2.3 *Circuits for Traveling Wave Crossed-Field Tubes*

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In this section we consider typical types of delay lines that have been used or considered for use in Injection Type tubes. Actually, most of the problems discussed are relevant to other types of tubes like the amplitron and, in general, to all tubes using a traveling wave rather than a standing wave, as in the magnetron. It is more difficult to obtain high rf voltages with a traveling wave than with a standing wave, and one of the more cogent necessities is the search for high impedance structures. On the other hand the high impedance circuits used in O-type tubes cannot easily be adapted to M-type tubes because they are frail and cannot sustain the heavy electron bombardment of the circuit characteristic of M-type tubes.

I. The Interdigital Line

A. General Relations

The interdigital line consists of two identical "combs" the "fingers" of which are interlaced as shown in Fig. 1(a). From the point of view of the

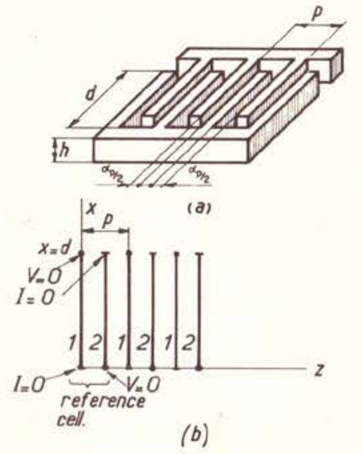

Fig. 1. Interdigital line. (a) In its practical shape the interdigital line is constituted by two combs; (b) theoretically, it is a set of parallel bars alternatively short-circuited to ground and open circuited.

bar line theory, it is a set of parallel bars of length d connected to the ground alternatively at one end or the other (Fig. 1(b)).

We have the boundary conditions

$$V_1(d) = V_2(0) = I_2(d) = I_1(0) = 0 \tag{1}$$

Using the relation (3) of the previous Section 2.2 by this author, leads to the relation

$$\begin{vmatrix} \cos kd & j \sin kd Z_{21} \\ j \sin kd Y_{12} & \cos kd \end{vmatrix} = 0 \tag{2}$$

or

$$\cos^2 kd = \frac{Y_{12}Y_{21}}{Y_{11}Y_{22}} \tag{3}$$

In the case where the combs are symmetrical one can show (1) that

$$\tan^2 \frac{kd}{2} = \frac{Y(\varphi/2)}{Y[(\varphi/2) + \pi]} \tag{4}$$

if Y is the characteristic admittance of the system considered as structure with one bar per cell. In the case where all the capacities may be neglected, except between adjacent bars (γ'), we have

$$Y(\varphi) = 4c\gamma' \sin^2 \frac{\varphi}{2} \tag{5}$$

and

$$\tan^2 \frac{kd}{2} = \tan^2 \frac{\varphi}{4} \tag{6}$$

The first solution is

$$\varphi = 2kd \tag{7}$$

This phase shift corresponds to the picture of a TEM wave flowing in zigzag fashion between the combs at light velocity.

From relation (21) of Section 2.1 by this author, the delay factor of the mth space harmonic will be

$$\frac{c}{v_{\mathrm{ph}}} = \frac{2d}{p} + \frac{m\lambda}{p} \tag{8}$$

remembering that p is the distance between adjacent fingers of the same comb, and d the length of the fingers.

B. Characteristics of Various Space Harmonics

In this paragraph we shall study the harmonics corresponding to $m = 0, m = -1, m = +1$. Their dispersion curves are shown in Fig. 2. The more important of them is for $m = -1$, which corresponds to a backward wave and is used in the crossed-field backward wave oscillator. It is seen

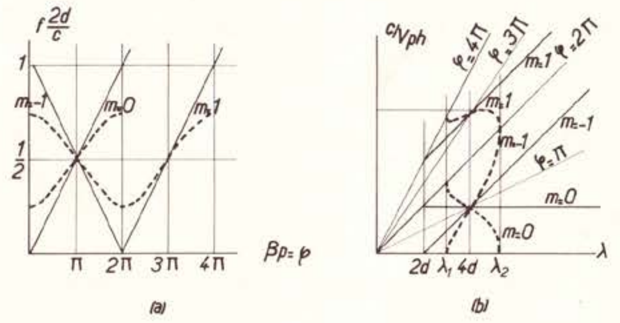

Fig. 2. Dispersion curve of an interdigital line. (a) Diagram frequency as a function of the phaseshift; (b) diagram: delay factor as a function of the wavelength. $m = 0$, forward antisymmetric wave; $m = -1$, backward symmetric wave (carcinotron); $m = 1$, forward symmetric wave. Without grounded plate the passband is from $\lambda = 2d$ to $\lambda = \infty$ (straight line curves); with a grounded plate near the fingers the passband is from λ_1 to λ_2 (dotted line curves).

from Eq. (8) that the pass band is defined by $2d \leqslant \lambda \leqslant \infty$; in fact, the useful bandwidth is only $4d < \lambda < 8d$.

The harmonic $m = 0$ is a forward mode and the corresponding delay factor is nearly constant. It could be used in forward amplifiers, but unfortunately it will be seen that its coupling impedance is zero at the middle of the interdigital line. Since the $m = 0$ and the $m = -1$ harmonics have the same range of phase velocities, such amplifiers would be very apt to oscillate on the backward waves. However, some modifications can be made which permit their use in an amplifier: essentially, the two combs are made strongly dissimilar so that the backward wave disappears in the regions where it could be harmful.

