# ELECTROMAGNETIC PROPAGATION ALONG CONTACTING DIELECTRIC TUBES

*Indexing term: Guided electromagnetic-wave propagation*

Experiments performed in the 50–80 GHz band show that single-mode propagation is possible near the contact line of two thin-wall dielectric tubes. The group delay $c/v_g$ is found to be almost a constant (3·3) from 55 to 75 GHz. Pulse spreading is therefore very low for that structure ($< 1·6$ ns/km$^{1/2}$). These experimental results are in good agreement with a semiclassical theory of propagation.

We report experiments performed in the 50 to 80 GHz band, showing that single-mode propagation is possible near the contact line of two thin-wall dielectric tubes.* We give a theory that explains the mechanism of operation. The theory presented is based on a semiclassical formulation of the laws of diffraction.[1-3] Let us consider a reactive surface, whose local wavenumber is $k$. $k$ is assumed to be a function of the transverse co-ordinate $y$, but to be independent of axial co-ordinate $z$. Waves are kept confined transversely if $k^2$ reaches a maximum at $y = 0$. If $k^2$ is quadratic in $y$, $k^2 = k_0{}^2 - \Omega^2 y^2$, where $k_0$ and $\Omega$ are constants and the modes are Hermite-Gauss.[1-3] The physical nature of the surface supporting the local waves is now specified.

The surface as a dielectric slab with variable thickness has been analysed before.[1,3] We consider two slabs of constant thicknesses $2d$, but of variable spacing $2D(y)$. When the two slabs are in contact ($D = 0$), the total slab thickness is $4d$; when the slabs are far apart ($D \gg \lambda$), they are uncoupled and the effective slab thickness is only $2d$. Because the wavenumber of a slab decreases with thickness, the local wavenumber of the pair of slabs decreases as the slab spacing $2D$ increases. This reduction in $k$ ensures confinement of the beams near the contact line $y = 0$. In our experiments, the slabs are, in fact, portions of thin-wall tubes (Fig. 1). Because the thickness of the tubes is small, there is only one H mode in the $x$ direction. The depth of the potential well $k^2(y)$, shown in Fig. 1, determines how many modes, with mode numbers $n = 0, 1, 2, \ldots$, can exist in the $y$ direction. If the well is sufficiently shallow, only one mode ($n = 0$) can propagate. We shall see that this is so for the dimensions that we have chosen.

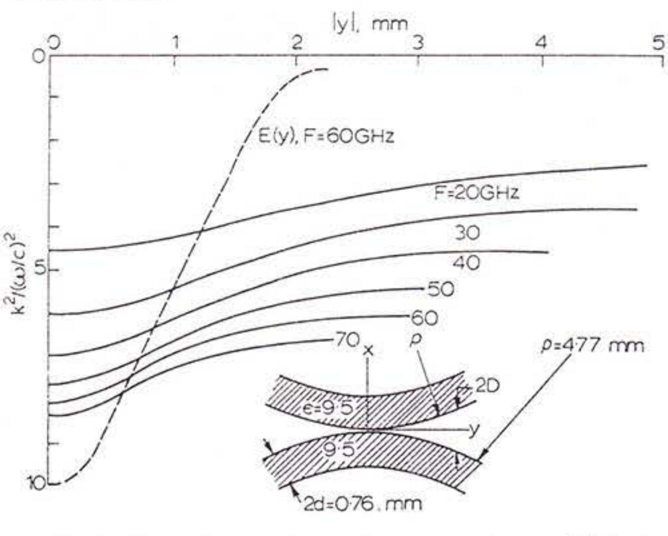


**Fig. 1** *Variation of local wavenumber k with distance |y| from contacting line between two dielectric tubes*
The broken line gives the variation of the electric field with $y$

Let us first evaluate the local wavenumber $k(y)$. Let $y$ be the axis tangent to the two dielectric tubes with radius $\rho$ and thicknesses $2d$ (Fig. 1). The propagation takes place along the $z$ axis. Locally, that is, at some value of the co-ordinate $y$, we have a system of two dielectric slabs both having a thickness $2d$, separated by a distance

$$2D = 2\rho[1 - \{1 - (y/\rho)^2\}^{1/2}] \quad \quad (1)$$

The wavenumber $k$ of the fundamental symmetrical H mode of two parallel dielectric slabs with relative permittivity $\varepsilon$ is easily found by matching the boundary conditions. It is given by the equation

$$(p/\kappa) \tanh(pD) \tan(2\kappa d) + \tanh(pD) - (\kappa/p) \tan(2\kappa d) + 1 = 0 \quad \quad (2a)$$

where

$$\kappa^2 \equiv \varepsilon(\omega/c)^2 - k^2 \quad \quad (2b)$$

and

$$p^2 \equiv k^2 - (\omega/c)^2 \quad \quad (2c)$$

Note that $D$, or $y$, can be written as an explicit function of the wavenumber $k$. The variation of the effective local permittivity $\{k/(\omega/c)\}^2$ obtained from eqn. 2 is plotted in Fig. 1 as a function of $y$ for different frequencies in the 40–80 GHz band. The next step in the evaluation of the axial wavenumber $\beta$ is to solve the eigenvalue equation

