis of geometrical optics, and the transformation of gaussian beams by nonorthogonal systems has been discussed by Suematsu and Fukinuki.[4]

The calculation of this paper is developed along the following lines: we start with an astigmatic ray pencil, which is defined by three real parameters; these are the axial positions of the tangential and sagittal focal lines and their angular orientation with respect to the transverse coordinate axes. It is well known that the field of a gaussian beam with simple astigmatism is formally described by the same expression as the astigmatic ray pencil, except that complex values are used for the positions of the focal lines. We formally obtain a gaussian beam with general astigmatism by attaching a complex value to the angular orientation of the focal lines. A gaussian beam with general astigmatism is then described by three complex beam parameters.

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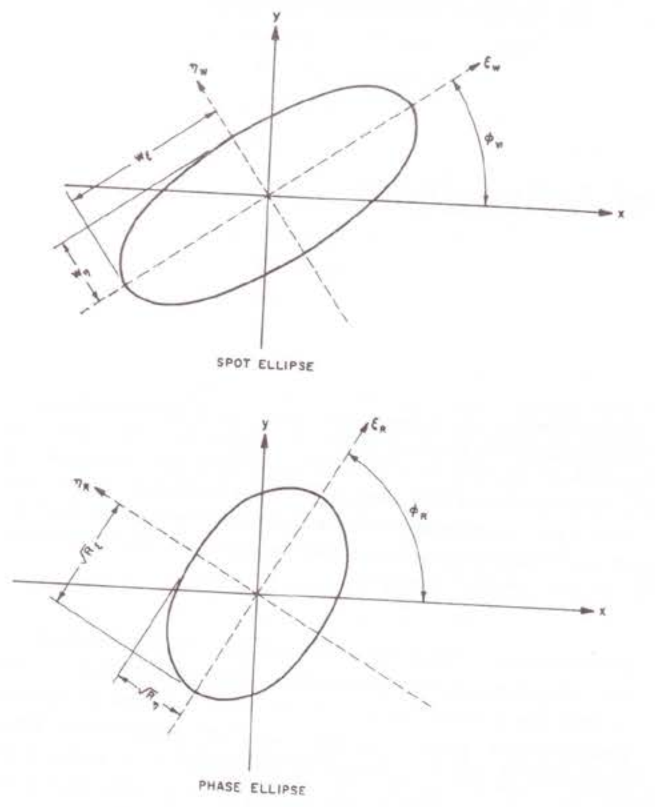

Fig. 1. Spot ellipse and phase ellipse.

In Secs. II, III, and IV we discuss the propagation in free space of a beam with general astigmatism. In Sec. V the transformation of the three complex beam parameters by a thin astigmatic lens is given. Expressions for the coupling between two gaussian beams with general astigmatism, which are of interest for beat experiments, are given in Sec. VI. Section VII reports experimental observations of beams with general astigmatism.

I. Solution for General Astigmatism

The propagation of laser beams in free space is governed by a Schroedinger-type wave equation[5]:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} - 2jk\frac{\partial\psi}{\partial z} = 0, \qquad (1)$$

where x, y, z is a rectangular coordinate system, ψe^{-jkz} the complex amplitude of the electric field, and the propagation constant is $k = 2\pi/\lambda$. A well known solution of this equation is given by

$$\psi(x,y,z) = (q_1q_2)^{-\frac{1}{2}}\left\{\exp\left[-j\frac{k}{2}\left(\frac{x^2}{q_1} + \frac{y^2}{q_2}\right)\right]\right\}, \qquad (2)$$

where q_1 and q_2 are the complex beam parameters for the xz and yz planes, respectively. They are given by

$$q_1(z) = q_{01} + z,$$
$$q_2(z) = q_{02} + z, \qquad (3)$$

where q_{01} and q_{02} are complex constants which describe the beam waist radii and the positions of the beam waists in the xz and yz planes.

The beam described by Eq. (2) is astigmatic. It has elliptical light spots, but both the ellipses of constant intensity and the ellipses (or hyperbolas) of constant phase are oriented along the x and y axes in every cross section of the beam. We call it a beam with simple astigmatism.

