CONTRIBUTION OF SHOT NOISE TO LASER DIODE LINEWIDTH

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Yamamoto and others have shown that when the pump (or injected current) fluctuations of a laser diode are suppressed, the optical power fluctuations are much reduced, but the laser linewidth remains essentially unchanged. We find that for a realistic extended laser model, the linewidth may in fact be importantly reduced. Our finding is based on a semiclassical theory. With full shot-noise in the injected current, the linewidth is proportional to $(1+\alpha^2)_{av}$, where $\alpha(z)$ denotes the phase-amplitude factor, and the average is evaluated along the diode z-axis. Without injected current fluctuations, one must subtract from this expression half the variance $(\alpha^2)_{av}-(\alpha_{av})^2$ of α. The linewidth reduction thus occurs only if α varies significantly along the diode.

The purpose of this paper is to examine the influence of the injected current fluctuations on the linewidth of a laser diode. Contrary to a previous report by Yamamoto and others [1–3], we find that this influence can be significant. This is because a laser diode should be considered as an extended structure. In ref. [3], the laser diode was considered as a single element device.

Let us first recall a few basic facts about laser oscillations. It is generally believed that well above threshold a laser generates a coherent state. For a coherent state, the (one-sided) spectral density $S_{\delta R}$ of the fluctuations of the rate of photon generation $R(t)$ is equal to $2R_0$ (full shot noise), where R_0 denotes the average rate. The spectral density $S_{\delta\Phi}$ of the phase deviation $\delta\Phi(t)$ is $1/2R_0$, and the product: $S_{\delta R}S_{\delta\Phi}$ is unity. The spectral density $S_{\delta\Phi}$ of a *laser* output field assumes the constant value $1/2R_0$ quoted above only at baseband frequencies f well above the "cold" cavity linewidth $\Delta\nu_c$ (which is on the order of 1 THz for laser diodes). Below that frequency, the fundamental phase noise is dominated by phase diffusion, which can be represented by a white spectrum $S_{\delta\nu}$ for the *frequency* deviation $\delta\nu(t)$. For simple tuned circuits, we have [4]: $S_{\delta\nu}=(\Delta\nu_c)^2/R_0$.

This behavior of phase fluctuations when $f\ll\Delta\nu_c$ is well known. What has been observed only recently [1] is that the output power fluctuations of a laser simply reflect those of the pump, that is, of the injected current in the case of a laser diode. Therefore a laser exhibits the amplitude fluctuations expected from a coherent state only when the arrival times of the injected electron–hole pairs are independent of each other (full shot noise). In contradistinction, it may happen that the electron–hole pair arrival times are correlated to the point that the current fluctuations are almost completely removed (e.g., when the injected current is controlled by a space-charge limited vacuum tube). Yamamoto et al. [1] have pointed out that under such circumstances the laser delivers an amplitude squeezed state rather than a coherent state. An experimental verification is reported in ref. [2]. These authors however assert that the laser linewidth is essentially unaffected when the injected current fluctuations are suppressed. We will show that this conclusion is valid only for a simplified laser model. Exact expressions for the laser linewidth are given for a realistic extended laser model, with and without short noise.

We shall use a semiclassical theory capable of reproducing the results of the full quantum theory as long as the photons are not detected individually. This theory differs somewhat from the semiclassical theory given in ref. [3] because here the carrier rate equation is explicitly introduced, see eq. (3) below. A white gaussian fluctuating current $i(t)$ is associ-

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ated with any conductance G, with spectral density [3]

$$S_i = 2h\nu|G| , \quad (1)$$

where the vertical bars denote absolute value. Fluctuations are therefore associated with positive conductances expressing absorption, as well as with negative conductances expressing stimulated emission (optical gain). The phase and amplitude fluctuations of any oscillator at low base-band frequencies f can be obtained from eq. (1), the usual circuit equations, and the carrier rate equation

$$J/e = G_d|V|^2/2h\nu , \quad (2)$$

where J is the injected current and $-e$ the electronic charge. G_d is the net active conductance (that is the difference between the conductance G_a expressing stimulated emission, and the conductance G_d expressing stimulated absorption). For simplicity, we restrict ourselves to the case where G_d is everywhere positive. $|V|$ is the modulus of the voltage V across the conductance. Eq. (2) asserts that the rate of photon generation equals the rate of electron–hole pair injection, an assumption valid when the quantum efficiency is close to unity and when the operating current is much larger than the threshold current. Eq. (2) is assumed to be valid at any instant of time. The injected current J consists of a dc bias current, which we consider to be prescribed by the external circuit, and a fluctuation part which has a spectral density equal to $2eJ$ (full shot noise), or 0 (no shot noise). At small base-band frequencies the oscillation frequency must be real, otherwise the amplitude fluctuations would be unbounded. This condition implies that some small variations of G_d occur. In general, when the conductance of an active medium varies by an amount g as a result of a change in carrier number there is also a small change b of the susceptance. The phase–amplitude coupling factor [5,6] α is defined as $-b/g$.

We have outlined above the basic concepts that are needed. For a single active element, eq. (2) is unnecessary [7,8]. For any number of active elements in parallel and $\alpha=0$, the oscillator linewidth is given in ref. [9] (note that the spectral densities used in that reference are different from the one used here, but they are equivalent to them when $\alpha=0$). For an extended structure such as a laser diode, it is necessary to introduce densities (of currents, conductances...). The mathematics is complicated because one must solve stochastic differential equations (the details are given in ref. [10]). But the final result is very simple for the configurations discussed in the present paper.

