BANDWIDTH OF DISTORTED MULTIMODE WAVEGUIDES EXCITED BY LASER SOURCES

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We show that the bandwidth of distorted multimode optical fibres excited by quasimonochromatic lasers is equal to the bandwidth of the undistorted fibre divided by the microbending loss measured with l.e.d. sources, to within a numerical factor. This simple result is derived for a slab model, but it may be general.

At the present time, many optical transmission systems use multimode optical fibres and light-emitting diodes (l.e.d.). One may wonder what improvement in transmission bandwidth is achieved when the l.e.d. source is replaced by a laser source. We shall consider the idealised situation where the laser source is quasimonochromatic and generates a single transverse mode that matches the fundamental mode of the multimode fibre. Some of the most recent injection lasers approach these conditions. In this case, the bandwidth would be almost unlimited if the fibre were free of distortion. With modern cabling techniques, the fibre distortions (mainly microbending) are fairly small but not negligible. The fundamental mode excited by the source therefore becomes coupled significantly to the higher-order modes, and the input pulse broadens because the group velocities of the various modes are usually not perfectly equalised. We will show that in fact the pulsewidth increases initially in proportion to the square of the fibre length. Thus, although distortions increase the fibre bandwidth when virtually all modes are excited (l.e.d. sources), they reduce the fibre bandwidth, at least initially, when only the fundamental mode of the fibre is excited (laser source). To our knowledge, the latter regime has not been investigated in detail. It is important in practice, whenever l.e.d. sources are being replaced by laser sources, to understand better their general behaviour. We shall treat in this letter a simple model of ray propagation that allows exceedingly simple calculations. These calculations have tutorial value because they help explain in a simple manner the basic mechanisms of pulse propagation. They also provide a formula, for the fibre bandwidth under laser excitation, whose requirements are two simple measurements. This formula may have greater generality than the derivation suggests.

Let us consider a step-index two-dimensional fibre with curvature law $C(z)$. This curvature is taken to be a Gaussian random process with white (uniform) spectrum. Because the fibre is highly multimoded, a ray treatment is appropriate. The laser excitation, matched to the fundamental mode of the fibre, is modelled by an axial ray.

From simple geometrical considerations, the angle $\theta(z)$ that a ray makes with the curved fibre axis at z is given by[1]

$$\theta(z) = \int_0^z C(z_1)\, dz_1 \tag{1}$$

if, as assumed earlier, $\theta(0) = 0$. The time of flight t_0 of an axial ray over a length L of fibre is simply L/u, where u denotes the group velocity in the slab material. In practice, we may set $u = c/n$, where n denotes the slab refractive index. For a ray that makes a small angle θ with respect to the fibre axis, the time of flight per unit length is

$$1/u \cos\theta \approx 1/u + \tfrac{1}{2}\theta^2/u \tag{2}$$

and therefore

$$t(L) = L/u + \frac{1}{2}u^{-1}\int_0^L \theta^2(z)\, dz \tag{3}$$

where $\theta(z)$ is given by eqn. 1.

We assume that the $C(z)$ curvature process has zero means and microscopic correlation, that is

$$\langle C(z)C(z')\rangle = \gamma\delta(z - z') \tag{4}$$

where $\langle\cdot\rangle$ denotes an ensemble average, γ the spectral density of the curvature process and $\delta(\cdot)$ the Dirac distribution. We thus assume that the spectral density of the process is independent of spatial frequency. In practice it is sufficient that the spectral density be a constant up to a spatial frequency of the order of $\sqrt{(\Delta)}/d$, where Δ denotes the relative index change, and $2d$ the slab thickness. Eqns. 1, 3 and 4 form the basis of all subsequent calculations.

The key point that simplifies our derivations is that $\theta(z)$ is Gaussian because $C(z)$ is assumed Gaussian. In fact, it is not even necessary that $C(z)$ be Gaussian. It suffices that $C(z)$, $C(z')$, $z \neq z'$ be independent. Because $\theta(z)$ is Gaussian, we have[2]

$$\langle\theta_1\theta_2\theta_3\theta_4\rangle = \langle\theta_1\theta_2\rangle\langle\theta_3\theta_4\rangle + \langle\theta_1\theta_3\rangle\langle\theta_2\theta_4\rangle + \langle\theta_1\theta_4\rangle\langle\theta_2\theta_3\rangle \tag{5}$$

where we have set for brevity $\theta(z_1) = \theta_1$, $\theta(z_2) = \theta_2$.