If we rotate this beam by an angle φ around the z axis, the expression describing its propagation assumes the form

$$\psi(x,y,z) = (q_1q_2)^{-\frac{1}{2}}\left(\exp\left\{-j\frac{k}{2}\left[\left(\frac{\cos^2\varphi}{q_1} + \frac{\sin^2\varphi}{q_2}\right)x^2 + \left(\frac{\sin^2\varphi}{q_1} + \frac{\cos^2\varphi}{q_2}\right)y^2 + \sin 2\varphi\left(\frac{1}{q_2} - \frac{1}{q_1}\right)xy\right]\right\}\right). \qquad (4)$$

It is, of course, still a solution of Eq. (1).

It is easy to verify that the expression given in Eq. (4) remains a solution of Eq. (1) even for complex values of φ. We find that a rotation of the beam by a complex angle $\varphi \equiv \beta + j\alpha$ leads to a gaussian beam with general astigmatism which has the configuration required to describe a fundamental mode of light propagating through nonorthogonal systems. It can also be observed that the real part β of φ represents a (real) rotation of the beam, which leaves unaffected the field pattern; in the following section, β will be set equal to zero, for simplicity.

We can rewrite the argument of the exponential in Eq. (4) in the form

$$-j\frac{k}{2}\,[Q_1x^2 + Q_2y^2 + \tan 2\varphi(Q_2 - Q_1)xy], \qquad (5)$$

where

$$Q_1 = (\cos^2\varphi/q_1) + (\sin^2\varphi/q_2)$$
$$Q_2 = (\sin^2\varphi/q_1) + (\cos^2\varphi/q_2), \qquad (6)$$

and the q parameters are of the same form as in Eq. (3). For φ real, we have the solution of simple astigmatism given in Eq. (2), and for $q_1 = q_2$ we have $Q_2 - Q_1 = 0$ and a gaussian beam with rotational symmetry. Notice that the field on axis ($x = y = 0$) does not depend on φ. From Eq. (4) it follows that the phase shift $\Phi(z)$ experienced on axis by a generalized astigmatic beam is, in vacuum (as for a simply astigmatic beam)

$$\Phi(z) = -\tfrac{1}{2}\,[\text{Phase of }(q_1) + \text{Phase of }(q_2)]. \qquad (7)$$

Equation (5) gives a simple mathematical description of a gaussian beam with general astigmatism. In the next sections we discuss the properties of such a beam in more detail.

II. The Beam Configuration for General Astigmatism

To study the beam configuration more closely, we have to introduce real-valued beam parameters. It turns out that the ellipses of constant intensity and the

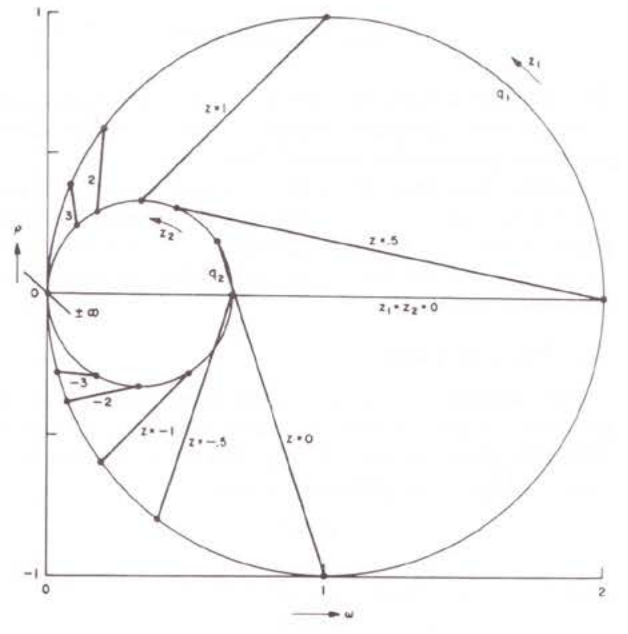

Fig. 2. Circle diagram for the parameters q_1 and q_2.

ellipses (or hyperbolas) of constant phase change their orientation along the z axis. Therefore, it is convenient to introduce rotating coordinate systems (ξ_w,η_w) and (ξ_R,η_R) whose axes are aligned with the major and minor axes of these ellipses

$$x = \xi_w\cos\varphi_w - \eta_w\sin\varphi_w = \xi_R\cos\varphi_R - \eta_R\sin\varphi_R,$$
$$y = \xi_w\sin\varphi_w + \eta_w\cos\varphi_w = \xi_R\sin\varphi_R + \eta_R\cos\varphi_R. \qquad (8)$$

