# Circuit theory of multielement laser diodes

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**A general theory of noise and small-signal modulation of multielement laser diodes in the saturated regime is established. Nonlinear elements are connected to the ports of a linear-optical circuit oscillating in a single electromagnetic mode. The laser amplitude and phase fluctuations are expressed simply in terms of the scattering matrix of the linear-optical circuit at any baseband frequency and for an arbitrary electronic feedback. The case of a single laser diode and a single detector is treated to demonstrate this method. The correlation between optical power and electrical voltage fluctuations is shown to disappear at large output powers, in agreement with recent experiments.**

An oscillator whose power or phase is modulated can be used to transmit information, but the transmission may be degraded by random fluctuations. A circuit theory has been proposed[1] that is capable of predicting the laser-diode noise properties on the basis of two simple concepts: (i) The law of particle-rate conservation: the annihilation of one electron–hole pair generates one photon and, conversely, one photon generates one electron–hole at any instant of time, if one assumes for simplicity that the quantum efficiency is unity and that the bias current is large so that carrier storage and recombination into nonoscillating modes can be neglected. (ii) White Gaussian (Nyquist-like) noise currents are associated with negative or positive conductances. These noise currents play an essential role in the above particle-rate conservation law.

These concepts were first applied to the case of nonlinear elements subjected to the same field. The frequency fluctuation was evaluated at zero baseband frequency in the absence of electronic feedback.[1] Similar results have been obtained for ring-type lasers with nonuniform $\alpha$ factors.[2] It was further shown[3,4] that when the current driving an optical amplifier is fed to an optical modulator, the amplitude fluctuations of light initially in the coherent state may be squeezed below the shot noise. For the configurations considered here, the results obtained from the concepts above, which require only the use of complex numbers, are the same results as those obtained from quantum optics even for nonclassical states of light. The theory presented here greatly generalizes the results in Refs. 1–4. Indeed, this theory is applicable to arbitrary optical and electrical configurations at any baseband frequency. Laser and detector diodes are treated equally. The amplitude and phase fluctuations are expressed in a simple manner in terms of the scattering matrix of the optical circuit. While some of the intermediate quantities introduced in this Letter may seem unusual in laser theory, the final expressions, e.g., Eqs. (13) and (14), in which $1/2\pi f_0$ is the so-called photon lifetime, are expressed in standard notation.

Consider an $n$-port linear-optical circuit characterized at an optical frequency $\nu$ by an $n \times n$ matrix impedance $\mathbf{Z}(\nu)$,

$$\mathbf{V} = \mathbf{Z}(\nu)\mathbf{I}, \tag{1}$$

where $\mathbf{V}$ denotes the column voltage and $\mathbf{I}$ denotes the column current. A simple example is shown in Fig. 1. An $\exp(-i2\pi\nu t)$ time dependence is implied.

Nonlinear elements (laser diodes and possibly detectors) with steady-state optical conductances $G_{0k}$ are connected at the ports of the optical circuit. The subscript 0 refers to steady-state values, and the subscripts $k = 1, \ldots n$ refer to the ports. The subscripts are omitted when no confusion may arise. A semiconducting element is submitted to an electrical voltage $U$ that is equal to the energy spacing between quasi-Fermi levels, with the absolute value of the electronic charge set equal to unity. There is a unique relationship between $U$ and the carrier number $N$ and between $N$ and the optical conductance $G$ at some optical frequency $\nu$. The so-called nonlinear gain, which would imply an explicit dependence of $G$ not only on $N$ and $\nu$ but also on the optical voltage $V$, is not considered here. For single-mode operation the dependence of $G$ on $\nu$ may be neglected, and thus $G$ can be considered a known (decreasing) function of $U$: $G = G(U)$. For laser diodes the condition $U > h\nu$ is satisfied, where $h\nu$ is the photon energy and $G < 0$, while for detector diodes $U < h\nu$ and $G > 0$. The diode series resistance, stray capacitance, and lead inductance are parts of the electrical circuit, while the optical cavities belong to the $n$-port optical circuit.

