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Focusing and Deflection of Optical Beams by Cylindrical Mirrors

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An optical system incorporating two closely spaced cylindrical mirrors is described. By properly orienting the mirrors in space, an incident beam can be focused at any desired point within a large volume. This simple periscopic system, which provides variable focal lengths and deflection angles, is applicable to optical or millimeter wave transmission systems lying along irregular paths. A paraxial ray theory of the system is given, as well as experimental results.

Introduction

Optical waveguides are currently being investigated as a means of transmitting communications from city to city or within cities.[1] They usually incorporate sequences of focusers (lenses or pairs of mirrors) whose separation is close, though not equal, to the confocal spacing. In most practical circumstances, it is necessary to bend or tilt in three dimensions the waveguide axis in order to circumvent natural obstacles or right-of-way restrictions. Although small tilts of less than, say, one degree, can be obtained by offsetting the focusers, large tilts are more easily obtained by rotating the mirrors in a periscopic configuration. It is consequently desirable to discuss periscopic systems providing variable deflections of the optical axes in three dimensions (to follow irregular paths) and variable effective focal lengths (to accommodate changes in focuser separation). In order to preserve the circular cross section of the beams, it is further required that the focusers behave as ordinary lenses with rotational symmetry, i.e., that they be free of astigmatism, to first order. It is the purpose of this paper to show that these various conditions are met in a periscopic configuration which incorporates two cylindrical mirrors of equal (and fixed) curvature. Changes in axial direction and focal length are both obtained through proper angular orientation of the two mirrors. Periscopic systems incorporating two cylindrical mirrors of unequal curvatures, capable of sharp focusing, but not of variable focal length and deflection angle, were proposed previously.[2,3] The flexibility of the configuration discussed in this paper results from the introduction of nonplanar optical axes. A configuration incorporating three mirrors with rotational symmetry has also been proposed.[4] In the case of optical waveguides, however, it is important to keep the losses down by minimizing the number of optical elements per refocuser. The problem of geometrical optics aberrations, on the other hand, is relatively unimportant since the effect of such aberrations is masked in practice by diffraction effects. It is worthwhile noting that the transformation of beams propagating in optical waveguides at a sufficient distance from the edges of the apertures can be obtained from the results of a paraxial ray theory.* One may therefore restrict oneself to a paraxial ray theory of curved mirrors under oblique incidence. General conditions for sharp focusing by two cylindrical mirrors are subsequently discussed, under the assumption that the separation between the

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* The wavefront and intensity pattern of a gaussian beam, in particular, can be described by a complex quadratic form in the two transverse variables which formally obeys the same laws of transformation as the wavefront of an astigmatic ray pencil. This transformation reads[5]

$$m' = (C + Dm)(A + Bm)^{-1},$$

where m and m' are 2×2 matrices describing the wavefronts at the input and output planes, respectively, and $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is the 4×4 ray matrix of the optical system. This relation generalizes the well known "$ABCD$ law" applicable to optical waveguides with rotational symmetry.[6]

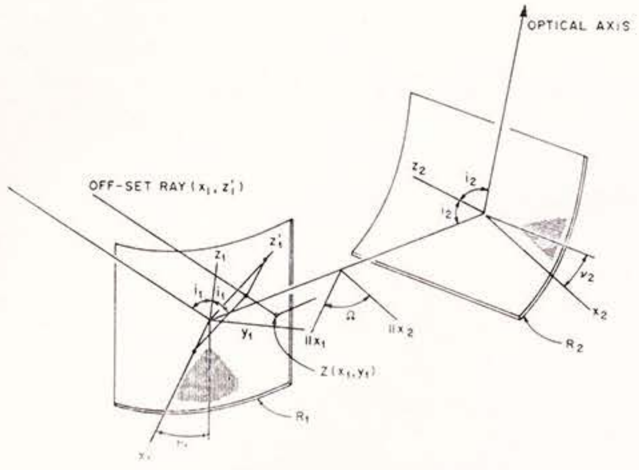

Fig. 1. Schematic representation of the two cylindrical mirror system. Notice that the optical axis, in general, does not lie in a plane; the angle between the planes of incidence at the two mirrors (or their normals x_1, x_2) is denoted Ω. The angles ν_1, ν_2, and Ω as shown in the figure are positive.

two mirrors is small compared with the focal length. Experiments made at visible wavelengths are also described.

