CLASSICAL THEORY OF SQUEEZED STATES OF LIGHT

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"From a modern view-point, nature possesses two levels: At the lower level is the electromagnetic field. At the upper level are material bodies, energy and forces. Only the upper level is accessible to observation" Freeman Dyson: From Eros to Gaia, Pantheon Books 1992 (Chap. 9: Comprehend Maxwell).

The classical theory of laser noise that we present is well in line with the above quotation. The basic concept is that laser noise is caused by atomic jumps between lower and upper levels, and that atoms subjected to classically-prescribed optical fields are independent of one-another. Electron jumps from one level to another are considered measurable, while the electromagnetic field is a complex quantity (analytic signal) that cannot be directly measured. Optical waves may anticipate emitting-atom fluctuations. Because this theory unlike previous semiclassical theories enforces energy conservation "non-classical" states of light (squeezed states) are accurately described. From our view-point photocurrents exactly reflect light fluctuations. No "detector shot-noise" should be considered. The phasor theory that attributes noise to the field spontaneously emitted in the oscillating mode by excited-state atoms, in contradistinction, cannot be understood consistently in semiclassical terms and is valid only for simple laser-oscillator models [1].

Note that the expression "photon number" (m) is employed in this paper as an abreviation for "electromagnetic energy divided by $\hbar\omega$" where $\hbar$ denotes the Planck constant divided by 2π, and ω the optical angular frequency, only narrow-band processes being considered. Similarly, "photon rate" refers to electromagnetic power divided by $\hbar\omega$. The photon rate Q absorbed by the detector and the photon rate R from excited-state atoms are complex quantities (active and reactive powers). (Real and imaginary parts of complex quantities will be denoted by primes and double primes, respectively). Only spectral densities are considered. General probability laws presumably require quantization of the optical field (positive P-distribution).

Our classical theory shows that high-power laser-oscillators with nonfluctuating pumps generate amplitude-squeezed states of light. We will show in detail how previous quantum-theory results [2] are recovered. Linewidth formulas applicable to lumped circuits are also discussed. For one-dimensional configurations the linewidth is inversely proportional to the modulus square of a complex round-trip transit time τ. The significance of this complex transit time can be discussed on the basis of variational formulas derived either from the Tellegen theorem or Maxwell equations.

Accidental fluctuations (due, e.g., to atmospheric propagation), 1/f noise and slow thermal fluctuations are not considered. A single-mode oscillation is considered.

We denote by J the pump rate (number of atoms in the emitting state or electrons injected per unit time). Below threshold ($J<J_{th}$, where J_{th} denotes the threshold value) the laser output consists of linearly amplified spontaneous emission. It is similar to frequency-filtered thermal light. The light distribution in the Re(E)-Im(E) plane (where E denotes the optical field) is gaussian and thus light intensity is exponentially distributed. The photo-electron statistics is superpoissonian (bunching).

When pumping is strong enough ($J>J_{th}$) the gain equals the loss and a classical stable oscillation can be sustained, which is only weakly perturbed by noise sources. The phase diffuses in the course of time with a coefficient essentially equal to the laser linewidth. If the pump fluctuations are at the shot-noise level (independent atom-injection times) the laser output may resemble a coherent state, with Poisson-distributed photons. If the pump does not fluctuate the laser output may be amplitude squeezed and exhibit subpoissonian photon statistics.

Nonfluctuating pumps can be realized in a number of ways: a) A fixed number of atoms may be excited by a pulse of light of appropriate duration and intensity. The subsequent atomic decay will, ideally, always give the same number of photons. b) It is possible to fabricate electron turnstiles that deliver exactly one electron every microsecond (for example). This regular electron flow can be employed as a quiet pump. c) Yamamoto made in 1986 the observation that the current generated by a battery and a (preferably cold) resistor is essentially nonfluctuating [2]. At room-temperature with a battery voltage of 10 volts, the spectral density of the electrical current fluctuations normalized to the shot-noise level is only 0.0025 according to the Nyquist formula. Usually the dynamic impedance of laser diodes of the order of 1Ω is negligible with respect to the external resistance, of the order of 1 kΩ. It is therefore in principle very easy to pump electrically a semiconductor laser noiselessly. Internal noise sources, however, remain.