Propagating waves $a$ and $b$ are defined with respect to the characteristic conductances that are equal to the steady-state values $G_0$ of the nonlinear elements, with the understanding that $G_0^{1/2} = i|G_0|^{1/2}$ when $G_0$ is negative:

$$a \equiv \tfrac{1}{2}(G_0^{1/2}V + G_0^{-1/2}I),$$
$$b \equiv \tfrac{1}{2}(G_0^{1/2}V - G_0^{-1/2}I). \tag{2}$$

The $b$ waves represent waves traveling either from a negative conductance to the optical circuit or from the optical circuit to a positive conductance, while the $a$

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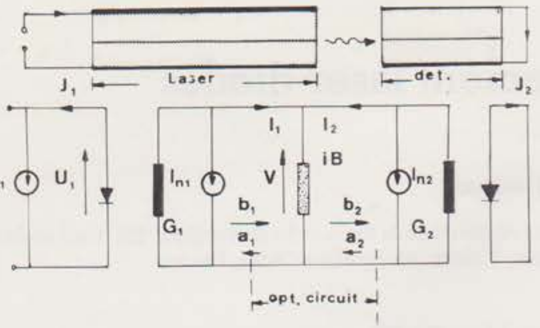


Fig. 1. (Top) Current-driven laser diode whose optical output is detected. (Bottom) Circuit representation of the laser diode (labeled 1) whose output optical power is converted into an electrical current by a detector diode (labeled 2). The laser diode is current driven with electrical current $-J_1$ and noise current $J_{n1}$. $I_1$ and $I_2$ are optical currents, and $V$ is the optical voltage across the circuit. $I_{n1}$ and $I_{n2}$ are Nyquist-like noise currents. $G_1 < 0$ and $G_2 > 0$ represent the optical conductance of the active laser material and the optical conductance of the detector, respectively. The laser-diode cavity, described by a susceptance $B(\nu)$, constitutes the optical circuit. The $b$ waves are forward-propagating waves, while the $a$ waves are weak backward-propagating waves.

waves are weak counterpropagating waves. In the steady state, $I_0 = -G_0V_0$, and thus $a_0 = 0$ and $b_0 = G_0^{1/2}V_0$.

The relationship between the $b$ and $a$ waves follows from Eqs. (1) and (2):

$$\mathbf{b} = \mathbf{S}(\nu)\mathbf{a}, \qquad \mathbf{S}(\nu) = 1 - 2[1 + \mathbf{G}_0^{1/2}\mathbf{Z}(\nu)\mathbf{G}_0^{1/2}]^{-1}, \quad (3)$$

where $\mathbf{1}$ denotes the unity matrix and $\mathbf{G}_0$ is a matrix with diagonal elements $G_{0k}$ [$\mathbf{Z}(\nu)$ was defined above]. $\mathbf{S}(\nu)$ is defined only for $\nu \neq \nu_0$. For a reciprocal optical circuit $\mathbf{S}$ is symmetric. However, because some of the $G_0^{1/2}$ are imaginary, $\mathbf{S}$ is not unitary for lossless-gainless circuits.

Consider now a first-order perturbation $\delta$, e.g., $\delta V \equiv V - V_0$. From Eqs. (2),

$$a \equiv \delta a = {}^1\!/_2(G_0^{1/2}\delta V + G_0^{-1/2}\delta I), \quad (4a)$$

$$\delta b = {}^1\!/_2(G_0^{1/2}\delta V - G_0^{-1/2}\delta I). \quad (4b)$$

If $I_n$ denotes the Nyquist-like noise current associated with $G_0$ (for the sign convention, see the Fig. 1), and $\delta G(1 - i\alpha)$ denotes the admittance change of $G_0$ under some change of carrier density, where $\alpha$ denotes the phase-amplitude coupling factor, it follows from Kirchhoff's law that

$$I + I_n + [G_0 + \delta G(1 - i\alpha)]V = 0, \qquad I_0 + G_0V_0 = 0,$$

$$\delta I + I_n + \delta G(1 - i\alpha)V_0 + G_0\delta V = 0. \quad (5)$$

Thus the amplitude of the counterpropagating wave in Eq. (4a) can be written as

$$-2a = [(1 - i\alpha)\delta G V_0 + I_n]G_0^{-1/2}. \quad (6)$$

Consider next the particle-rate conservation law, in which we set absolute value of the electronic charge equal to unity. At high powers, carrier storage and spontaneous recombination can be neglected, thus

$$J + \mathrm{Re}(V^*I) = 0, \qquad J_0 = G_0|V_0|^2, \quad (7a)$$

$$\delta J = -\mathrm{Re}(\delta V^*I_0 + V_0^*\delta I) = J_0\,\mathrm{Re}(2\delta b/b_0), \quad (7b)$$

in which the expression for $\delta b$ from Eq. (4b) is used.