Paraxial Ray Theory

Consider a curved mirror such as R_1 shown in Fig. 1 and an incident optical axis. Let $x_1y_1z_1$ be a rectangular coordinate system, with z_1 directed along the inner normal to the mirror and x_1 perpendicular to the plane of incidence. The x_1 axis is oriented in such a way that the y_1 component of the ray direction vector is positive. The position of an incident ray, offset with respect to the incident optical axis, is defined by coordinates x_1 and z_1', the z_1' axis being perpendicular to both the x_1 axis and the incident optical axis. If $z_1 = Z_1(x_1,y_1)$ denotes the equation of the mirror surface, and i_1 the angle of incidence (see Fig. 1), simple geometric considerations show that, within the paraxial approximation, the extra path length experienced by a ray x_1,z_1', as a result of the mirror deformation, is

$$\Delta_1(x_1,z_1') = 2\cos i_1\, Z(x_1,z_1'/\cos i_1). \tag{1}$$

This change in path length is evaluated by considering rays parallel to the optical axis, incident on an area of the mirror which has been displaced along the mirror normal. To be sure, the rays can be oblique with respect to the optical axis, but a small tilt introduces a negligible change in path length at the mirror. For the case of a cylindrical mirror of equation

$$Z_1(x_1, y_1) = (1/2R_1)(x_1 \sin\nu_1 - y_1 \cos\nu_1)^2, \tag{2}$$

where R_1 denotes the mirror radius and ν_1 the angle between the mirror generatrix and the x_1 axis, one obtains, from Eqs. (1) and (2),

$$\Delta_1 = (\cos i_1/R_1)[x_1 \sin\nu_1 - (z_1'/\cos i_1)\cos\nu_1]^2. \tag{3}$$

By rotating the coordinate system x_1, z_1' by an angle ϵ_1 given by

$$\tan\epsilon_1 = \cos i_1 \tan\nu_1, \tag{4}$$

one finds that Δ_1 is equal to the path length that would be introduced by a cylindrical lens of focal length

$$f_1 = (R_1/2)(\cos i_1 \sin^2\nu_1 + \cos^2\nu_1/\cos i_1)^{-1}. \tag{5}$$

The orientation of this equivalent cylindrical lens with respect to the z_1' axis is defined by the angle ϵ_1, given by Eq. (4). Similar results hold, of course, for the second mirror, to which the index 2 henceforth refers. The coordinate system, however, must be rotated by an angle Ω (corresponding to the angle between the planes of incidence at the two mirrors) when going from the first to the second mirror.

As is well known, two cylindrical lenses provide a sharp focusing of incident homocentric ray pencils when they have equal focal lengths and are oriented 90° to one another. These two conditions, when applied to the lenses equivalent to the two mirrors, can be written from Eqs. (5) and (4),

$$\frac{R_1}{2}\left(\cos i_1 \sin^2\nu_1 + \frac{\cos^2\nu_1}{\cos i_1}\right)^{-1} = \frac{R_2}{2}\left(\cos i_2 \sin^2\nu_2 + \frac{\cos^2\nu_2}{\cos i_2}\right)^{-1} = f, \tag{6}$$

where f is the equivalent focal length of the system, and

$$\Omega = \tan^{-1}(\tan\nu_1 \cos i_1) + \tan^{-1}(\tan\nu_2 \cos i_2) \pm (\pi/2). \tag{7}$$