The $m = +1$ harmonic is also a forward wave, but the associated field has the same distribution along the fingers as in the case for $m = -1$. The dispersion is not zero but can be made small if we use a ground plate near the line (see Fig. 2(b), dotted line). The main trouble in its use in an amplifier structure is the rather low value of its coupling impedance and the great variation of it in the pass band. However, it has been used in the early crossed-field amplifier.

C. Coupling Impedance

The simple results which follow relate to an interdigital line with fingers of rectangular cross section and a field uniform in the gaps.

If U is the potential between adjacent bars, one obtains easily

$$E_m| = \frac{2U}{p} \frac{\sin[(\alpha/2)(\varphi + 2m\pi)/2]}{(\alpha/2)(\varphi + 2m\pi)/2} \times \begin{cases} \cos kx, & m \text{ odd} \\ \sin kx, & m \text{ even} \end{cases} \tag{9}$$

(The origin of the x axis is assumed here to be the middle of the line.) On the other hand, the interdigital line can be considered as a bifilar line of characteristic admittance $c\gamma'$. Then the power flowing is

$$P = \tfrac{1}{2} c\gamma' U^2 \tag{10}$$

and the coupling impedance of the mth harmonic is

$$\mathcal{R}_m = \frac{2}{\gamma'/\epsilon_0} \frac{\varphi}{(\varphi + 2m\pi)^2} \left\{ \frac{\sin[(\alpha/2)(\varphi + 2m\pi)/2]}{(\alpha/2)(\varphi + 2m\pi)/2} \right\}^2 \begin{pmatrix} \cos kx \\ \text{or} \\ \sin kx \end{pmatrix}^2 \tag{11}$$

So, for $m = -1$ (backward wave space harmonic), and for $\varphi = \pi(\lambda = 4d)$, $\alpha = \frac{1}{2}$, $x = 0$, we have

$$\mathcal{R}_{-1} = \frac{0.6}{\gamma'/\epsilon_0} \tag{12}$$

and its mean value over the width d of the structure is

$$\mathcal{R}'_{-1} = \frac{0.54}{\gamma'/\epsilon_0} \tag{13}$$

Now, we also have

$$\frac{1}{\gamma'/\epsilon_0} = \frac{p}{4h} \tag{14}$$

if we neglect the fringing fields. For $h = 2p$ we have

$$\mathcal{R}'_{-1} = 0.067 \tag{15}$$

D. Special Cases

In some cases, the picture of the wave traveling with a velocity c between the combs is not sufficiently accurate, and it is necessary to return to the more general expression of the dispersion, Eq. (3).

1. *Interdigital Line with Ground Plate**

This case can be encountered when, at very low frequencies, it is impossible for a large distance between the body of the tube and the line to exist or when we wish to increase the dispersion of the fundamental in order to decrease the dispersion of the wave $m = +1$. Then, from (4),

$$\tan^2 \frac{\pi d}{\lambda} = \frac{\gamma_0 + 4\gamma' \sin^2(\varphi/4) + \cdots}{\gamma_0 + 4\gamma' \cos^2(\varphi/4) + \cdots} \tag{16}$$

where γ_0 is the capacity between a finger and the ground, and γ' is the capacity between adjacent fingers.

The effect of γ_0 is to decrease the pass band on the low and high frequencies sides, so that the cutoff wavelengths λ_1 and $\lambda_2(\varphi = 0, \varphi = 2\pi)$, always satisfy the relation

$$\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{2d} \tag{17}$$

2. *Case of Tape Fingers*

Such thin structures give high coupling impedances (but not theoretical maximum), and could be useful when the heat dissipation has no importance. We make use of formula (21) of Section 2.2 by this author and (4) of this section and we obtain

$$\tan^2 \frac{\pi d}{\lambda} = |\tan \varphi/4| \tag{18}$$

Around $\varphi = \pi$, the dispersion of the fundamental is greater than previously. One may be surprised to find for $\lambda = 2d$ an infinite group velocity which is physically impossible; this arises because the bar line theory fails in such cases, since it neglects the unavoidable perturbations at the end of the fingers.

* Fig. 2, dotted lines

3. *Case of a Small Geometrical Shift of One Comb with Respect to the Other in the Direction of Propagation*

In this case the interdigital line is no longer symmetrical and we must return to the more general expression of the dispersion relation (3).

One can see that the shift involves a little stop band centered around $\varphi = \pi$; for a line of finite length one observes an increase of the VSWR at the input of the line at $\lambda = 4d$ (in cold tests) and a jump of the starting current (in O-type backward wave oscillators, in particular)(*1*, *2*).

If the drift is ϵ, the stop band Δf is found to be

$$\frac{\Delta f}{f} \simeq \frac{8}{\pi}\frac{\epsilon}{\alpha p} \tag{19}$$

II. The Ladder Line

A. Main Propagation Modes

Let us consider an array of cylindrical conductors limited at $x = -d$ and $x = 3d$ by perfectly conducting planes; then the bar line theory applies exactly and leads to a fixed frequency condition

$$4kd = K\pi, \qquad K \text{ integer} \tag{20}$$

Such a structure does not propagate in a finite pass band (one can say that Poynting's vector in the z direction is everywhere zero). On the contrary, if we put a ground plate near the bars of a ladder, covering only part of the length of the bars, the discontinuities at the edges of the plate permits propagation (*9*).