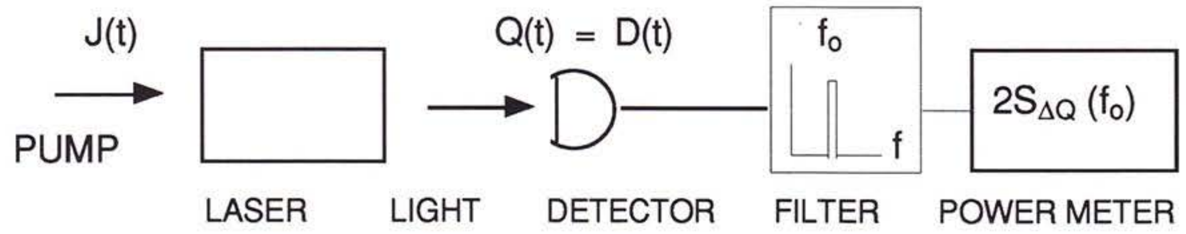

Figure 1: measurement of intensity-noise (double-sided) spectral density. J denotes the pump rate, Q the photon rate emitted by the laser and D=Q the photo-electron rate. The filter centered at frequency f_0 has a 1 Hz bandwidth.

Figure 1 shows a typical arrangement for measuring the amplitude noise spectral density. The laser output is entirely collected by a detector that we assume ideal (every incident photon being converted into an electron). The photo-electron rate D(t) is analyzed with a spectrum analyser, schematically represented by a 1-Hz bandwidth filter centered at a frequency f_0 that can be scanned. After time-integration, twice the (double-sided) amplitude-noise spectral density is obtained as a function of baseband frequency f_0 (abbreviated to f in the following). From a mathematical standpoint, the spectral density is best defined as the Fourier transform of $\langle\Delta D(t)\Delta D(0)\rangle$, where the signs $\langle . \rangle$ denote an ensemble average, and $\Delta D \equiv D-\langle D\rangle$ is a real, zero-mean, stationary, ergodic process.

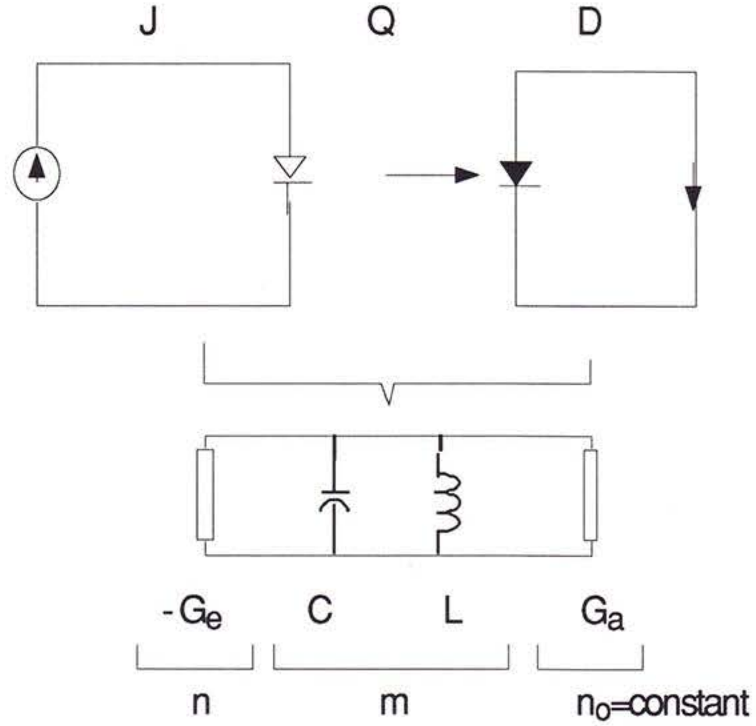

Figure 2: Squeezed-state generation and observation. The laser oscillator (white triangle) is driven by a pump rate J. It generates a photon rate Q, converted into a photocurrent D=Q by the detector (black triangle). The laser oscillator comprises a negative conductance $-G_e$ and an L-C resonator. n denotes the number of emitting atoms (or the number of electrons in the conduction band for semiconductors) and m the photon number. The detector conductance G_a is equal to the steady-state value of G_e.

Figure 2 represents the laser diode driven at pump rate J. The output photon rate Q equals the detected rate D. The laser oscillator is split into a tuned L-C circuit containing m photons, and a negative conductance $-G_e$ containing n atoms (or electrons). It is essential to consider also the absorber of radiation, i.e., the detector in the present configuration. The separation between the laser and the detector turns out to be irrelevant. If part of the laser radiation goes into free-space, light is supposed to be absorbed at infinity. In any event the aborber is cold and frequency-insensitive. The number n_o of atoms in the absorber is mentioned in the figure only for the sake of completeness. It is considered constant and needs not be considered in the following.