The angles of orientation $\varphi_w(z)$ and $\varphi_R(z)$ of the two coordinate systems, i.e., of the constant intensity and phase curve axes, with respect to the original coordinate system are given by

$$\tan 2\varphi_w = [(\rho_1 - \rho_2)/(\omega_1 - \omega_2)]\,th2\alpha \qquad (9a)$$
$$\tan 2\varphi_R = -\,[(\omega_1 - \omega_2)/(\rho_1 - \rho_2)]\,th2\alpha. \qquad (9b)$$

Here we have put

$$\varphi = j\alpha,$$
$$1/q_i = \rho_i - j\omega_i = 1/(jp_i + z_i),\; i = 1,2, \qquad (10)$$
$$\rho_i(z) = z_i/(z_i^2 + p_i^2)$$
$$\omega_i(z) = p_i/(z_i^2 + p_i^2)$$

using real valued parameters ρ_i, ω_i, p_i, and z_i. The constant quantity $2p_i(\equiv b_i)$ is usually called the confocal parameter, and $z_i = z\text{-}z_{0i}$ is a linear function of the z coordinate. For simply astigmatic beams, z_{0i} is interpreted as the position of the beam waist, but this physical interpretation no longer holds for general astigmatism.

Using the rotating coordinates, Eq. (5) can be written in the form

$$\psi = \lambda\,(q_1q_2)^{-\frac{1}{2}}\left\{\exp\left[-\left(\frac{\xi_w^2}{w_\xi^2} + \frac{\eta_w^2}{w_\eta^2}\right) - j\frac{k}{2}\left(\frac{\xi_R^2}{R_\xi} + \frac{\eta_R^2}{R_\eta}\right)\right]\right\}. \qquad (11)$$

The principal beam radii or spot sizes w_ξ and w_η are given by

$$\lambda/\pi w_{\xi,\eta}^2 = \tfrac{1}{2}\,\{\omega_1 + \omega_2 \pm [(\omega_1 - \omega_2)^2ch^22\alpha + (\rho_1 - \rho_2)^2sh^22\alpha]^{\frac{1}{2}}\} \qquad (12)$$

and the principal radii of curvature of the phase front R_ξ and R_η are obtained as

$$(R_{\xi,\eta})^{-1} = \tfrac{1}{2}\,\{\rho_1 + \rho_2 \pm [(\rho_1 - \rho_2)^2ch^22\alpha + (\omega_1 - \omega_2)^2sh^22\alpha]^{\frac{1}{2}}\}. \qquad (13)$$

The form of Eq. (11) is similar to that of a beam with simple astigmatism. But in the latter case the coordinates ξ and η are fixed, while for general astigmatism they change their orientation along the z axis.

In each beam cross section, the ellipses of constant intensity are described by w_ξ and w_η, which are the principal axes of what we shall call the "spot ellipse." Correspondingly $(R_\xi)^{\frac{1}{2}}$ and $(R_\eta)^{\frac{1}{2}}$ are the principal axes of the "phase ellipse" (or "phase hyperbola"). This is illustrated in Fig. 1.

Equation (12) describes the expansion of the beam. It replaces the hyperbolic expansion law for stigmatic beams, or beams with simple astigmatism.

The relative orientation $\varphi_w - \varphi_R$ of the spot ellipse and the phase ellipse is obtained as

$$\tan 2(\varphi_w - \varphi_R) = sh2\alpha ch2\alpha\left\{\frac{\rho_1 - \rho_2}{\omega_1 - \omega_2} + \frac{\omega_1 - \omega_2}{\rho_1 - \rho_2}\right\}. \qquad (14)$$

The two ellipses can only be aligned (i.e., $\varphi_w - \varphi_R = 0$ or $\pi/2$) for the special cases of $\alpha = 0$ or $q_1 = q_2$, which are the cases of simple astigmatism or perfect stigmatism. In other words: for general astigmatism ($\alpha \neq 0$) the spot ellipses and phase ellipses are *never* aligned. Similarly one finds that the spot ellipses of beams with general astigmatism never degenerate into circles.