The total deflection angle θ experienced by the optical axis as a result of the reflections on the two mirrors is given, from elementary trigonometry, by

$$\cos\theta = \cos 2i_1 \cos 2i_2 - \cos\Omega \sin 2i_1 \sin 2i_2. \tag{8}$$

Notice now that the angle $\theta = 0$ (no deflection) can be achieved only if $i_1 = i_2 \equiv i$. This condition is henceforth assumed to hold. It is also assumed, for simplicity, that $R_1 = R_2 \equiv R$. Equation (6) then shows that we must have

$$\nu_1 = \pm\nu_2. \tag{9}$$

Let us consider first the solution $\nu_1 = \nu_2 \equiv \nu$ and rewrite Eqs. (6), (7), and (8), taking into account the above assumptions. One obtains

$$f = R/2\left(\cos i \sin^2\nu + \frac{\cos^2\nu}{\cos i}\right)^{-1}, \tag{10}$$

$$\Omega = \pi/2 + 2\tan^{-1}(\tan\nu \cos i), \tag{11}$$

$$\cos\theta = \cos^2 2i - \cos\Omega \sin^2 2i. \tag{12}$$

For a given value of R, we may consequently choose the effective focal length f of the system and the total deflection angle θ, and find the necessary values for i, ν, and Ω from Eqs. (10), (11), and (12). Figure 2 shows the range of values of f and θ which can be obtained in that way, i being restricted to angles between 10° and 60° to avoid interferences between the two mirrors. This figure shows that for $\theta = 0$ (no deflec-

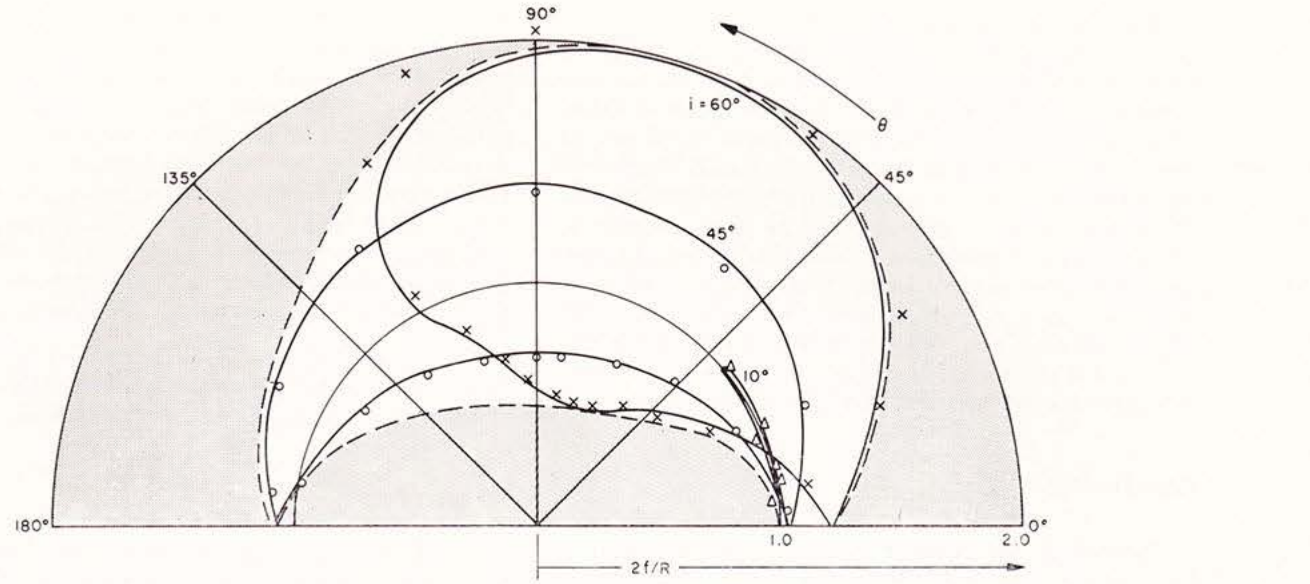

Fig. 2. The clear area shows the values of the system focal length (f) and deflection angle (θ), which can be achieved simultaneously by properly adjusting the cylindrical mirror orientations when the incidence angles are restricted to a 10–60° range. The theoretical curves (plain lines) and experimental points, are shown for three angles of incidence: 10°, 45°, and 60°. R is the radius of both mirrors. By rotating the clear area about the horizontal axis, the volume in which an incident collimated beam can be focused is generated.

tion), the system focal length f can be varied between $0.5R$ and $0.6R$. For larger deflection angles such as 75°, f can be varied in a larger range ($0.25R$ to R).

