Dispersion of tubular modes

J. ARNAUD

Equipe de Microoptoélectronique de Montpellier, U.S.T.L., 34060 Montpellier Cedex, France

Abstract

I show that a theoretical expression for the dispersion $(d^2\beta/d\omega^2)$ of tubular modes in dispersive multimode square-law fibers coincides with the wave-optics result.

Introduction

Tubular modes can propagate in multimode optical fibers, and can serve as independent communication channels [1]. The dispersion $(d^2\beta/d\omega^2)$ of each of these modes is comparable to that of a simple-mode fiber and thus very large transmission capacities would be allowed. Over short distances, mode coupling has been shown to be moderate. It is therefore of interest to calculate precise values for the modal dispersion of these modes. Such a result has been obtained by Barthelemy from a quasi-geometrical optics approach [2]. We wish to show here that for the special case of square-law profiles ($\kappa = 1$), the same result can be obtained from wave optics in the limit that the azimuthal mode number (μ) is very large. Although this agreement is expected from the correspondence principle, the detailed verification may be of interest because it gives an estimate of the error made in using geometrical optics.

The ray optics result

Tubular modes can be represented approximately by helical rays that remain at a constant distance, r, from the fiber axis. Barthelemy was able to excite such modes and to measure their dispersion ($d^2\beta/d\omega^2$, $\beta =$ propagation constant), using an original interferometric technique. Different tubular modes can be excited at the same time, and their differences both in time of flight (t), and dispersion ($dt/d\lambda_0$) can be exhibited in a plane. In the reconstruction procedure, the average time of flight (t) of a pulse is proportional to the transverse distance, while optimum focusing takes place at an axial distance proportional to

dispersion $(dt/d\lambda_0)$. In principle, all the modes of a multimode fiber could be displayed as points in a plane, but experimentally this has been done so far only for a few modes. Considering the uncertainty in the profile dispersion, the agreement obtained between measured and calculated values (a factor of 2) is fair.

We assume a square-law profile $n(r) \sim r^{2\kappa}$, $\kappa = 1$ but allow for an arbitrary dispersion of the material. For reasons of clarity, we assume that the material on axis is non dispersive $(dn/d\lambda_0 = 0)$. If it is in fact dispersive $(\lambda_0 \neq 1.3 \ \mu m$ for silica), to first order, it is sufficient to add that dispersion to the modal dispersion presently discussed.

We thus assume that

$$U(r, \omega) \equiv 1 - n(r, \omega)/n_0 = \Delta(\omega)(r/r_c)^2 \tag{1}$$

where U is a normalized index profile, $n_0 = n(0, \omega)$ is supposed to be a constant. $\Delta(\omega)$ denotes the relative index change and r_c the core radius. As is well known, the optimum profile is a power $2\kappa_0$ of r, where

$$\kappa_0 = 1 + \Delta^{-1}\dot{\Delta}; \quad \dot{\Delta} = \omega d\Delta/d\omega \tag{2}$$

For most dopants, κ_0 is a function of ω. We shall need the first derivative of κ_0 denoted: $\dot{\kappa}_0 = \omega d\kappa_0/d\omega$.

Let us now review Barthelemy quasi-ray-optics theory for the case considered. For helical rays, the relationship between the azimuthal mode number μ and radius r is

$$(\mu/k_0)^2 = 2R^2 dU/dR; \quad R \equiv r^2; \quad k_0 = \frac{\omega}{c}n_0 \tag{3}$$

and the relative time of flight is

$$\tau \equiv t/t_0 - 1 = RdU/dR - \kappa_0 U \tag{4}$$

where t_0 is the time of flight of pulses on axis, a constant according to our assumption. Next, we look for dispersion $\dot{t}$, or $\dot{\tau}$, keeping μ (but not R) a constant as the frequency varies. We get from Eqns. (3) and (4)