$$d^2E/dy^2 + k^2(y)E = \beta^2 E \quad \quad (3)$$

where $k(y)$ is the local wavenumber just obtained. An approximate solution of eqn. 3 is obtained by noting that, for small $|y|$, $k^2(y)$ can be written as

$$k^2 \simeq k_0{}^2 - \Omega^2 y^2 \quad \quad (4)$$

Within this square-law approximation, the fundamental solution of eqn. 3 is well known:

$$E = \exp(-\tfrac{1}{2}\Omega y^2) \quad \quad (5a)$$

and

$$\beta^2 = k_0{}^2 - \Omega \quad \quad (5b)$$

At 60 GHz, for instance, we find from the curves in Fig. 1 that $k_0{}^2 = 12·68$ mm$^{-2}$ and $\Omega = 1·23$ mm$^{-2}$. Thus $\beta = 3·4$ mm$^{-1}$ and the beam halfwidth is $\Omega^{-1/2} = 1·1$ mm. Once the variation of $\beta$ with $\omega$ has been obtained, the group delay $c/v_g$ follows by differentiation. The variation of $c/v_g$ with frequency is shown in Fig. 2D. To check the accuracy of the above square-law approximation, eqn. 3 has been solved numerically with the initial conditions $E = 1$ and $dE/dy = 0$ at $y = 0$. The parameter $\beta$ is varied until $E \to 0$ as $y \to \infty$. At 60 GHz, the exact value of $\beta$ is found by this numerical technique to be 3·395 mm$^{-1}$, in close agreement with the one obtained from the square-law approximation: $\beta = 3·4$ mm$^{-1}$. The variation of $E$ with $y$, obtained by numerical integration, is shown in Fig. 1 as a broken line.

The experimental setup is shown in Fig. 2A. Two 254 mm-long alumina tubes, outer diameter = 9·55 mm, wall thickness = 0·75 mm and relative permittivity = 9·5, are attached side by side. They are held between two brass plates having small (0·76 mm) coupling apertures for measurement purposes. The electric field $E$ is along the $y$ axis. Figs. 2B and 2C show the response of the resonator when the generator frequency is swept from 50 to 80 GHz. The perfect regularity in spacing of the resonances demonstrates that only one mode propagates, at least up to 70 GHz. The variations in amplitude of the resonances are due to the equipment. That the tubes can be touched with the hand without significantly affecting the resonance shows that the field is concentrated near the contact line between the two cylinders.


**Fig. 2A** *Experimental setup*

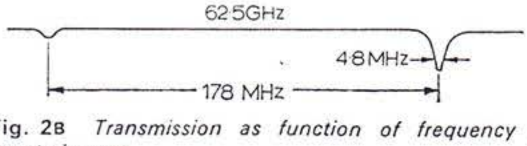


**Fig. 2B** *Transmission as function of frequency over free spectral range*


**Fig. 2C** *Transmission from 55 to 75 GHz*

* E. A. J. Marcatili suggested earlier that optical beams could be kept con[...] near the contact line of quartz tubes (private communication). To our knowl[...] no theory or experiments have been reported

Fig. 2B shows two adjacent resonances separated by a frequency $\Delta F = 178$ MHz. The group velocity is obtained at any frequency from the measured $\Delta F$ with the help of the formula $v_g = 2L\Delta F$, where $L = 254$ mm is the tube length.

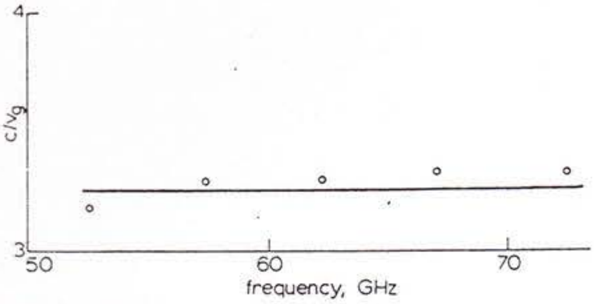


Fig. 2D *Group delay*
○ experiment
— theory

The variation of $c/v_g$ as a function of frequency is shown in Fig. 2D. We observe that the propagation is almost dispersion free. Because the variation of $c/v_g$ over a 10 GHz band does not exceed 0·05, the pulse spreading would not exceed

$$\{c^{-1}\,\Delta(c/v_g)/\Delta\omega\}^{1/2} = 1{\cdot}6\ \mathrm{ns/km}^{1/2}$$

The $c/v_g$ curve measured for a 1·6 mm-diameter rod made with the same material exhibits a much larger group dispersion: $c/v_g = 5{\cdot}3$ at 50 GHz and $c/v_g = 4{\cdot}2$ at 70 GHz.

In conclusion, we have found that propagation along a system of two contacting dielectric tubes can be understood on the basis of a semiclassical theory. This configuration can be made single mode and almost dispersion free. It is therefore attractive for the transmission of microwave signals over moderate distances.


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