It is interesting to note that the product

$$\tan 2\varphi_w \tan 2\varphi_R = -th^22\alpha \qquad (15)$$

is independent of z.

To illustrate the propagation properties of a beam with general astigmatism as predicted by the relations above, let us consider a typical numerical example. For this we choose a wavelength of 1 μm, and beam parameters with the values $p_1 = 0.5$ m, $p_2 = 1.5$ m, $z_{01} = 0.5$ m, and $z_{02} = -0.5$ m. The evolution of the two q parameters of Eq. (10) can be traced with the aid of a circle diagram[5] as shown in Fig. 2. This is a chart in the complex plane of j/q_i with the axes ω_i and ρ_i. The values corresponding to q_1 and q_2 as z varies are located on two corresponding circles. Parameter values obtained for given values of z are shown connected by a straight line. The tangent of the angle of this connecting line with respect to the $0w$ axis is equal to $(\rho_1 - \rho_2)/(\omega_1 - \omega_2)$. This is the value used to determine the ellipse angles φ_w and φ_R in Eqs. (9). The connecting line is seen to tumble through a total angle of 2π radians in the interval of $z = -\infty$ to $z = \infty$, and its angle increases with increasing z. From a similar inspection of the circle chart, it follows, quite generally, that (for

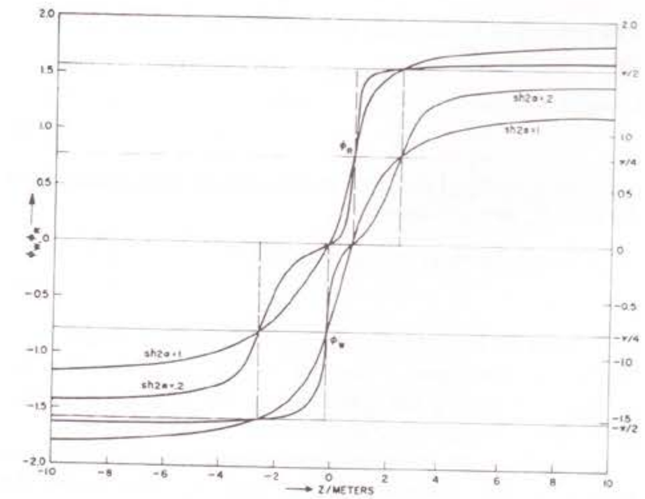

Fig. 3. Plot of φ_w and φ_R for $\lambda = 1 \mu m$, $p_1 = -0.5$ m; $p_2 = 1.5$ m, $z_{01} = 0.5$ m, and $z_{02} = -0.5$ m; $sh2\alpha = 0.2$ and 1.0 (i.e., $\alpha = 0.10$ and 0.44), respectively.

positive α) both φ_w and φ_R increase with increasing z, and each angle changes by a total of π radians from $z = -\infty$ to $z = +\infty$. Figure 3 shows a plot of φ_w and φ_R for the specific parameters chosen above and for values of $sh2\alpha = 0.2$ and $sh2\alpha = 1.0$ which satisfy the stability condition [Eq. (17)] discussed in the following section. Note that the fast changes of φ_w and φ_R occur in the confocal regions of q_1 and q_2, and in the vicinity of z values where $\omega_1 = \omega_2$ and $\rho_1 = \rho_2$, respectively. The dashed lines are approached for very small values of α.

Figure 4 shows a plot of the principal axes w_ξ and w_η of the spot ellipse for our numerical example. Entered for comparison are plots of w_1 and w_2 which are the beam radii of gaussian beams with the parameters q_1 and q_2, respectively ($\alpha = 0$). Note that w_ξ is always smaller than w_1 and w_2, and that w_η is always larger. From Eq. (12) it can be shown quite generally, that the spot ellipses of a beam with general astigmatism ($\alpha \neq 0$) are always more elongated than those of the corresponding beam with simple astigmatism ($\alpha = 0$). We have mentioned before a somewhat related finding, namely that one never gets a circular spot for general astigmatism.

