# Comments and Corrections

## Comments on "LaGuerre–Gaussian Periodically Focusing Beams in a Quadratic Index Medium"

J. ARNAUD

In a recent paper,[1] Newstein and Rudman give the phase of Laguerre-Gauss beams in focusing media and write, "The solutions in cylindrical coordinates, in particular the phase shifts, appear not to be available in the literature." I would like to indicate that the complete Laguerre-Gauss solution, including the phase, is given in my book [1] in (2.209g) for a focusing constant ($\Omega(z)$ in my notation) that varies arbitrarily with $z$. To obtain an explicit solution, it suffices to be able to solve the paraxial ray equation. For the case of a constant $\Omega$ considered in the above paper,[1] this is easily done [1, eq. (2.54)]. My result derives from a very general expression based on the complex ray representation of Gaussian beams.

Specifically, the phase 8 of the on-axis field of a circularly symmetric Gaussian beam is twice the value for two-dimensional Gaussian beams [1, eq. (2.22)], and therefore opposite to the phase of the complex ray $q(z) \equiv q_r(z) + iq_i(z)$:

$$\tan(\theta) = -q_i(z)/(z). \quad (1)$$

A medium with constant quadratic focusing is defined as [1, eq. (2.8)]

$$k^2(r, z) \equiv k_0^2[1 - \Omega^2 r^2/2]^2 \cong k_0^2[1 - \Omega^2 r^2]. \quad (2)$$

Note that for any $z$-invariant medium, it is straightforward to go from the field solution applicable to the paraxial form (first expression in the above equation) to the exact form (second expression). The solution of the ray equation is [1, eq. (2.53)]

$$q \equiv q_r + iq_i = \xi_0 \cos\Omega z + i(k_0\Omega\xi_0)^{-1}\sin\Omega z \quad (3)$$

where $\xi_0$ denotes the beam radius at the $1/e$ point of the power density.

Therefore, from (1) and (3),

$$\tan\theta = -(k_0\Omega\xi_0^2)^{-1}\tan\Omega z. \quad (4)$$

For a Laguerre–Gauss beam, the form [1, eq. (2.209f), (2.209g)] shows that the phase given in (4) is multiplied by $2\alpha + \mu + 1$ where $\alpha = 0, 1, 2, \cdots$ denotes the radial mode number and $\mu$ the azimuthal mode number.

The result [(15) of the above[1]] coincides with our (4) when the appropriate changes of notation are made.

Manuscript received June 8, 1987.

The author is with the Equipe de Microoptoelectronique de Montpellier, CNRS 392, USTL Montpellier, 34060 Cédex, France.

IEEE Log Number 8717805.

[1]M. Newstein and B. Rudman, *IEEE J. Quantum Electron.*, vol. QE-23, pp. 481–482, May 1987.

### References

[1] J. Arnaud, *Beam and Fiber Optics*. New York: Academic, 1976.