EFFECT OF POLARISATION ON PULSE BROADENING IN MULTIMODE GRADED-INDEX OPTICAL FIBRES

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A formulation of ray optics that takes the polarisation of electromagnetic waves into account shows that the maximum difference in time delay between corresponding HE and EH modes is, for almost any multimode graded-index fibre, of the order of 10 000 $(\Delta n/n)^2/V$ ns/km, where V is the V-number of the fibre and Δn is the variation of the refractive index between axis and cladding. The effect of polarisation on pulse broadening is found to be negligible if the V-number of the fibre is much larger than about 20.

The laws of scalar ray optics predict that the impulse response width of a circularly symmetric fibre with the optimum profile excited by a quasimonochromatic lambertian source is about[1]

$$\Delta\tau_{min} = 625\,(\Delta n/n)^2 \quad \text{ns/km} \qquad (1)$$

For example, if $\Delta n/n = 0{\cdot}01$, $\Delta\tau_{min} = 62$ ps/km. The best experimental results are at least one order of magnitude higher than the value predicted by eqn. 1. In this letter, we investigate the possible effect of polarisation on pulse broadening; that is, the consequences of the fact that corresponding HE and EH modes do not have exactly the same group velocity. For simplicity, the effect of material dispersion is neglected and we assume that the fibre is isotropic, circularly symmetric, uniform and overmoded. The following calculation is based on a form of ray optics that takes polarisation changes into account.

In an inhomogeneous isotropic medium the electric field vector rotates along the ray, with respect to the binormal, at a rate equal and opposite to the rotation of the binormal itself. We can describe qualitatively the situation by saying that the electric field tends to remain as parallel to itself as is permitted by the requirement that it remain perpendicular to the ray. Let us recall that the binormal of a ray is always perpendicular to the ray and to the gradient of the refractive index.[2] For circularly symmetric fibres with polar coordinates (r, ϕ, z) the gradient of n is a purely radial vector. On that basis, and with the help of the Hamilton equations in Reference 1, I have obtained a simple and exact expression for the azimuthal rate of rotation of the field with respect to the ray binormal (or, equivalently, with respect to the ray principal normal), which reads

$$d\theta/d\phi = -k_z k(r)/(k_z^2 + l_z^2/r^2) \qquad (2)$$

where $k(r) \equiv (\omega/c)\,n(r)$ and k_z and l_z are the axial components of the wavenumber and angular momentum of the ray, respectively. k_z and l_z are two constants of motion of the ray. Within the WKB approximation, k_z is the propagation constant of the corresponding mode and $l_z \equiv \mu$ is the azimuthal mode number. An $\exp[i(k_z z + \mu\phi - \omega t)]$ variation of the field is assumed.

Within the weakly guiding approximation, we have $k_z \simeq k(r) \simeq k(0) \equiv k_0$ and $\mu^2/r^2 \ll k_z^2$. Thus $d\theta/d\phi$ is very close to -1. At the maxima of the ray radius, the principal normal of the ray coincides with the radial direction. Because the direction of the field is refered to the principal normal, the variation of θ over a ray period is just opposite to the change in azimuthal angle ϕ. This means that, within the weakly guiding approximation, the direction of the field is, in fact, fixed in space. A rotation different from zero is predicted, however, by the exact expression (eqn. 2) if we go beyond the weakly guiding approximation.

Because the state of polarisation of a mode, by definition, must be independent of ϕ and z, the only permissible states of polarisation are circular, clockwise and counterclockwise. For definiteness we now assume that $\mu > 0$. These circularly polarised modes suffer a phase shift equal (clockwise polarisation) or opposite (counterclockwise polarisation) to $\phi+\theta$ in eqn. 2. The change in axial wavenumber is obtained by dividing the variation of the phase shift $\phi+\theta$ over a ray period by the ray period Z, defined as the distance between adjacent maxima of r. In general, we must integrate $\pm Z^{-1}(d\phi + d\theta)$ in eqn. 2 over a period. Because the effect is maximum for helical rays, we shall evaluate in the next paragraph the perturbation for that special case.

For the special case where the ray is helical, that is, has a constant radius (radial mode number $\ll$ azimuthal mode number), the variation of ϕ over a ray period is equal to π, provided that the profile $k(r)$ is continuous. Thus the variations in axial wavenumber are, from eqn. 2,

$$\begin{aligned}\Delta k_z &= \pm(\pi/Z)(1 + d\theta/d\phi)\\ &= \pm(\pi/Z)\{k_z[k_z - k(R)] + \mu^2/R\}/(k_z^2 + \mu^2/R) \qquad (3)\end{aligned}$$

where $R \equiv r^2$. The + sign in eqn. 3 is applicable to $EH_{\mu-1}$ modes and the − sign to $HE_{\mu+1}$ modes.