III. Limitations on the Value of α

In Sec. I a complex angle of rotation $\varphi \equiv \beta + j\alpha$ was used to generate solutions of general astigmatism. To restrict the solutions to beams with intensities decreasing with distance from the optic axis (z axis), we have to impose restrictions on the value of α. We have to postulate that the principal axes w_ξ and w_η of the spot ellipse given in Eq. (12) are always real.

This condition can be written in the form

$$ch^2 2\alpha \leq \frac{(z_1 - z_2)^2 + (p_1 + p_2)^2}{(z_1 - z_2)^2 + (p_1 - p_2)^2} \quad (16)$$

or, more compactly

$$ch2\alpha \leq \left|\frac{q_1 - q_2^*}{q_1 - q_2}\right|. \quad (17)$$

This condition is independent of z, which means that a confined beam remains confined near the optic axis as the beam propagates through free space.

Notice that, from Eq. (17), if q_1 and q_2 are both real, α must be equal to zero, and we get the astigmatic ray pencil solution. In that case the beam reduces to two focal lines, perpendicular to each other, at two planes (generally distinct).

IV. The Far Field

The divergence of an astigmatic beam and the geometry of the far field are easily determined from the relations given in Sec. II by letting z approach infinity. From Eq. (10) we see that

$$\rho_i \to 1/z_i \quad (18)$$

and

$$\omega_i \to p_i/z_i^2$$

as $z \to \infty$. The far field orientations φ_w and φ_R of the spot ellipse and the phase ellipse follow from Eqs. (9a), (9b),

$$\tan 2\varphi_w \to -\frac{z_1 - z_2}{p_1 - p_2} th2\alpha, \quad (19a)$$

$$\tan 2\varphi_R \to \frac{p_1 - p_2}{z_1 - z_2} th2\alpha. \quad (19b)$$

The relative orientation of the ellipses is given, in the limit of large z, by

$$\tan 2(\varphi_w - \varphi_R) = -sh2\alpha\, ch2\alpha \left(\frac{z_1 - z_2}{p_1 - p_2} + \frac{p_1 - p_2}{z_1 - z_2}\right). \quad (20)$$

For the far field angles $\theta_\xi \equiv (w_\xi/z)_{z\to\infty}$ and $\theta_\eta \equiv (w_\eta/z)_{z\to\infty}$, we obtain from Eq. (12)

$$1/\theta_{\xi,\eta}^2 = (\pi/2\lambda)\{p_1 + p_2 \pm [(p_1 - p_2)^2 ch^2 2\alpha + (z_1 - z_2)^2 sh^2 2\alpha]^{\frac{1}{2}}\}. \quad (21)$$

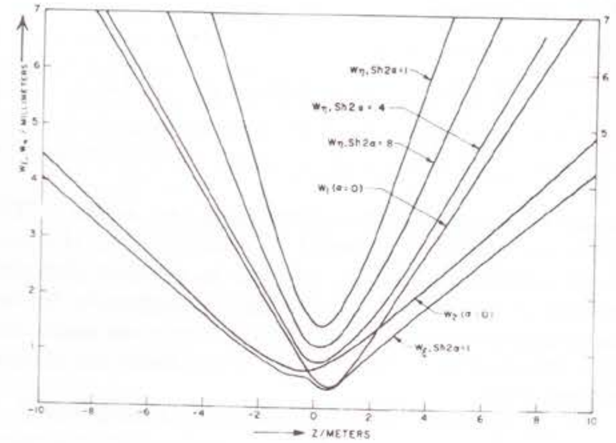

Fig. 4. Plot of the principal axes of the spot ellipse, w_ξ and w_η, for $\lambda = 1\ \mu m$, $p_1 = 0.5$ m, $p_2 = 1.5$ m, $z_{01} = 0.5$ m, and $z_{02} = -0.5$ m, $sh2\alpha = 0.4$, 0.8 and 1.0 (i.e., $\alpha = 0.20$, 0.37 and 0.44), respectively.