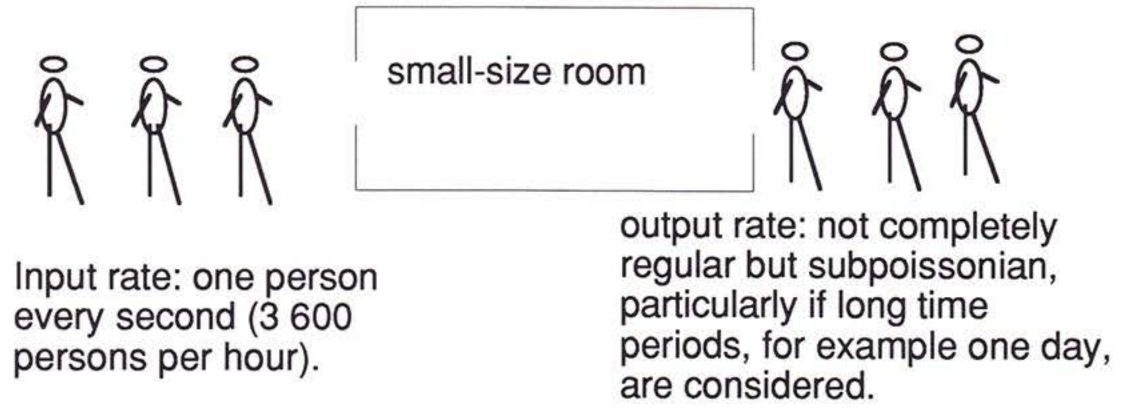

Figure 3: This figure illustrates the fact that regular pumps entail subpoissonian photocurrents. Note that long measurement time-intervals correspond to baseband frequencies f small compared with the so-called "cold-cavity linewidth". This conclusion holds irrespectively of the detailed internal mechanisms (e.g., spectral-hole burning) as long as negligible internal loss occurs.

Before going into detailed calculations, let us explain with a picturesque model why nonfluctuating pumps entail nonfluctuating photoelectrons. Figure 3 shows people entering, one every second, in a small-sized room. An hour later exactly 3 600 people have entered. Almost exactly 3 600 people also have left the room because not many people can stay in the room. We can be pretty sure that this number deviates from the mean by much less than the square-root of 3 600, namely 60. The output rate thus is "subpoissonian". This would be even more so if a longer measuring time such as one day were considered. Going back to the physical situation we can assert that if the laser pump rate is nonfluctuating the spectral density of the outgoing electrons vanishes at low frequency (corresponding to long measuring times). This simple picture applies only if no electron is lost through the process of spontaneous atomic or carrier recombination, and no photon is lost by dissipation or scattering. Under these ideal conditions, incomplete population inversion or gain compression are irrelevant. The spectral density increases as a function of baseband frequency f and reaches the shot-noise level at very large frequencies.