The solution $\nu_1 = -\nu_2$ of Eq. (9) corresponds, from Eqs. (7) and (8), to $\Omega = \pi/2$ and $\cos\theta = \cos^2 2i$. For a given value of θ, f can consequently be varied by varying ν, the incident angles staying constant. This solution covers, in particular, the area enclosed by the curve $i = 10°$ in Fig. 2, which is not covered by the previous solution. Both solutions are consequently of interest.

Experiments

A mechanical setup, shown in Fig. 3, was constructed to verify the above results. Adjustment and direct dial reading with precision better than 1° are provided for all relevant angles (i_1, i_2, ν_1, ν_2, and Ω). The variations of i_1 and i_2, however, are restricted to a 7–60° range. A deformable parallelogram orients the first mirror in such a way that the reflected beam is always directed to the second mirror. Notice also that the whole system can rotate about the incident optical axis to provide beam deflections in two dimensions. The radii of curvature of the 2 in. × 2 in. × ½ in. (5 cm × 5 cm × 1.3 cm) mirrors are 6 m, and they were made with an accuracy better than 2 μm. The focal

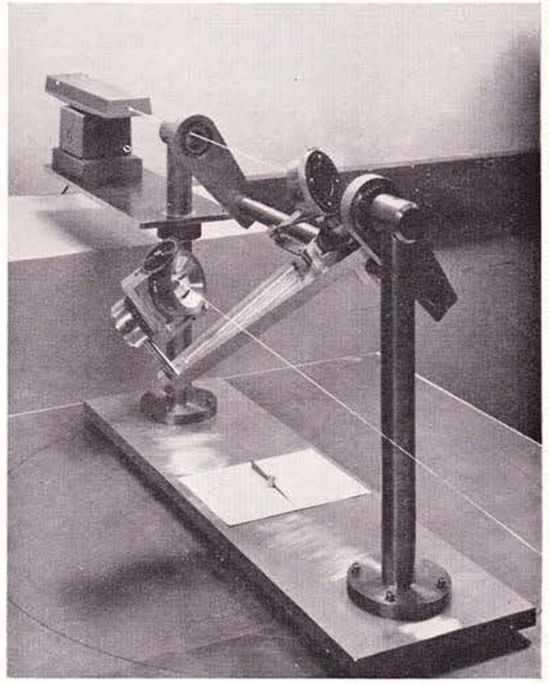

Fig. 3. Photograph of the experimental setup. The laser beam shows the axis of the optical path. The actual experiments were made with incoherent point sources collimated by lenses. The whole assembly can be rotated about a vertical axis, for convenience of measurement.

length of the system is measured by sending a collimated ray pencil onto the system and measuring the position of the focus with respect to the midpoint between the two mirrors. A small astigmatic difference of 20 cm to 40 cm, which is of the order of the mirror separation (26 cm), is observed. This astigmatic difference would be masked by diffraction effects and would therefore be negligible for long focal length systems such as those considered for communication purposes. For shorter focal lengths and fixed object planes, perfect focusing can be achieved by introducing a small correction to the orientation angles given before. The experimental values obtained for f and θ, shown in Fig. 2 for $i = 10°, 45°$, and $60°$, are in good agreement with the calculated values.

Conclusion

Theoretical calculations, as well as experiments, have shown that a system of two cylindrical mirrors properly oriented in space can provide an optimum focusing of incident beams with variable focal lengths and deflection angles. Cylindrical optical mirrors, although somewhat more difficult to manufacture than spherical mirrors with the techniques presently available, can be made with an acceptable accuracy. At millimeter wavelengths, cylindrical mirrors of circular cross section are easy to make by bending aluminum plates with bending moments exerted along the plate edges [rms errors of less than 0.1 mm have been measured on 4 ft $\times$ 4 ft (0.1 m $\times$ 0.1 m) plates, bent with 40-m radii]. As in any periscopic system, the two mirrors, once properly oriented, must, of course, be held rigidly with respect to one another. The positioning of the periscopic system as a whole is just as critical as the positioning of lenses in conventional optical waveguides, and must be controlled by similar devices.

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