We shall now consider the symmetric structure of Fig. 3. We can have

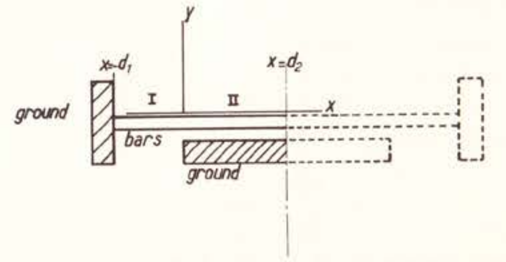

Fig. 3. Ladder line with a grounded plate; only the symmetric wave is considered. The origin of the coordinates is at the discontinuity brought by the grounded plate.

symmetrical modes for which the structure can be cut in its middle without any important changes, and antisymmetrical modes for which the middle of the bars can be grounded without any important changes.

Let us consider the symmetrical modes. We have a two-sections system: for $x < 0$, section I is shortened at $x = -d_1$ and of characteristic admittance Y^{I}; and for $x > 0$, section II is open at $x = d_2$ and of characteristic admittance Y^{II}. The boundary conditions at $x = 0$ lead to the dispersion condition.

$$-jY^{\mathrm{I}} \cot kd_1 = -jY^{\mathrm{II}} \tan kd_2 \tag{21}$$

$$\tan kd_1 \tan kd_2 = \frac{Y^{\mathrm{I}}}{Y^{\mathrm{II}}} \tag{22}$$

The left-hand term is a function of λ only, and the right-hand term a function of φ only. If we take the capacities into consideration we have

$$\tan kd_1 \tan kd_2 = \frac{4\sum_{l=1}^{\infty} (\gamma^l)^{\mathrm{I}} \sin^2 l\varphi/2}{\gamma_0^{\mathrm{II}} + 4\sum_{l=1}^{\infty} (\gamma^l)^{\mathrm{II}} \sin^2 l\varphi/2} \tag{23}$$

and if we neglect all the capacities except $\gamma_0{}^{\mathrm{II}}$ and $(\gamma')^{\mathrm{I}}$ and assume $d_1 = d_2 = d$

$$\tan^2 kd = \frac{4(\gamma')^{\mathrm{I}}}{\gamma_0^{\mathrm{II}}} \sin^2 \frac{l\varphi}{2} \tag{24}$$

The low frequency solution corresponds to a forward wave (*3*) which is used in crossed-field amplifiers. One may see from relation (24) that two lines having the same capacities (which remain unchanged by an homothety in a plane transverse to the bars) and the same d, must have the same $\varphi(\lambda)$. This is proved experimentally in Fig. 4 with good accuracy.

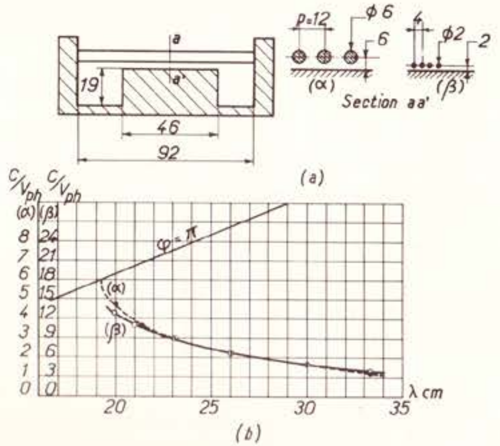

Fig. 4. Dispersion curves measured on two ladder lines which are homothetical only in a plane perpendicular to the bars. One sees that the delay factors are nearly exactly in the same ratio as is claimed by the bar line theory. All dimensions in millimeters.

B. Filter Analogy and Dispersion Improvements

If we make the assumptions leading to relation (24) in the previous section, the ladder line structure can be represented by a π filter structure (L, C) and one can apply the general result of relation (9) of Section 2.1 by this author, which shows that the dispersion would be only

$$v_{\mathrm{ph}}/v_{\mathrm{g}} = \tan(\varphi/2)/\varphi/2 \tag{25}$$

with purely capacitive or purely inductive reactances. In fact, we must use distributed reactances, then $v_{\mathrm{ph}}/v_{\mathrm{g}}$ is greater.

Furthermore, the previous assumption is not right because we have at least one coupling capacity γ'^{II} which increases the dispersion. This harmful capacity may be decreased by milling corrugations in the ground plate in order to introduce screens between the bars. Practically, one can obtain values of $v_{\mathrm{ph}}/v_{\mathrm{g}}$ less than 1.7 for L-band tubes.

C. Coupling Impedance

The present computation of the magnitude of the space harmonics assumes a constant field in the gaps. It is difficult to compute directly the value of P; instead, we compute the value of the stored energy by relation (5) of Section 2.2, and multiply it by v_{g}. Then we have

$$\mathcal{R} = \frac{1}{(\gamma_0^{\mathrm{II}}/\epsilon_0)\{1 + (4\gamma'^{\mathrm{II}}/\gamma_0{}^{\mathrm{II}}) \sin^2 \varphi/2\}} \left(\frac{\sin \varphi/2}{\varphi/2}\right)^2 \varphi \frac{v_{\mathrm{ph}}}{v_{\mathrm{g}}} \left\{\frac{\sin(\alpha/2)\varphi}{(\alpha/2)\varphi}\right\}^2 \tag{26}$$

The useful width of this line being $l = 2d$.