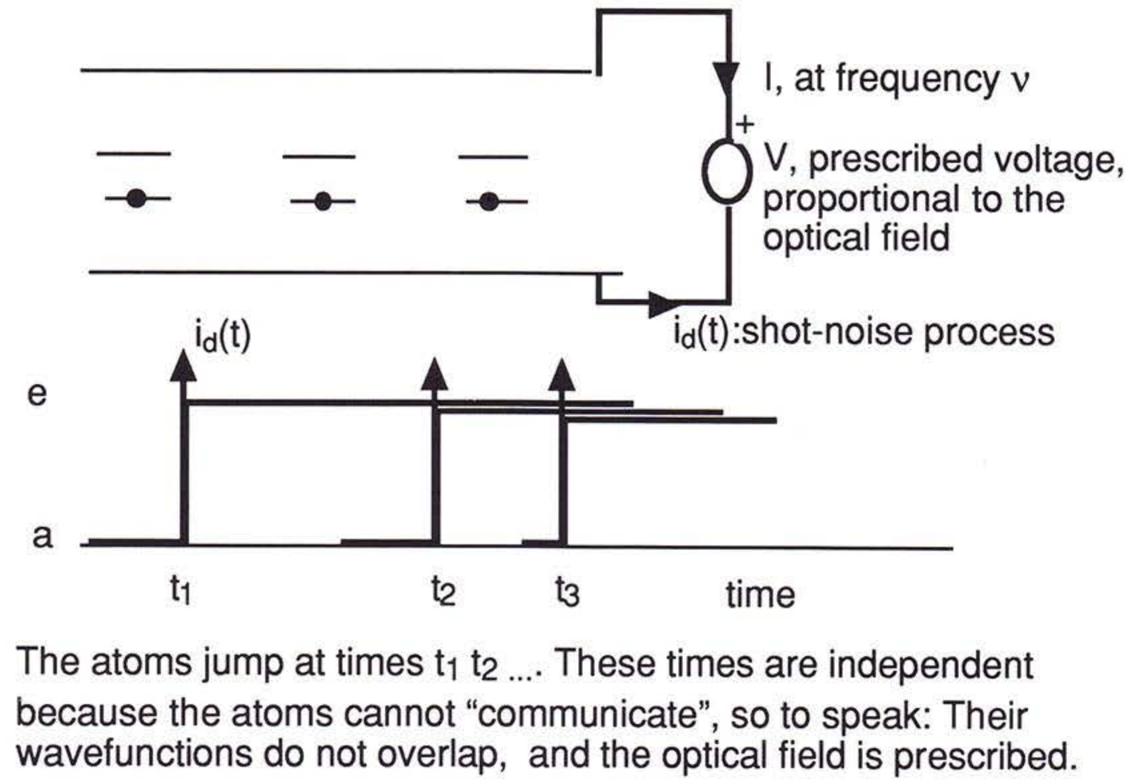

Figure 4: A number of atoms initially in the absorbing state are submitted to the same prescribed optical field, proportional to the voltage V. They undergo quantum jumps at independent times. As a consequence, the induced current I_d consists of a sequence of δ-functions at Poisson-distributed times. The same principle applies to atoms in the emitting state, and to detuned atoms.

The fundamental noise sources can be derived from a simple intuitive principle: atoms submitted to a classical prescribed optical field are independent. This is so because the atoms cannot communicate with one another (so-to-speak) neither directly because the wave functions do not overlap, nor through induced field fluctuations. When the number of atoms is large this implies that the quantum jumps are Poisson distributed, and for the one-photon processes considered the intrinsic rate fluctuation q(t) is at the shot-noise level, that is $S_q=Q$, where Q denotes the average absorbed rate, the signs <> being omitted when no confusion may arise. The same principle applies to emitters. However, for high-power oscillators it suffices to consider the absorber. We have

$$Q = \frac{m}{\tau_p} + q, \qquad S_q = Q \tag{1}$$

where the first term is the deterministic contribution. In terms of the circuit element shown in Fig. 2 we have $m=CV^2$, V being the normalized voltage across the circuit, and $\tau_p \equiv C/G$, $G=G_a \approx G_e$. At high power the number of atoms n is negligible because they quickly decay as a result of stimulated emission. Therefore, the atom number rate equation simplifies to

$$\frac{dn}{dt} = J(t) - R(t) \approx 0 \Rightarrow \Delta R = \Delta J = 0 \tag{2}$$

if the pump is nonfluctuating.

Accordingly, the rate equation for the photon number m simplifies

$$\frac{dm}{dt} = R(t) - Q(t) = \langle R \rangle - Q(t) \tag{3}$$

Considering small deviations from the steady-state denoted by Δ, and going to the baseband angular frequency $\Omega \equiv 2\pi f$ [with an $\exp(j\Omega t)$ time-dependence implied], Eqs.(3) and (1) reads:

$$\Delta Q = -j\Omega\, \Delta m = j\Omega\tau_p\, (q - \Delta Q), \qquad S_q = Q \tag{4}$$

This equation shows that ΔQ is proportional to the intrinsic fluctuation q. Accordingly, the spectral density of ΔQ reads

$$X \equiv \frac{S_{\Delta Q}}{Q} = \frac{F}{1+F}, \qquad F \equiv (\Omega\tau_p)^2 \tag{5}$$

This result verifies that X vanishes at zero baseband frequency and reaches the shot-noise level (X=1) at high frequencies.

At moderate power the atom number n cannot be neglected in comparison to the photon number m. The first expressions in Eqs.(1)-(3) remain applicable. We now need the detailed expression of the photon rate R from emitting atoms

$$R = A\, n\, m + r, \qquad S_r = R = Q \tag{6}$$

where A is a constant. The intrinsic fluctuation r is independent of q but has the same statistics. The deterministic term Anm is (as was the case for Q) proportional to the photon number m, but it also depends on the atom number n. For simplicity R is assumed to be proportional to n. The normalized amplitude-noise spectral density is shown as the upper curve in Fig.5, for n=m. At lower powers (n>>m) the relaxation peak is more pronounced.