From the ray equations we have, for helical rays,

$$\pi/Z = \mu/(Rk_z) \qquad (4a)$$

$$k_z^2 = d[Rk^2(R)]/dR \qquad (4b)$$

$$\mu^2 = -R^2\,dk^2(R)/dR \qquad (4c)$$

For example, if the profile is not too different from a square-law:

$$k^2(R) = k_0^2 - k_1^2 R \qquad (5)$$

we have, substituting eqn. 5 in eqn. 4,

$$\pi/Z = k_1/k_z \qquad k_0^2 - k_z^2 = 2k_1\mu \qquad \mu = k_1 R \qquad (6)$$

In a first approximation, Δk_z for near-square-law profiles is, substituting eqn. 6 in eqn. 3,

$$\Delta k_z \simeq \pm\tfrac{1}{2}\mu k_1^2/k_0^3 \qquad (7)$$

Eqn. 7 can be rewritten as

$$\Delta k_z \simeq \pm\tfrac{1}{2}\mu k_0\,\delta^2/(k_0 a)^2 \qquad (8)$$

if we define

$$\delta^2 \equiv 1 - k_s^2/k_0^2 \simeq 2\Delta n/n \qquad (9)$$

and a denotes the core radius and k_s the cladding wavenumber.

These expressions coincide with a result obtained by Matsuhara from a totally different approach[3] (a degenerate perturbation of the scalar wave equation applicable to square-law media). It is also interesting that, for a step-index fibre, the numerical calculations of Biernson and Kinsley[4] for $\mu = 3$ lead to values of $2\Delta k_z \equiv k_{zEH_{21}} - k_{zHE_{41}}$ which differ from eqn. 8 only by a multiplicative factor of 0·9. The discontinuity of the index, however, introduces depolarisation effects that are not accounted for by the present theory. Thus exact agreement is not expected. For step-index fibres, the description of EH and HE modes in terms of circularly polarised components has been discussed by Kapany and Burke.[5] We see now that this description is general.

In fibre optics, we are more interested in relative times of flight than in propagation constants. The relative time of flight of a mode is defined as the ratio of the time of flight of the mode to that of plane waves on axis. The difference in relative time of flight between an electromagnetic mode and the corresponding scalar mode is obtained by differentiating Δk_z in expr. 8 with respect to ω. Neglecting material dispersion, we obtain

$$\Delta\tau \simeq \mp\tfrac{1}{2}\mu\delta^2/(k_0 a^2) \qquad (10)$$

If we replace μ in expr. 10 by its maximum value for propagating modes, we obtain

$$\Delta\tau_{max} \simeq \pm(\Delta n/n)^2/V \qquad (11)$$

where

$$V \equiv (2\Delta n/n)^{1/2}\,k_0 a \qquad (12)$$

is the V-number of the fibre. It is interesting to compare $2\Delta\tau_{max}$ from expr. 11 with the minimum impulse width in eqn. 1 predicted by scalar ray optics. $2\Delta\tau_{max}$ is found to be comparable to the minimum impulse width $\Delta\tau_{min}$ when $V = 16$. I have calculated the r.m.s. impulse width σ for a fibre with $\Delta n/n = 0{\cdot}005$, core radius = 20 μm, $\lambda = 1$ μm, from scalar ray optics and from eqn. 2 based on vector ray optics, as a function of the exponent κ of r^2 in the expression of the index profile. The result is shown in Fig. 1. The minimum value of σ predicted by vector ray optics is 20% higher than the value predicted by scalar ray optics. If

$\Delta n/n = 0{\cdot}01$, the error introduced by the scalar approximation is only 8%.

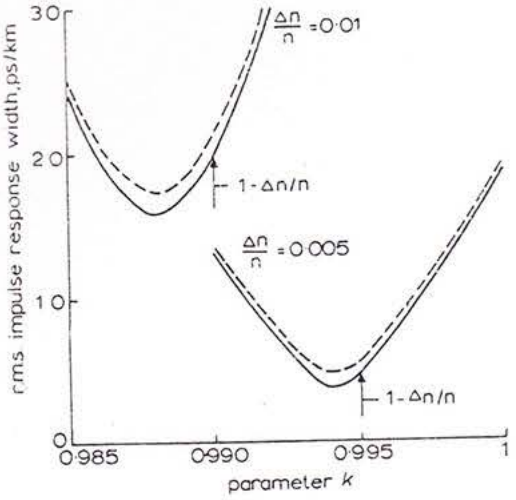

Fig. 1 *Variation of r.m.s. impulse response width of multimode graded-index fibre as function of exponent κ of r^2 in profile law*

Plain lines: calculated using scalar ray optics. Broken lines: calculated using vector ray optics (the present theory). Two values of the relative change of refractive index are considered: $\Delta n/n = 0{\cdot}01$ and $0{\cdot}005$. In both cases the core radius is equal to 20 μm and the wavelength is 1 μm

$[n(r)/n(0)]^2 = 1-2(\Delta n/n)(r_{\mu m}/20)^{2\kappa}$

In conclusion, using a simple vector ray-optics model of propagation in circularly symmetric graded-index multimode fibres, one can obtain the difference in time of flight between the EH and HE modes that degenerate into a given scalar mode in the limit of small wavelengths. The scalar approximation is found to be sufficiently accurate for most applications if the fibre V-number is much larger than about 20. Otherwise, a correction needs to be introduced.

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