D. Tape Structure

The computation of the tape structure is easy for infinitely small pitch. Pierce (*4*) obtained for the dispersion the simple expression:

$$\tan kd_1 \tan kd_2 = \frac{2}{1 + \coth(\varphi b/p)}, \tag{27}$$

where b is the distance between the bars and the ground. A more accurate expression is given by Butcher (*5*) for finite pitch.

E. The "T" Structure

Often, for reasons due to heat expansion, the bars must be cut in their middle. This measure modifies very little the dispersion of the symmetrical mode, but the dispersion curve of the antisymmetrical mode comes then very close to that of the symmetrical mode if the capacity between the end

of the bars is small (this capacity is a true capacity and not, as previously, a capacity per unit length; it is greater for low delay factors). In practice, it is necessary to increase this capacity to shift the antisymmetrical mode away from the pass band of the tube, towards the lower frequencies. Another possibility is to use the T-shaped bars as is shown in Fig.

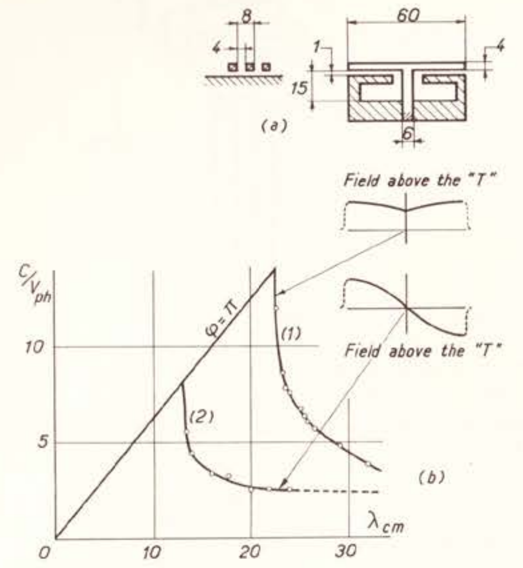

Fig. 5. (a) "T" structure. (b) Such a structure could be used instead of a ladder line after the dispersion curve (1); an antisymmetric mode (2) can propagate. All dimensions in millimeters.

5(a). Experiments show that an antisymmetric mode still propagates, but with a delay factor very different from that of the symmetric mode Fig. 5(b). It can be eliminated by strapping of the two base-plates.

III. The Helix and the Vane Type Line

A. The Helix

This structure has been extensively studied, mainly for the O-type TWT. Its application to crossed-field tubes involves many difficulties.

First, the helix must have rectangular windings so as to show a planar surface to the interaction space. Secondly, better cooling than in O-type TWT is necessary. Finally, experiments show that the supports of the helix are rapidly metallized and eventually short circuit the rf wave.

One can alleviate partially these drawbacks by maintaining each winding (Fig. 6) of the helix by a quarter-wavelength stub. An approximate computation can be made in this case assuming that the only important

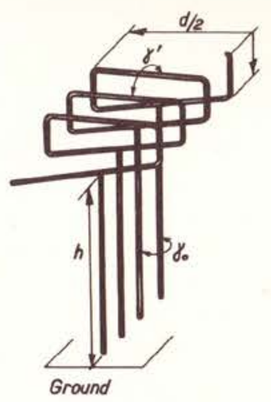

FIG. 6. Helix structure supported by stubs. γ' is the capacity per unit length between adjacent turns of the helix, and d the length of a turn. γ_0 is the capacity per unit length between adjacent stubs and h their height; the backward propagation disappears for $h = d$.

capacities are the capacity γ' between the adjacent rings, and the capacity γ_0 between the adjacent stubs. Then the phase shift φ is given by

$$\cos\varphi = \cos kd + \frac{\gamma_0}{2\gamma'}\cot kh \sin kd \tag{28}$$

where d is the total length of a winding and h is the height of the stubs. This expression shows that it is possible to suppress the backward mode (which is dangerous for the stability of the tube), if $h = d$. Then, the dispersion is greater than that of the free helix, but reasonable bandwidth can still be obtained.

One more remark can be made about the dispersion of the free helix. If we compute the dispersion curve of the free helix by the bar line theory, the dispersion is found to be zero even if we take into account all the capacities; in fact, the dispersion results from the coupling between opposite parts of the windings. This coupling is greater when the rings have a rectangular shape and should be suppressed by a screening plate inside the helix.

B. The Vane Type Line

The vane type line consists of a set of rectangular parallel fins of height h and width l short-circuited at their bottom by a conducting plane (Fig. 7). For the main mode we have TEM waves propagating transversally inside the vanes and a delayed wave above the vanes propagating the power. Such a structure is used in the magnetrons where the circuit is

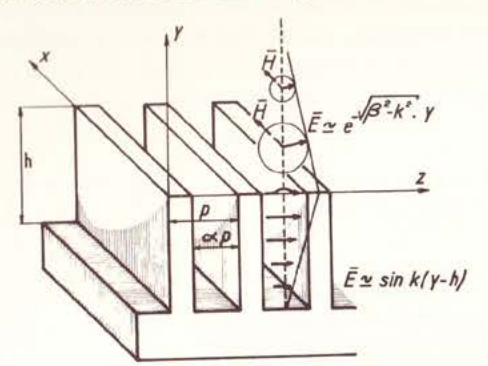

FIG. 7. Vane type line. The pitch is p, the gap width αp. The electric field varies as a sine wave in the vanes and exponentially above the structure ($y > 0$); it is assumed to be uniform along Ox.

closed on itself and operates as a cavity. Consequently, the dispersion of this structure has importance only as far as mode separation is concerned.