$$\dot{\tau} = -U\left[\ddot{\kappa}_0 + \tfrac{1}{2}(\kappa_0 - 1)(\kappa_0 - 3)\right] \simeq -U\left[\dot{\kappa}_0 - (\kappa_0 - 1)\right] \tag{5}$$

which is a special case ($\kappa = 1$, $\dot{n}_0 = 0$ in Eqn. (11)) of Ref. [1]. The approximate expression in Eqn. (5) is for $\kappa_0 \simeq 1$. Note, incidentally, that if we were considering only the dispersion of the material at the radius r of the mode we would get a different result, namely

$$\dot{\tau}' = -U\left[\dot{\kappa}_0 + \kappa_0(\kappa_0 - 1)\right] \simeq -U\left[\dot{\kappa}_0 + (\kappa_0 - 1)\right] \tag{6}$$

which coincides with the correct result in Eqn. (5) only when the optimum profile is the square-law profile ($\kappa_0 = 1$). This is the case only at $\lambda_0 \approx 0.9 \ \mu m$ for germania doped silica.

The wave optics result

Let us now consider the exact (within the paraxial-wave approximation) Laguerre-Gauss modes. According to Ref. [3], pp. 104–106, the mode-generating concept (or more elementary methods) leads to the following expression for the propagation constant β of stationary modal solutions (radial mode number $\alpha = 0$)

$$\beta = k_0 - (\mu + 1)\Omega \tag{7}$$

where $\Omega = \sqrt{2\Delta}/r_c$. The calculation of $\dot{\tau}$ from Eqn. (7) is lengthy but straightforward. Some details are given below. First note that:

$$\begin{aligned}\dot{\tau} &\equiv \omega d\tau/d\omega, \quad \tau = t/t_0 - 1 \\ t_0 &= n_0 L/c, \quad t = Ld\beta/d\omega\end{aligned} \tag{8}$$

Thus, using Eqn. (7)

$$\dot{\tau} = (c\omega/n_0)d^2\beta/d\omega^2 = -(c\omega/n_0)(\mu + 1)d^2\Omega/d\omega^2 \tag{9}$$

Because μ is kept constant, and n_0, for the sake of simplicity, is kept a constant as before. Now, since $\Omega = \sqrt{2\Delta}/r_c$ we have first

$$\frac{d\Omega}{d\omega} = \left(1/r_c\omega\sqrt{2\Delta}\right)\dot{\Delta} = \left(1/r_c\omega\sqrt{2\Delta}\right)\Delta(\kappa_0 - 1) \tag{10}$$

where $\dot{\Delta} = \omega d\Delta/d\omega$, and Eqn. (2) has been used. The second derivative involves three terms

$$\frac{d^2\Omega}{d\omega^2} = \left(\sqrt{\Delta}/r_c\sqrt{2}\ \omega^2\right)\left[\dot{\kappa}_0 + \frac{1}{2}(\kappa_0 - 1)^2 - (\kappa_0 - 1)\right]. \tag{11}$$

If we substitute the result in Eqn. (11) into Eqn. (9), use the expression for μ in Eqn. (3)

$$\mu = k_0\sqrt{2RU}; \quad k_0 = \frac{\omega}{c}n_0 \tag{12}$$

and Eqn. (1), we obtain

$$\dot{\tau} = -U\left[\dot{\kappa}_0 + \tfrac{1}{2}(\kappa_0 - 1)(\kappa_0 - 3)\right] \tag{13}$$

in agreement with Eqn. (5), provided 1 can be neglected compared with μ (high azimuthal mode number). Note, however, that in the experiments reported in Ref. [2] the neglect of 1 compared with μ involves fairly large errors, of the order of 30%.

References

[1] S. Berdagné and P. Facq, Mode division multiplexing in optical fibers. *Applied Optics* 21 (11) (1982) 1950–1955.
[2] A. Barthelemy, Dispersion of tubular modes propagating in multimode optical fibers. *Opto and Quantum Electr.* 15 (1983) 225–230.
[3] J. Arnaud, *Beam and Fiber Optics*, Academic Press, New York (1976).