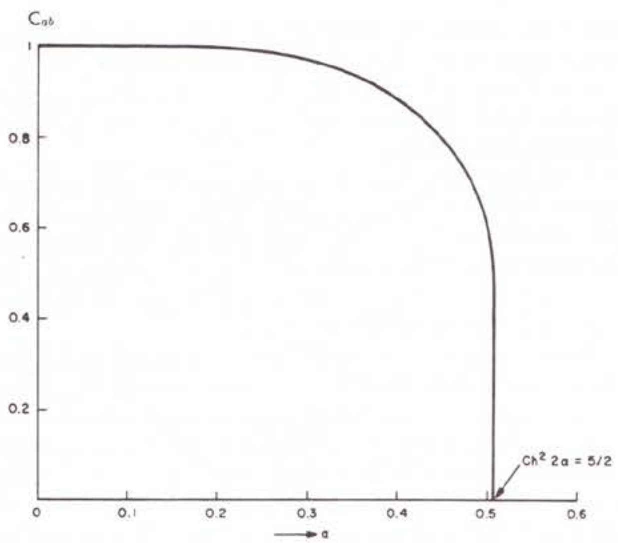

Fig. 5. Coupling factor between a beam with simple astigmatism ($\alpha = 0$) and a beam with general astigmatism (α). For both beams $p_1 = 0.5$ m, $p_2 = 1.5$ m, $z_{01} = 0.5$ m, $z_{02} = -0.5$ m, $\beta = 0$.

V. Beam Transformation by Lenses

We have shown that a gaussian beam of general astigmatism can be described by two complex beam parameters q_1 and q_2 and by a complex rotation angle $\varphi \equiv \beta + j\alpha$ around the z axis. This section considers how these three complex parameters are transformed when the beam passes through an optical system. For the case of an optical system with rotational symmetry, it is easy to show that q_1 and q_2 are transformed according to the known *ABCD* law (Ref. 5) and that φ remains unchanged. The transformation of gaussian beams through arbitrary optical systems is formally the same as the transformation of astigmatic ray pencils except for the fact that q_1, q_2, and φ are complex rather than real quantities. It is, consequently, easily expressed as a function of the elements of the 4 × 4 ray matrix of the system. This transformation, obtained before from a different method,[4] is not reproduced here. In this section, we use the wave optics point of view to calculate the transformation of a beam through a thin astigmatic lens, arbitrarily oriented. Together with the transformation laws in free space given before, this result also allows the (computer) analysis, step by step, of the transformation through any optical system. Similarly, the phase shift along the optical axis can be obtained by using Eq. (7).

A thin astigmatic lens with focal lengths f_1 and f_2 oriented at an angle ν with respect to $0x$ introduces a phase shift

$$\frac{k}{2}\left[\left(\frac{\cos^2\nu}{f_1} + \frac{\sin^2\nu}{f_2}\right)x^2 + \left(\frac{\sin^2\nu}{f_1} + \frac{\cos^2\nu}{f_2}\right)y^2 + \sin 2\nu\left(\frac{1}{f_2} - \frac{1}{f_1}\right)xy\right] \equiv \frac{k}{2}[F_1x^2 + F_2y^2 + \tan 2\nu(F_2 - F_1)xy], \quad (22)$$

where we have defined the new lens parameters F_1 and F_2 similarly as the quantities Q_1 and Q_2 in Eq. (6).

The laws of transformation through the lens are simplified by using the beam parameters Q_1, Q_2, and φ rather than q_1, q_2, and φ. If we denote by a prime the parameters of the outgoing beam, we get from Eqs. (5) and (22)

$$Q_1' = Q_1 - F_1,$$

$$Q_2' = Q_2 - F_2,$$

$$\tan 2\varphi' = \frac{\tan 2\varphi(Q_2 - Q_1) - \tan 2\nu(F_2 - F_1)}{(Q_2 - Q_1) - (F_2 - F_1)}. \quad (23)$$

If, after this transformation, we want to go back to the original q_1, q_2, φ parameters (which are more convenient for the transformation in free space), we may use the relations inverse of Eqs. (6)

$$2/q_{1,2} = Q_1 + Q_2 \pm [(Q_1 - Q_2)/\cos 2\varphi]. \quad (24)$$

The relations of Eq. (23) can be used to derive quantities that are invariants of the thin lens transformation. An obvious example of such an invariant is

$$(q_1^{-1} + q_2^{-1}) - (q_1^{-1} + q_2^{-1})^*. \quad (25)$$

One can use these invariants and show, after some algebraic manipulations, that the condition of Eq. (17) is an invariant of the lens transformation. In other words: a confined beam remains confined after passing through an astigmatic lens as one would expect on physical grounds.