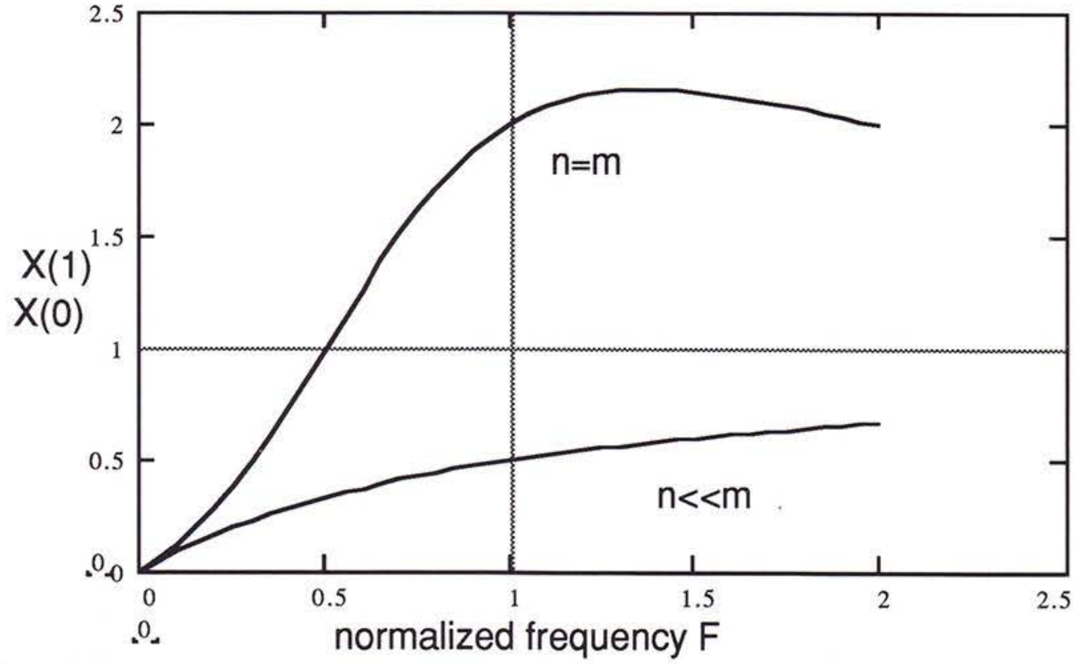

Figure 5: Spectral density of intensity noise normalized to the shot-noise level, $X \equiv S_{\Delta Q}/Q$, as a function of the baseband frequency normalized by the cold-cavity linewidth, $F \equiv (\Omega\tau_p)^2$. The parameter a=n/m=0 at very high power when n<<m. In that case one needs only the expression of the absorbed rate Q. The result at moderate powers (a=1 or n=m) exhibits a relaxation.

From our viewpoint the origin of noise is stimulated rather than spontaneous emission. We have assumed above that the laser oscillator operates much above threshold, so that the rate S of spontaneous decay is negligible in comparison with the stimulated decay rate: S<<R=Q. Just above threshold (S>Q) spontaneous decay adds noise. But whether this decay is radiative and part of it goes into the oscillating mode or not is not relevant.

We have so-far considered only amplitude noise. To treat phase noise one should consider that $Q \equiv Q'+iQ''$ is complex, as discussed earlier (Q' representing active power and Q'' reactive power). Here, "i" is the symbol for imaginary numbers with an $\exp(-i\omega t)$ time dependence implied. In the general situation, a bicomplex (i,j) notation is convenient. It consists in the replacement of the (Laplace transform) $p \equiv -i\omega$ in the expression of a causal response $Y(p)$ by p_1+p_2 where $p_1=-i\omega$ and $p_2=j\Omega$. $Y(p_1+p_2)$ can be expressed as a sum of four terms proportional respectively to 1, p_1, p_2 and $p_1 p_2$. This bicomplex notation is not employed here.