As a non-reentrant circuit, the vane line has very great dispersion if the delay factor is high. Then, one can obtain a reasonable pass band (say 10%) only for delay factors of about 2. This would involve sole-to-line voltage of some 400 kv.

The dispersion can be decreased by creating a magnetic coupling between adjacent vanes; one can introduce a capacitive coupling to a plate at the upper end of the vanes but the structure is then very near to the ladder structure previously discussed.

For small pitch, the dispersion curve is easily obtained by matching the TEM wave in the vanes and the delayed wave about them; we have

$$c/v_{\mathrm{ph}} = \sqrt{1 + (\alpha \tan kh)^2} \tag{29}$$

The dispersion is a minimum when the vane thickness is zero; then,

$$c/v_{\mathrm{ph}} = 1/\cos kh \tag{30}$$

On the other hand, the coupling impedance is nearly independent of α and of the frequency; then one has, approximately,

$$\mathfrak{R} \simeq 2 \tag{31}$$

In conclusion, the vane line seems to be of interest only in the case of very high peak powers; it has the advantages of a high dissipation and a good uniformity of the field.

IV. A Biperiodic Structure: The Multiple Interdigital Line

We shall apply the general relation (12) from Section 2.2, by this author, to a particular structure, called the multiple interdigital line (*6*). We can consider it as a set of infinite parallel bars in the same plane above a ground plate. The bars are periodically grounded, as shown in Fig. 8(b). When all

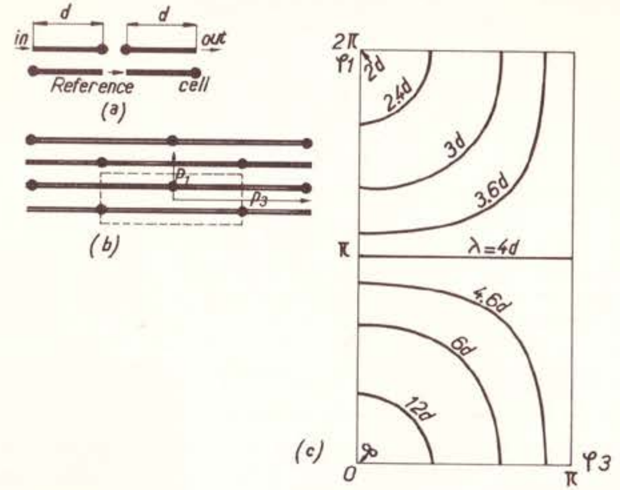

FIG. 8. Biperiodic interdigital line. (a) Reference cell (black points indicate that this end of the bar is grounded). (b) Two dimensional array with periods p_1 and p_3. (c) Curves relating φ_1 and φ_3 at a given wavelength. The projections of β upon p_1 and p_3 are φ_1/p_1 and φ_3/p_3.

sections of one bar are excited in phase by an rf source, the infinite structure behaves as a simple interdigital line, for this current is zero at the middle between two adjacent grounded points of a bar, and one can cut the structure perpendicularly to the bars at these points without modifying propagation. But in the general case it is not so, and the dispersion must be calculated with relation (12) from Section 2.2. We have one pitch $\mathbf{p}_1$ only in the direction normal to the bars ($\mathbf{p}_2 = \infty$), but we have another fundamental phase shift φ_3 in the x direction. In the case where $\mathbf{p}_1$ and $\mathbf{p}_3$ are not perpendicular, use of the concept of reciprocal lattice after L. Brillouin (*7*) must be made.

Considering the fundamental cell shown in Fig. 8(a) we see that we have to apply relation (12) of Section 2.2 once for the section d_1 and once for the section d_2. Then the eigenvalue of the matrix, relating the input to the output current and voltage, is equal to $2\cos\varphi_3$. We shall write the dispersion equation in a very simple case when we take account of only the capacity between two adjacent bars, and when $d_1 = d_2 = d$; then,

$$\cos kd = \cos\frac{\varphi_1}{2}\cos\frac{\varphi_3}{2} \tag{32}$$

For a given frequency (for a given k) we obtain a relation between φ_1 and φ_3, which is shown in Fig. 8(c). A "plane wave" is defined by its propagation constants $\beta_1 = \varphi_1/p_1$ and $\beta_3 = \varphi_3/p_3$. Generally the array is limited in the x direction. For instance, we have reflecting planes, normal to the bars, including between them n cells in the $\mathbf{p}_3$ direction. Then, the condition

$$n\varphi_3 = K\pi \qquad K \text{ integer}$$

involves a relationship between φ_1 and ω, and the structure behaves as a uniperiodic one for each value of K.

V. The Infinitely Thin Zigzag Line

This structure is the complementary of the infinitely thin interdigital line (in the sense of the Babinet's principle) and its parameters are easily deduced from the parameters of the interdigital line (Eq. (18), Figs. 1 and 2).