VI. Beat Signal from Gaussian Beams with General Astigmatism

When two optical beams are incident on a large square law detector the intermediate frequency current that they generate is proportional to their coupling factor

$$c_{ab} = \int_{xy\ \text{plane}} E_a E_b^* ds, \quad (26)$$

E_a and E_b being the fields of the two beams, which are taken as having the same polarization and a *small* frequency difference. The normalized coupling factor

$$C_{ab} = c_{ab}/c_{aa}^{\frac{1}{2}} c_{bb}^{\frac{1}{2}}, \quad (27)$$

as it departs from unity, expresses a difference in intensity distribution or wavefront shape between the two beams. The square of the modulus of C_{ab} (power coupling factor) is of interest for the problem of mode conversion. It was given before for the case of simply astigmatic Hermite-gaussian beams.[7] For the case of two gaussian beams with general astigmatism, we get, substituting the expression of Eq. (4) of the fields into Eq. (26) and integrating

$$c_{ab}^{-2} = (q_{1a} - q_{2b}^*)(q_{1b}^* - q_{2a}) - (q_{1a} - q_{2a}) \times (q_{1b} - q_{2b})^* \cos^2(\varphi_a - \varphi_b^*), \quad (28)$$

c_{ab} being independent of z, as expected.

The normalized coupling factor C_{ab} has been plotted in Fig. 5 for the sets of parameters considered before: for both beams we take $p_1 = 0.5$ m, $p_2 = 1.5$ m, $z_{01} =$

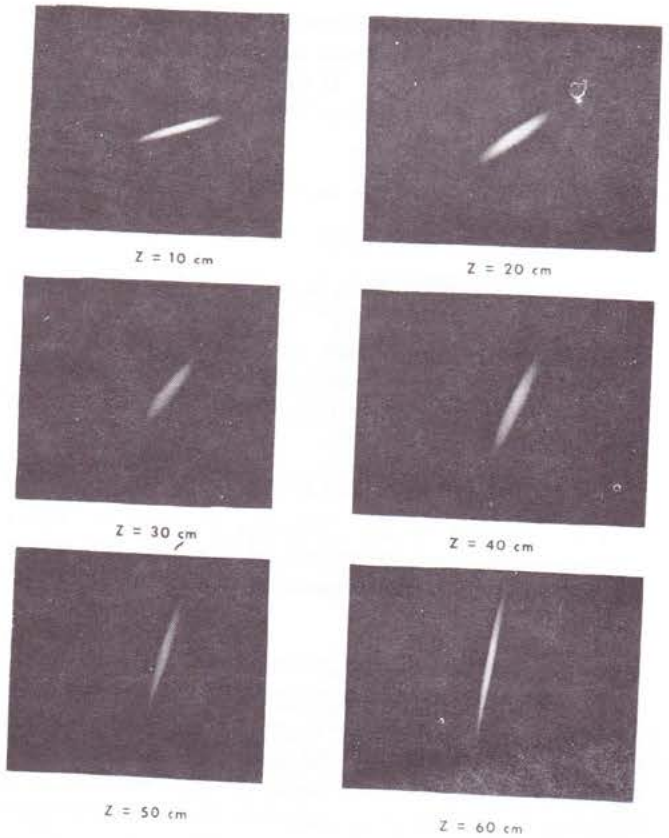

Fig. 6. Photograph of the spot ellipse at various distances z from the second cylindrical lens ($z = 0$). The beam parameters at $z = 0$ (calculated from the measured beam waist before the first cylindrical lens) are: $q_1 = -665 + j\,0.61$ mm, $q_2 = 0.00005 + j\,0.19$ mm, $\varphi = 0.27 \times 10^{-7} + j\,0.47 \times 10^{-3}$.

0.5 m, $z_{02} = -0.5$ m, and $\beta = 0$. For beam a, α is taken as equal to 0. C_{ab}, which is real-valued in that special case, is plotted as a function of the value of α assumed by beam b.