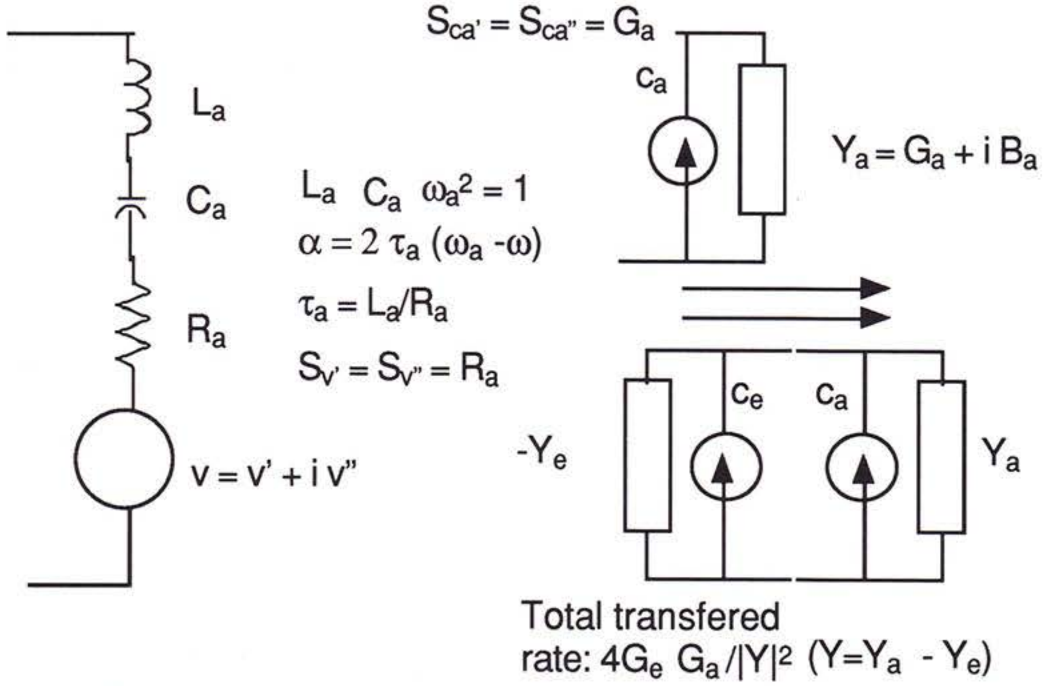

Figure 6: Equivallent circuit for absorbing atoms (series representation on the left, parallel representation on the right). The condition that atoms submitted to prescribed classical fields are independent requires that the complex noise sources (voltage, current or rate) have the spectral densities shown on the figure. The schematic on the lower right explains the principle of spontaneous emission.

The intrinsic noise $q \equiv q'+iq''$ is also complex with q' and q'' uncorrelated and having both spectral densities equal to Q'. This follows from the intuitive concept explained earlier, namely that (possibly detuned) atoms submitted to prescribed optical fields must be independent. Two alternative representations are given in Fig.(6). On the left, a series circuit model is shown with a white voltage noise source $v(t)$. In the following we use the parallel circuit representation on the right of the figure, with a complex current source $c(t)$.

If we consider absorbing atoms in free-space (i.e. surrounded by absorbers), the rates radiated by noise sources relating to the absorbing atoms and by noise sources relating to surrounding atoms cancel out so that no spontaneous absorption occurs, as one expects. But for emitting atoms the two contributions add up (see Fig.6). Our formalism thus predicts the phenomenon of spontaneous emission.

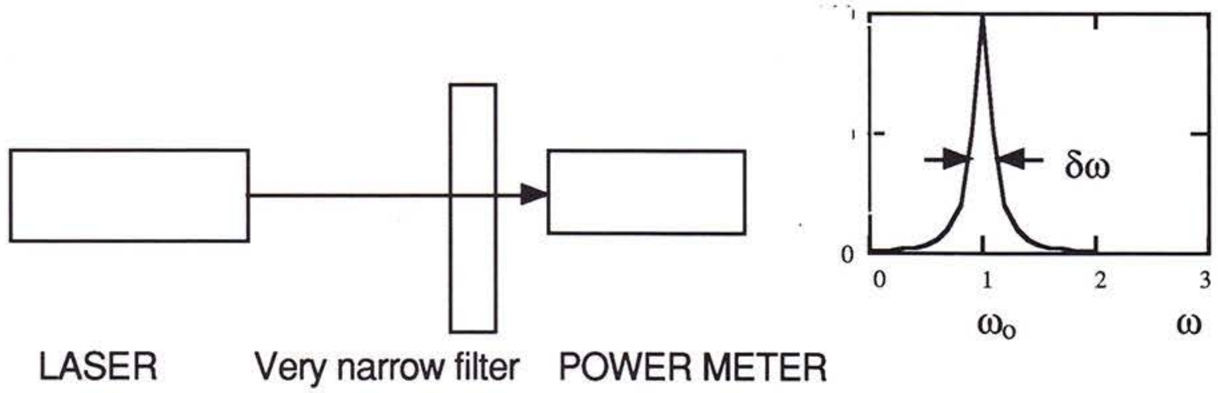

Figure 7: Direct measurement of the laser linewidth with the help of a scanned narrow optical filter. The laser linewidth $\delta\omega$ is full width at half power.