Let us assume that the zigzag line is supported by a dielectric of constant ϵ and of infinite extent in the negative y direction. Such a structure could be used theoretically for an M-type amplifier (*8*). The inhomogeneity of the medium should involve E_x and H_x components, but they can be neglected if one restricts oneself to delay factors high before 1. Then, the dispersion curve is given by

$$\frac{c}{v_{\mathrm{ph}}} = \frac{\lambda}{2\pi p}\varphi$$

$$\left|\tan\frac{\varphi}{4}\right| = \tan^2\frac{kd'}{2}$$

if we put

$$d' = d\sqrt{\frac{1+\epsilon}{2}}$$

and

$$\mathfrak{R} = \frac{EE^*kd}{2\beta^2 P\sqrt{\mu_0/\epsilon_0}} = \frac{2}{1+\epsilon}\frac{8}{\varphi^2}\frac{\sin(\varphi/4)\ \ kd'}{(2/\sin kd') - \sin kd'}$$

In the case where a ground plate supports the dielectric, the thickness of the dielectric being a, it is possible to cool the zigzag line by the transmission of heat through the dielectric; in this case φ becomes approximately ($\beta a \ll 1$):

$$\varphi = 2kd\sqrt{\epsilon}$$

and the coupling impedance

$$\mathfrak{R} = \frac{1}{\sqrt{\epsilon}}\frac{8\sin^2(\varphi/4)}{\varphi}\frac{\epsilon_0}{\gamma_0}$$

γ_0 being the capacity per unit length between the wire of the zigzag line and the ground plate; neglecting fringing fields one has

$$\frac{\gamma_0}{\epsilon_0} = \epsilon\frac{(\alpha - 1)p/2}{a}$$

A zigzag line could also be supported by quarter-wavelength stubs as indicated in Fig. 6 for the helix.

VI. Main Parameters of the Interdigital Line, Half Ladder Line, and Vane Type Line

A. Introduction

A knowledge of the output power, gain, efficiency, and bandwidth of a tube permits the approximate determination of the following parameters of the delay structure.

1. The delay factor $\tau = c/v_{ph} = \beta/k$.
2. The mean coupling impedance $\mathcal{R}' = E_m E_m^* kl/2\beta^2 P$.

E_m is determined from the mth space harmonic of the electric field component directed along Oz, for $y = 0$ and averaged over the width l of the line, the power flow in the structure being P.

3. The dispersion factor v_{ph}/v_g.

Three scalings of the delay structures are possible; the first two are obvious but the third holds only for bar type structures (interdigital lines and ladder lines, in particular).

(a) Scaling in width: if many identical structures are used in parallel to increase the width of the beam, $\mathcal{R}'$ is the same.

(b) Scaling in wavelength: all dimensions are multiplied by the ratio of the wavelength λ/λ_0; then, τ, $\mathcal{R}'$, v_{ph}/v_g remain constant, the attenuation per delayed wavelength is multiplied by $\sqrt{\lambda_0/\lambda}$. λ_0, τ_0 denote operating wavelength and delay factor of a given line, respectively. λ and τ denote the operating wavelength and delay factor of the scaled line.

(c) Scaling in delay factor: all dimensions perpendicular to the bar direction are multiplied by the reverse of the ratio τ/τ_0 of delay factors without any change in the Ox direction; $\mathcal{R}'$ and v_{ph}/v_g remain constant. The attenuation per delayed wavelength is multiplied by τ/τ_0.

Three structures will be reviewed in the next paragraphs: the interdigital line, the half ladder line, and the vane type line, which are the more commonly used in M-type traveling wave tubes.

B. The Interdigital Line

This structure is used mainly for backward wave oscillators or amplifiers on the space harmonic $m = -1$.

$$\tau = \frac{\lambda}{2\pi p}|\varphi - 2\pi| = \frac{\lambda}{p} - \frac{2d}{p} \tag{33}$$

p is the distance between two corresponding points of the same comb, d is the length of a finger, and $\varphi = 2kd$, the fundamental phase shift. The group velocity is given by

$$c/v_g = 2d/p \tag{34}$$

Generally, the voltage of a carcinotron is varied less than by the ratio 2:1 to avoid too severe variations of the applied power; then the electronic band is

$$\frac{\Delta f}{f} = \frac{2\pi - \varphi}{3\pi} \tag{35}$$

A wide electronic tuning band implies $\varphi = 2kd \ll 2\pi$. In the case where the line is comprised of two combs, it is best to avoid the wavelength $\lambda = 4d(\varphi = \pi)$, which corresponds often to a small stop band.

$$\mathcal{R}' = \frac{p/2h}{1 + 0.5(p/2h)}F(\varphi) \tag{36}$$

h being the height of the fingers and

$$F(\varphi) = \frac{\varphi}{(2\pi - \varphi)^2}\left(\frac{\sin\varphi/4}{\varphi/4}\right)^2\left\{\frac{\sin(2\pi - \varphi/8)}{(2\pi - \varphi/8)}\right\}^2 \tag{37}$$

See Table I for results.

TABLE I

$\varphi = 2kd$	0	0.2π	0.4π	0.6π	0.8π	π	1.2π	1.4π	1.6π	1.8π	2π
$F(\varphi)$	0	0.016	0.041	0.079	0.14	0.24	0.42	0.79	1.15	5.7	∞

In Eq. (36) it is assumed that the gap between adjacent fingers $\alpha p/2 = p/4$. Equation (36) becomes inaccurate for high values of $p/2h$. If the capacity per unit length between adjacent fingers γ' is known, the term $2\epsilon_0/\gamma'$ has to be used instead of

$$\frac{p/2h}{1 + 0.5(p/2h)}$$

Table III can be used, but it may be noticed that the pitch of the interdigital line is twice the pitch used in this table.