We notice on this curve that the coupling factor vanishes abruptly as α reaches the limiting value given in Eq. (17); this results from the large area occupied by beam b when this critical value of α is approached.

VII. Experiments

Experiments were made at a wavelength of 6328 Å to verify some aspects of the outlined theory.

A coherent optical beam in the fundamental mode was sent through two cylindrical lenses. The first lens was aimed at generating a simply astigmatic beam, and the second lens, oriented at an angle of 45° with respect to the first lens, transformed this beam into a beam with general astigmatism. The beam waist of the input beam was positioned at a distance of 500 mm from the first lens. The focal lengths of the first and second lens were, respectively, 250 mm and 200 mm and their separation was 500 mm.

The spot ellipse was observed at various positions along the beam axis and photographed. Figure 6 shows how the spot ellipse is transformed as the distance z from the second lens is increased. At $z = 0$ the spot ellipse is close to a horizontal focal line. Away from that plane, it rotates and becomes close to a vertical focal line. Further away, the rotation goes on as the beam expands and, eventually, the far field limit angle is reached.

A similar evolution of an astigmatic beam was observed for several q parameters of the input beam and the orientation φ_w of the spot ellipses was measured. Figure 7 shows the values of φ_w as a function of z for an input beam of $q = 500 + j\,16.4$ mm at the first lens (beam waist radius $= 57\ \mu$m).

This experimental result has been checked against the theory given before in a two-step procedure. First, the parameters of the beam (with general astigmatism) at the output of the second lens are calculated from the value of q and the lens separation and focal lengths given before. This is made by programming Eqs. (6), (23), and (24) and using a computer. The numerical results are given in caption of Fig. 6 and 7. The spot ellipse orientation φ_w is then calculated as a function of z from Eqs. (9a) and (10). The agreement between the theoretical and experimental curves, shown in Fig. 7, is very good. Because the value given to the imaginary part of q is small compared with the system dimensions, the curve given in that figure stays close to the geometrical optics limit corresponding to a homocentric input ray pencil. Computer calculations show that a significant departure from this limit occurs when the imaginary part of q is comprised between 300 mm and 1500 mm. When the imaginary part of q is much larger than 1500 mm, the input beam becomes close to a collimated ray pencil and another geometrical optics limit is approached.

VIII. Conclusions

We have discussed the propagation and diffraction of coherent light beams in nonorthogonal optical systems. A simple mathematical description for the fundamental mode was obtained. It corresponds formally to a conventional astigmatic gaussian beam rotated around its axis by a complex-valued angle. Three complex parameters are necessary to describe such beams of general astigmatism; these are the two (conventional) complex beam parameters q_1 and q_2 and the complex rotation angle φ. Restriction to confined beams imposes an upper bound on the imaginary part of φ. A beam with general astigmatism has a gaussian intensity profile with elliptical light spots in each beam cross section and elliptical or hyperbolical curves of constant phase. The axes of the light spots and the phase curves are oriented at oblique angles with respect to each other and change their orientation as the beam propagates and diffracts in free space. The rotation of the spot ellipse of such beams in free space has been observed experimentally, and is in good agreement with the outlined theory.

We have also treated the transformation of an astigmatic beam by a thin astigmatic lens and the coupling between two astigmatic beams relevant to beat experiments.

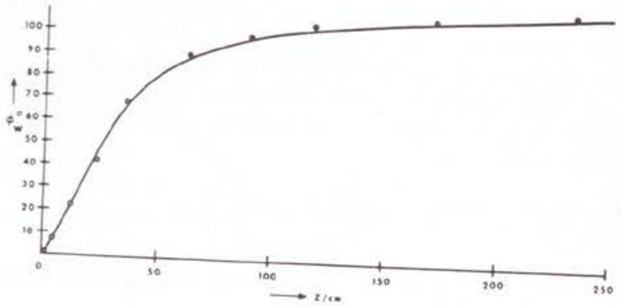

Fig. 7. Theoretical and experimental (circles) values for the orientation of the intensity ellipse (φ_w) as a function of the axial distance z. The beam parameters at $z = 0$ are $q_1 = -660 + j\,52.1$ mm, and $q_2 = 0.406 + j\,16.4$ mm, $\varphi = 0.00202 + j\,0.041$.

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