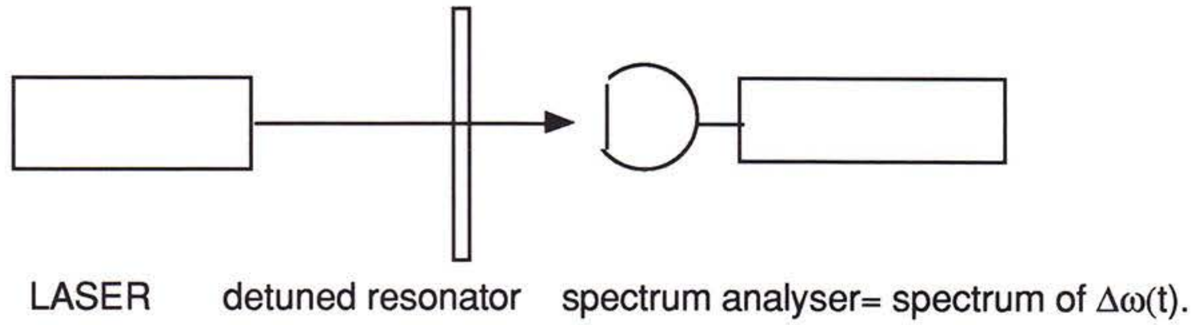

Figure 8: Alternative method of measuring the laser linewidth, applicable above threshold when the amplitude fluctuations contribute negligibly. The linewidth $\delta\omega$ is the spectral density of the instantaneous frequency deviation $\Delta\omega(t)$ in the small baseband-frequency limit. The later is measured with the help of a detuned optical filter acting as a frequency-to-amplitude convertor.

Fig.7 exhibits a direct method of linewidth measurement. Figure 8 is applicable above threshold, when the amplitude fluctuations contribute negligibly to linewidth. One first measures the spectral density of the instantaneous optical frequency fluctuation $\Delta\omega(t)$. For gaussian processes the linewidth $\delta\omega$ is the low-frequency limit of that spectral density.

It is straighforward to recover from the principles just explained detuned-atom linewidth formulas first derived by Lax from quantum theory in 1966 [3]

$$Q\,\delta\omega = \frac{1+\alpha^2}{(\tau_e+\tau_p)^2+(\tau_e-\tau_p)^2\,\alpha^2} \qquad \text{(below threshold)}$$

$$Q\,\delta\omega = \frac{1}{2}\,\frac{1+\alpha^2}{(\tau_e+\tau_p)^2} \qquad \text{(above threshold)}$$

where the detuning (or phase-amplitude coupling factor) $\alpha \equiv 2\tau_e(\omega_e-\omega_o)$, where ω_o is the actual oscillation frequency. The polarization relaxation time τ_e is defined as τ_a in Fig.6. The "photon lifetime" τ_p is equal to C/G_a.

The above-threshold linewidth formula can be generalized to the case of any number of atomic species. For a cold nondispersive absorber, complete population inversion of the emitting elements, and poissonian pumps we obtain

$$Q\,\delta\omega = \frac{1}{2}\,\frac{1}{\tau_p^2}\,(1+\alpha^2)_{av}, \qquad a_{av} \equiv \frac{\Sigma_k\, J_k\, a_k}{\Sigma_k\, J_k}$$