In summary, the fluctuation $\delta J_k$ of the electrical current in the $k$ laser or detector diodes is related to $\delta b_k$ through Eqs. (7), $\delta b_k$ are related to $a_k$ through the scattering matrix $\mathbf{S}$, and $a_k$ is related to $\delta G_k$ through Eq. (6). Finally, $\delta G_k$ is linearly related to the electrical voltage $\delta U_k$ across the diode since $\delta G/G = \gamma u$ and $\gamma \equiv (U/G)(dG/dU)$, where $u \equiv \delta U/U$. On the other hand, the electrical circuit that connects the laser diodes and detectors imposes a relationship between the electrical voltages $\delta U_k$ and the electrical currents $\delta J_k$. Self-consistency solves the general problem, at least formally.

Deterministic or random real functions of time, such as $\delta G(t)$ or the real part of the noise current $I_n(t)$, can be expressed as a Fourier series with a Fourier (or baseband) frequency $f$ over some time interval $T$. Sinusoidal time variations at frequency $f$ entail variations of the optical voltages and currents at frequencies $\nu_0 + f$ and $\nu_0 - f$. A conventional treatment is possible but cumbersome. It is more convenient to denote slow time variations by $\exp(j2\pi ft)$, as is typically done in electrical engineering, through the use of a bicomplex $i, j$ notation, with the understanding that $i^2 = j^2 = -1$, that $ij$ cannot be simplified, and that $\mathrm{Re}(a + jb + ic + ijd) = a + jb$ for any real numbers $a, b, c, d$.

Combining Eqs. (3), (6), and (7b), with the subscripts 0 omitted, the general solution is then concisely written

$$\delta J = J\,\mathrm{Re}(2\delta b/b), \quad (8a)$$

$$2\delta\mathbf{b} = \mathbf{S}(\nu_0 + ijf)2\mathbf{a}, \quad (8b)$$

$$-2a = (1 - i\alpha)(\delta G/G)b + c + is,$$

$$c + is \equiv I_nG^{-1/2}, \quad (8c)$$

$$\delta G/G = \gamma u, \qquad \gamma \equiv (U/G)(dG/dU)_{U=U_0},$$

$$u \equiv \delta U/U, \quad (8d)$$

$$\delta\mathbf{U} = \mathbf{Z}_e(\delta\mathbf{J} - \mathbf{J}_n). \quad (8e)$$

Equations (8b) and (8e) are written in matrix form, while Eqs. (8a), (8c), and (8d) are applicable to each port. Equation (8e), in which $J_n$ are electrical noise currents, expresses the linear relationship between electrical voltages and currents imposed by the electrical power supplies, which may be interconnected. The above linear inhomogeneous system of equations, Eqs. (8), gives the fluctuations $\delta J_k$ of the optical powers in terms of independent white Gaussian sources whose spectral densities are given below.

In order to obtain the *phase* fluctuation $\delta\Phi$ it suffices to write

$$i\delta\Phi_c \equiv i\delta\Phi + {}^1\!/_2\delta J/J = \delta b/b. \quad (8f)$$

The real part of Eq. (8f) coincides with Eq. (8a).

Note that the optoelectrical relation between $\delta J$ and $\delta U$ that follows from Eqs. (8a)–(8d) should be modeled in general by a nonreciprocal electrical circuit, even if the optical circuit itself is reciprocal. The nonreciprocity stems from the fact that signals are better transmitted downstream than upstream with respect to the mean photon flux.

Two kinds of Gaussian random processes appear in Eqs. (8): the normalized Nyquist-like complex current $c + is$ and the electrical noise current $J_n$. If the optical circuit is lossless-gainless and if there are no electrons in the conduction band (detectors with $G_0 > 0$) or no electrons in the valence band (ideal laser diodes with $G_0 < 0$) in the relevant range of energy, then the (double-sided) spectral densities of the $c(t)$ and $s(t)$ processes are, setting $h\nu_0 = 1$ for brevity,

$$S_{cc} = S_{ss} = 1, \qquad S_{cs} = 0. \quad (9)$$

Root-mean-square quantities are implied. Equation (9) expresses the fluctuation–dissipation theorem for quantum noise. According to the theory of Haus,[5] when the optical circuit contains *linear* dissipative or active elements at $T = 0$ K, terms proportional to the Hermitian part of $Z$ should be included in Eqs. (9).