The specific laser model considered is the one-dimensional ring-type laser shown in fig. 1, with only the clockwise wave oscillating. The active layer is driven by electrons originating from a cathode whose emission is either temperature-limited (full shot noise) or space-charge limited (negligible shot-noise). This configuration may not be practical, and it is shown here for the sake of illustration. It is assumed that the current spreading and ambipolar diffusion lengths are much smaller than the diode length. Therefore any two distinct elementary lengths of the laser can be considered as independent from each other. They are coupled only by the electromagnetic field. The bias injected current density $J(z)$ is supposed to be independent of the laser dynamics. This is the case when the voltage drop outside the active layer is much larger than the voltage drop inside the active layer, a rather common situation, particularly for long wavelength lasers. The theory given in ref. [10] is valid for arbitrary distributed losses along the laser length. But in the present paper we assume that all of the loss is located at $z=0$ and due to a partially reflecting mirror of power reflectivity R. In other words, all the generated power P is extracted out of the laser.

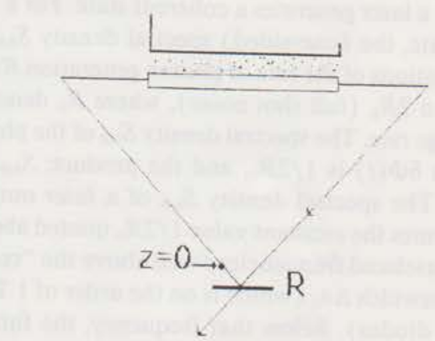

Fig. 1. Ring-type laser incorporating one mirror of power reflectivity R. Only the clockwise wave propagates. The injected current density J, the α-factor and the spontaneous emission factor n_{sp} are arbitrary functions of z, but distributed loss is neglected. The injected current originates from an extended cathode, shown above the active layer, which may be either temperature limited [full shot noise, see eq. (3)], or space-charge limited [negligible shot noise, see eq. (4)].

We have given above the principles of our calculation and clarified the laser diode model. Let us now give the expressions for the laser linewidth $\Delta\nu = \pi S_{\delta\nu}$. When full shot noise is present we have

$$2\pi\Delta\nu P/h\nu = \tau^{-2}[(1-R)^2/2R]\{[n_{sp}(1+\alpha^2)]_{av}\} , \quad (3)$$

where P denotes the output power and τ the round-trip time. n_{sp} is the so-called spontaneous emission factor. In terms of the conductances G_a and G_b defined earlier, $n_{sp}=G_a/(G_a-G_b)$. n_{sp} is unity at low temperatures when stimulated absorption is negligible ($G_b\approx 0$), and on the order of 2 at room temperature. Both n_{sp} and α may very arbitrarily with z, the coordinate along the laser length. The averaging in eq. (3) is performed not over z itself but over the coordinate $\zeta(z)$ defined as the reciprocal of the power gain from $z=0$ to some z-value.

When the injected current fluctuations are suppressed, one must substract half the ζ-variance of α from the terms in braces in eq. (3), that is

$$2\pi\Delta\nu P/h\nu = \tau^{-2}[(1-R)^2/2R] \times\{[n_{sp}(1+\alpha^2)]_{av} - [(\alpha^2)_{av} - (\alpha_{av})^2]/2\} . \quad (4)$$

It is remarkable that expressions formally identical to eqs. (3) and (4) are obtained when any number of active elements are connected in parallel, the averagings being evaluated in that case with the injected bias currents as weighting factors.

Going back to the ring-laser, let us assume that there are two laser amplifiers along the path, labeled by 1 and 2 in that order, with power gains G_1, G_2, factors α_1, α_2 and n_{sp1}, n_{sp2}, respectively. For that case, the ζ-averaging of any quantity a_k, $k=1, 2$, is defined as

$$a_{av} \equiv (1-R)^{-1} \times[a_1(1-1/G_1)+a_2(1/G_1-R)] , \quad (5)$$

where we have used the fact that the oscillation condition requires that $G_1G_2R=1$.

For the sake of illustration, consider the following numerical values

$$n_{sp1}=n_{sp2}=1, \quad G_1=G_2=2, \quad \alpha_1=6, \alpha_2=0 . \quad (6)$$

The terms in braces in eqs. (3) and (4) that we call A are, respectively, using the definition of averaging in eq. (5) and the numerical values in eq. (6),

$A=24$ (full shot noise) ,

$A=21$ (no shot noise) .

If the α factors are reversed, namely $\alpha_1=0$, $\alpha_2=6$, the linewidths are proportional to

$A=13$ (full shot noise) ,

$A=9$ (no shot noise) .

In a single laser with a homogeneous active layer, the n_{sp} and α factors may vary along the length under the condition of large series resistances, because the local gain is nonuniform. These factors may also vary if there are compositional changes along the diode.

In conclusion, we have reported the results of a theory of laser oscillator phase noise that rests only on the semiclassical expression in eq. (1) of the zero-point fluctuation of the electromagnetic field, and usual electrical engineering formulas. In our opinion, the frequently used concept of "number of photons in the cavity" (see e.g. ref. [11]) is ambiguous and unable to lead to the formulas reported here. The Petermann K-factor [12,13] has not been considered in this paper. Thus the above formulas are restricted to index-guided lasers.

Specifically, we have found that suppression of the injected current fluctuations may significantly reduce the laser diode linewidth. Conceptually, our result is in full agreement with the findings of Yamamoto et al. [1]. The discrepancy arises only from the fact that we have considered laser diodes with nonuniform α-factors.

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