The thermal dissipation power of the structure made of copper, with a maximum temperature of 750°C and with a useful length L' (in meters) is

$$P_W = 3 \times 10^5 L'\frac{h}{d} \tag{38}$$

The attenuation per delayed wavelength Λ of such a line, at $T = 30$°C, is (λ in meters)

$$\delta_{db/}{}^{\Lambda} = 31 \times 10^{-4}\frac{\varphi}{(\varphi - 2\pi)^2}\frac{\tau}{\sqrt{\lambda}} \tag{39}$$

C. The Half Ladder Line

This structure is used at the fundamental $m = 0$ in the forward wave amplifiers and on the space harmonic $(m = -1)$ for backward wave oscillators. The delay factor is

$$\tau = \frac{\lambda}{2\pi p}|\varphi + 2m\pi| \tag{40}$$

φ being obtained from

$$\tan kd = \frac{\nu_1 \sin\varphi/2}{\sqrt{1 + \nu_2^2\sin^2(\varphi/2)}} \tag{41}$$

if $d_1 = d_2 = d$ is the half-length of a bar, and with

$$\nu_1^2 = \frac{4\gamma'^{\mathrm{I}}/\epsilon_0}{\gamma_0^{\mathrm{II}}/\epsilon_0} \tag{42}$$

$$\nu_2^2 = \frac{4\gamma'^{\mathrm{II}}/\epsilon_0}{\gamma_0^{\mathrm{II}}/\epsilon_0} \tag{43}$$

γ_0 and γ' can be obtained from Table III. The index I refers to the grounded end of the bars and the index II to the opened end of the bars.

$$\mathcal{R}' = \frac{1}{\gamma_0^{\mathrm{II}}/\epsilon_0\{1 + \nu_2^2\sin^2(\varphi/2)\}}\left(\frac{\sin\varphi/2}{\varphi/2}\right)^2\varphi\left(\frac{\sin kd}{kd}\right)^2\frac{v_{ph}}{v_g}\theta^2 \tag{44}$$

v_{ph}/v_g has to be deduced from (41). θ^2 is a space harmonic decomposition factor which is given for $m = 0$ in particular cases in Table III. When the ground plate is smooth, the following expression can be used

$$\theta = \sin\left[\frac{\alpha}{2}(\varphi + 2m\pi)\right]\Big/\frac{\alpha}{2}(\varphi + 2m\pi) \tag{45}$$

αp being the gap between adjacent fingers.

Let us consider the simplest case where $\nu_2^2 = 0$ and $kd \ll 1$; the lumped circuit theory applies with

$$L = \frac{d}{c^2\gamma'^{\mathrm{I}}} \qquad C = \gamma_0^{\mathrm{II}}d$$

$$\sin\frac{\varphi}{2} = \frac{kd}{\nu_1}$$

$$\frac{v_{ph}}{v_g} = \frac{\tan\varphi/2}{\varphi/2}$$

$$\mathcal{R}' = \frac{1}{\gamma_0^{\mathrm{II}}/\epsilon_0}G(\varphi)\left(\frac{\sin kd}{kd}\right)^2\theta^2$$

with

$$G(\varphi) = \left(\frac{\sin\varphi/2}{\varphi/2}\right)^2\varphi\left(\frac{\tan\varphi/2}{\varphi/2}\right)$$

See Table II for results.

TABLE II

kd/ν_1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
φ/π	0	0.063	0.128	0.195	0.26	0.33	0.41	0.49	0.59	0.71	1
v_{ph}/v_g	1	1	1	1.03	1.06	1.09	1.16	1.26	1.45	1.84	∞
$G(\varphi)$	0	0.2	0.4	0.61	0.82	1.03	1.29	1.58	2	2.6	∞

Usual values for this type of structure are $\varphi = \pi/2$, $\tau = \lambda/4p$, $v_{ph}/v_g = 2$, and $\mathcal{R}' = 0.1$ to 0.2 in 15% bandwidth.

To increase the width of such a line it is possible to put two or more structures in parallel. This led to the T-line, and the π-line or the double T-line. In a first approximation τ, v_{ph}/v_g, and $\mathcal{R}'$ are the same as for the half ladder line.

D. The Vane Type Line

This structure has a small bandwidth, except for very low delay factors, if it is used as a forward wave structure $(m = 0)$. Then, for infinitely thin vanes and small pitch,

$$\tau = 1/\cos kh$$

h being the height of the vanes. Such a structure propagates from $\lambda = 4h$ to $\lambda = \infty$, for the lower pass band. The width can be increased arbitrarily except for the problem of multimoding. For usual delay factors,

$$\frac{v_{ph}}{v_g} \simeq 1 + 2.5\tau$$

and $\mathcal{R} \simeq 2$.

E. Experimental Results for the Capacities per Unit Length and the Space Harmonic Decomposition Factor θ

An analog method can be used, the shape of the cross section of the bars being sketched on a conductive paper.

The γ_{mn} capacity is given by the current flowing from the mth bar into the ground, the nth bar being at the potential +1 and all the others being connected to the ground.