If the electrical noise current $J_n$ is simply the shot noise associated with the average current $J$, it is uncorrelated with the $c$ and $s$ processes, and the spectral density of $j_n \equiv |J|^{-1/2} J_n$ is

$$S_{j_n} = 1. \quad (10)$$

We now apply the general theory to the configuration represented in Fig. 1, which consists of a laser diode whose output optical power is detected without any optical loss: $-J_1 = J_2 \equiv J > 0$ and $-G_1 = G_2 \equiv G > 0$. In spite of its simplicity, this configuration has apparently never been fully solved. The laser cavity is modeled by a parallel inductance–capacitance ($LC$) circuit resonating at $\nu_0$, whose susceptance $B(\nu)$ is to first order

$$B(\nu_0 + \delta\nu) = -4\pi C\delta\nu. \quad (11)$$

The $\mathbf{Z}$ matrix is easily obtained from Ohm's law, $I_1 + I_2 = iBV$, where $V$, taken as real, denotes the optical voltage across the circuit, and the expression for $\mathbf{S}$ follows from Eqs. (3):

$$\mathbf{S}(\nu_0 + \delta\nu) = i(f_0/\delta\nu)\begin{bmatrix} -1 & i \\ i & 1 \end{bmatrix} - 1, \qquad 2\pi f_0 \equiv G/C. \quad (12)$$

From the general result in Eqs. (8), the normalized fluctuations $j_k \equiv |J_k|^{-1/2}\delta J_k$, $k = 1, 2$, of the laser and detector electrical currents are

$$j_1 = -(1 + z)(u' + s_1) + zc_2, \quad (13a)$$

$$j_2 = z(u' + s_1) + (1 - z)c_2, \quad (13b)$$

$$z \equiv -jf_0/f, \qquad u' \equiv \gamma_1 u_1 J^{1/2}, \quad (13c)$$

where we have assumed that the detector conductance does not depend on the light level $\gamma_2 \approx 0$; $\gamma_1 \equiv \gamma$ and $u_1 \equiv u$ have been defined in Eq. (8d).

If the electrical source driving the laser has infinite impedance at frequency $f$, $j_1$ is independent of the fluctuations of the electrical voltage across the diode: $j_1 = j_{n1}$. The normalized voltage fluctuation $u'$ across the diode can therefore be expressed in terms of $j_{n1}$, $s_1$, and $c_2$ from Eq. (13a). The normalized detected optical power fluctuation $j_2$ is similarly expressed when $u'$ is eliminated from Eqs. (13a) and (13b). If we further assume that the electrical source driving the laser diode generates shot noise and that stimulated absorption in the laser material can be neglected, the spectral densities of the uncorrelated processes $j_{n1}$, $s_1$, and $c_2$ are unity, according to Eqs. (9) and (10).

Recalling that $z$ in Eqs. (13) is imaginary with respect to $j$, the spectral densities of the normalized optical power fluctuation $j_2$, and relative voltage fluctuation $u \equiv \delta U_1/U_1$ across the laser diode are found to be

$$S_{j_2j_2} = 1, \qquad S_{uu} = 2/J\gamma^2, \qquad S_{j_2u} = 0, \quad (14)$$

at *any* baseband frequency $f$. The first relation in Eqs. (14) expresses the fact that the output photon rate fluctuations are the same as the assumed injected electron–hole pair fluctuations. This result was previously obtained from quantum mechanics by Yamamoto *et al.* [see the curve labeled "ideal laser" in Fig. 4a of Ref. 6].

The other two relations in Eqs. (14) seem to be new. The last one shows that the correlation coefficient between the laser electrical voltage fluctuations and the detected current fluctuations vanishes in the limit of large injected currents.

Our theoretical result agrees with measurement: a correlation coefficient smaller than 0.1 has been measured[7] for an index-guided laser diode. The bias current was only 1.3 times the threshold current, however, and a more complete expression than the one given in Eqs. (14) is perhaps needed for a precise comparison.

At moderate power levels one must take into account carrier storage and spontaneous recombination. This amounts to adding to the left-hand side of Eq. (7b).

$$\delta(dN/dt + N/\tau_s) = (j2\pi f + 1/\tau_s)N_U\delta U,$$

$$N_U \equiv (dN/dU)_{U=U_0}, \quad (15)$$

where $N$ is the carrier number in the diode considered, $\tau_s$ is the spontaneous lifetime, and $N_U$ is a parameter characteristic of the active material. The spectral density of $J_n$ is now proportional to $|J| + N/\tau_s$, since spontaneous emission and carrier injection are independent. Near the relaxation frequency the intensity noise is much larger than shot noise, and our theory then agrees with conventional rate equations.[8]

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