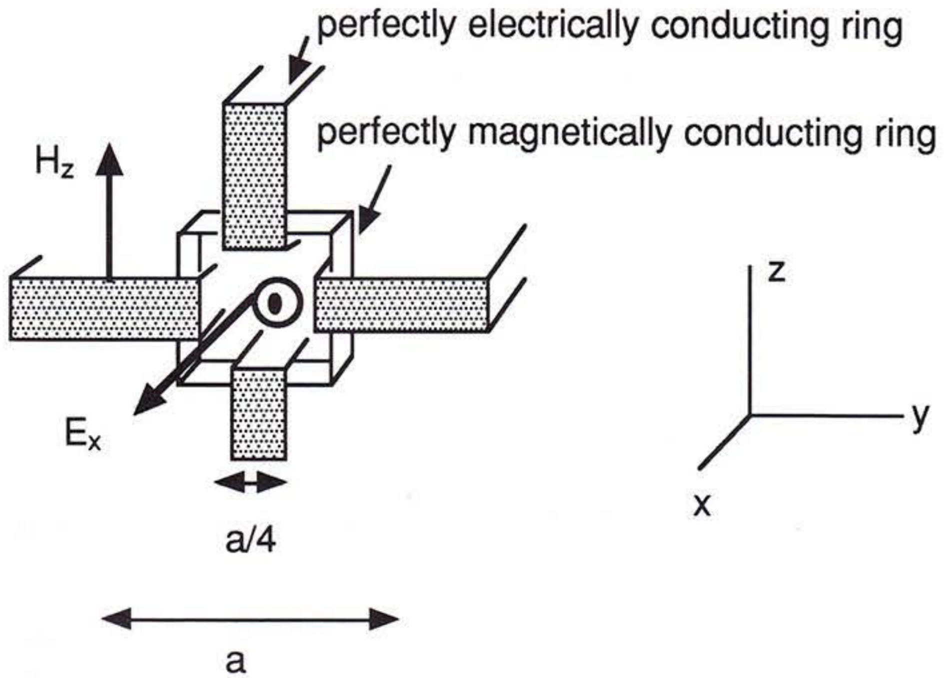

Figure 9: Circuit representation of free space. Each perfectly conducting magnetic (electric) ring is intertwined to four perfectly conducting electric (magnetic) rings. It can be seen by inspection that the Maxwell equations are obeyed. An atom (with electric-dipole allowed transition) is shown in the middle of the magnetic loop generating the E_x field.

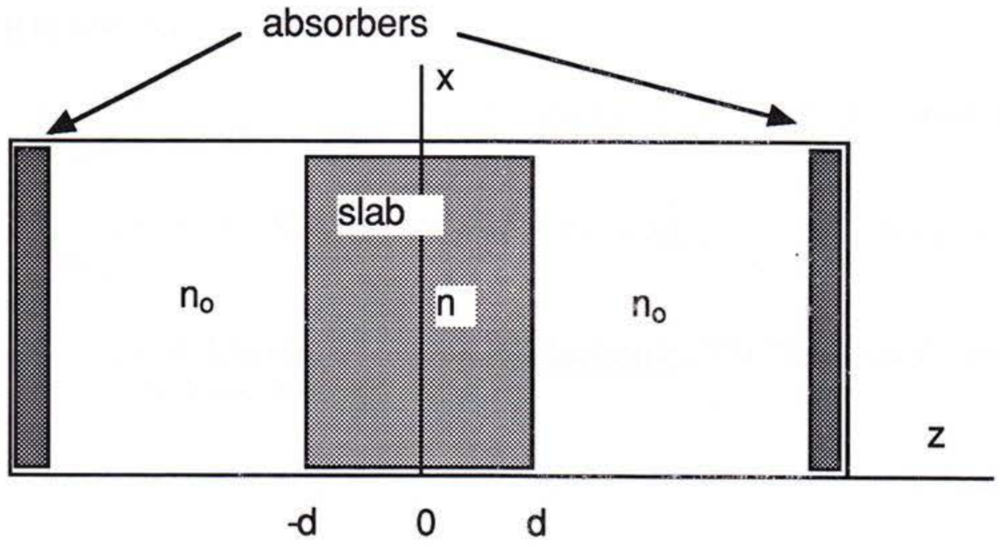

Figure 10: Dielectric slab of refractive index n (whose imaginary part expresses gain) and thickness 2d, immersed in a medium of refractive index n_o. Perfect absorbers are supposed to be located at a large distance from the slab. Propagation may be wiewed in two ways. a)with $k_x=0$, the figure represents a Fabry-Pérot resonator with the optical power leaking along the z direction at the index discontinuities, $z=\pm d$. b) with large k_x values, propagation essentially occurs along the x-axis, and the wave decays exponentially in the outer medium. For small d-values (weak guidance) the laser linewidth is enhanced by a Petermann K-like factor.

The most general situation is that of an arbitrary circuit involving emitters and absorbers. Indeed a continuous medium admits a circuit representation as shown in Fig.9 for free space. An important configuration is the dielectric slab shown in Fig.10.

Below threshold, a general formula for bianisotropic media has been given that involves only the resonating field [1]. For example, for a ring-type resonator involving in sequence a gain G_1, a reflection $R_1 \equiv 1/L_1$, a gain G_2 and a reflection $R_2 \equiv 1/L_2$, the result reads, with τ the complex round-trip time

$$|\tau|^2\,Q\,\delta\omega = (1+R_1R_2)(G_1+G_2+L_1+L_2) - R_1R_2(G_1+G_2)(L_1+L_2) - 4$$

Above threshold only special configurations have been treated analytically. Numerical methods are in general required.

We conclude by saying that the popular "phasor" method that attributes noise to the power spontaneously emitted in the oscillating mode is accurate only for the simplest laser oscillator models. Unlike the classical theory that I presented here, the phasor model is unable to recover, e.g. detuned-atom linewidth formulas derived from quantum theory.

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