To determine θ, it is sufficient to connect one bar at the potential +1,

TABLE III

Line no.	α	a	b	e	h	ϕ	$\frac{\gamma_0}{\epsilon_0}$	$\frac{\gamma'}{\epsilon_0}$	$\theta(0)$	$\theta(0.6\pi)$	$\theta(\pi)$	$\frac{\gamma_0}{\epsilon_0}+\frac{4\gamma'}{\epsilon_0}\sin^2\frac{0.6\pi}{2}$
1						0.5		1.8				4.7
2			0.075			0.45	5.5	0.87	0.95	0.77	0.69	7.8
3		0.25	0.25	0.15		0.5	5.2	0.55	0.84	0.67	0.63	6.6
4		0.125	0.375	0.15		0.5	6.2	0.26	0.72	0.67	0.67	6.9
5		−0.075	0.5	0.15		0.5	7.1	0.08	0.5	0.5	0.63	7.2
6	0.6	0.2	0.5	0.15	0.4		5.2	0.55				6.6
7	0.6	0	0.57	0.15	0.4		5.8	0.27				6.5
8	0.6	0.4	0.5	0.15	0.8		6.6	0.82	0.91			8.7
9	0.6		0.1		0.4		6.6	0.83				8.8
10	0.6		0.1		0.8		7.2	1.4	0.95			10.9
11	0.5		∞		0.2			.25				3.3
12	0.5		0.5		1		2.1	2.5	0.98			8.7
13	0.5		∞		1			3				7.9
14	0.5		∞		0.5			2				5.3
15	0.5		∞		0.25			1.5				4

the others being at the ground. With the potential $V(z)$ at the upper level of the structure, θ is obtained from

$$\theta(\varphi) = \frac{\varphi}{\sin\varphi/2}\int_0^\infty V(z)\cos\varphi\frac{z}{p}\,dz$$

Since $V(z)$ decreases rapidly with z, three of four cells are sufficient. Besides, the bars are generally symmetrical and the paper can be cut at the middle of the bar which is at the potential +1.

It may be noticed that $\lim_{\varphi\to 0}\theta(\varphi)$ is not unity, in general. If the field with a potential distribution $V_n = e^{j\varphi/2}e^{-jn\varphi}$ is $E(z, \varphi)$

$$\lim_{\varphi\to 0}\theta(\varphi) = \lim_{\varphi\to 0}\frac{1}{2j\sin\varphi/2}\int_0^p E(z,\varphi)e^{j\varphi z/p}\,dz$$

$$= -j\int_0^p \frac{\partial E}{\partial\varphi}(z, 0)\,dz + \frac{1}{p}\int_0^p E(z, 0)z\,dz$$

This limit is unity in the cases described in Section 2.2 on the "Theory of Bar Lines" by this author; it is ½ in the case of a corrugated ground reaching the upper level of the bars.

In various cases $\theta(\varphi)$, γ_0 (capacity per unit length between a bar and the ground), and γ' (capacity per unit length between two adjacent bars) have been determined by this method.

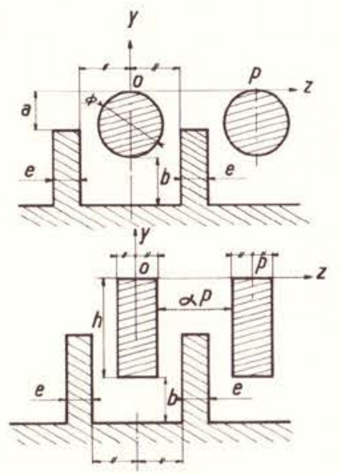

FIG. 9. Bar lines with corrugated ground plate.

In Table III the dimensions refer to Fig. 9. The last column gives the quantity

$$\left(\frac{\gamma_0}{\epsilon_0}+\frac{4\gamma'}{\epsilon_0}\right)\sin^2\frac{\varphi}{2}$$

for a commonly used value of the phase shift $\varphi = 0.6\pi$; this quantity is proportional to the characteristic admittance of the structure. (All symbols used in the table are dimensionless, the dimensions a, b, . . . being referred to a pitch of unity.)

List of Symbols

f	frequency
c	velocity of light
ω	angular frequency
$k\ (=\omega/c)$	propagation constant
$\lambda\ (=c/f)$	free space wavelength
ϵ_0	vacuum permittivity
μ_0	vacuum permeability
v_{ph}	phase velocity
β	delayed propagation constant
$\mathbf{p}_i(i = 1, 3), p$	pitch or periodicity of line
$\varphi_i(i = 1, 3), \varphi\ (=\beta p)$	fundamental phase shift
$\tau_0, \tau = \beta/k = c/v_{\text{ph}}$	delay factor
$v_g = pd\omega/d\varphi$	group velocity
l	line width or integer
P	rf power
W	stored energy
E	electric field
$\mathcal{R}, \mathcal{R}_m$	coupling impedance
$\mathcal{R}'$	mean coupling impedance
a, b	distances in Oy direction
e	thickness of screening plates
U	potential between adjacent bars
d	length of bars
V	potential
I	current
h	height of fingers or of stubs
θ	space harmonic decomposition factor
ϕ	diameter of wire or bar
l, m, n, K	integers
L'	length of a line
$\|\|Y_{r,R}\|\|$	generalized characteristic admittance
$\|\|Z_{r,R}\|\|$	generalized characteristic impedance
γ'	capacity per unit length between adjacent fingers
α	ratio of the gap width to the pitch (ladder line, vane line). Ratio of the gap width to the half pitch (interdigital line)
δ	attenuation

ϵ	relative dielectric constant or small distance
d'	modified length of a bar
L	self inductance
C	capacitance

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