The average time of arrival (or pulse centre) is

$$\langle t(L)\rangle = (1/2u)\int_0^L \langle\theta^2(z)\rangle\, dz \tag{6}$$

where the constant term L/u has been dropped for brevity.

From eqn. 1 we have

$$\langle\theta\theta'\rangle = \int_0^z\int_0^{z'} \langle C(z_1)C(z_2)\rangle\, dz_1\, dz_2 = \gamma \min(z, z') \tag{7}$$

where min (a, b) is a or b, whichever is the smaller. In particular, $\langle\theta^2(z)\rangle = \gamma z$. After integration, eqn. 6 is

$$\langle t(L)\rangle = \gamma L^2/4u \tag{8}$$

The square of the r.m.s. pulse width σ is, from eqns. 6 and 1,

$$\sigma^2 = \langle t^2\rangle - \langle t\rangle^2 = (1/2u)^2\int_0^L\int_0^L [\langle\theta^2\theta'^2\rangle - \langle\theta^2\rangle\langle\theta'^2\rangle]\, dz\, dz' \tag{9}$$

However, using eqn. 5

$$\langle\theta^2\theta'^2\rangle - \langle\theta^2\rangle\langle\theta'^2\rangle = 2\gamma^2[\min(z, z')]^2 \tag{10}$$

and after integration we find

$$\sigma(L) = \gamma L^2/\sqrt{(12)}\,u \tag{11}$$

a relation that exhibits the L^2 initial behaviour of σ as announced earlier.

It is not difficult to treat similarly the case where the laser excites a plane wave at an angle $\theta(0) = \theta_0$. We find

$$\sigma(L) = u^{-1}(\gamma^2 L^4/12 + \gamma\theta_0^2 L^3/3)^{1/2} \tag{12}$$

If the first term in eqn. 12 is negligible, we note an $L^{3/2}$ behaviour.

In deriving the previous results, it has been assumed that all the rays are totally reflected, that is, the microbending loss with laser excitation is assumed negligible. If, however, all the modes are equally excited, a situation encountered with l.e.d. sources, the microbending loss $\mathscr{L}$ in decibels is given by the formula[3]

$$\mathscr{L} = 2{\cdot}65\gamma L/\Delta \tag{13}$$

We assume that cabling is good enough that $\mathscr{L}$ be not large compared with unity. This excess loss can be measured by comparing the loss data for the cabled and uncabled fibre.

The expression for the r.m.s. pulse width of the undistorted fibre under the same conditions (that is, with l.e.d. sources), on the other hand, is given by

$$\sigma_0 = L\Delta/\sqrt{(12)}\,u \tag{14}$$

This formula is applicable to a rectangular pulse of width $L\Delta/u$. The σ_0 parameter should be measured on a fairly short piece of fibre (e.g. a few hundred meters) so that distortions play a negligible role. Note also that the l.e.d. output should be filtered to reduce chromatic dispersion as much as possible.

Then, from eqns. 11, 13 and 14, we find that the fibre σ under laser excitation can be written

$$\sigma/\sigma_0 = \text{excess loss in dB}/2{\cdot}65 \tag{15}$$

This formula is valid provided that the excess loss is less than about 1 dB.

Let us assume, for example, that the excess loss in dB due to cabling, measured with an l.e.d. source, is $\mathscr{L} = 0{\cdot}1$ dB. Then eqn. 15 tells us that the fibre bandwidth under ideal laser excitation will be 26 times the bandwidth measured on the straight fibre with all modes excited. More specifically, if $\Delta = 0{\cdot}01$, $L = 10$ km and $n = 1{\cdot}5$ we have $\sigma_0 = 145$ ns. With a laser source, the bandwidth is $\approx 1/4\sigma = 46$ MHz.

The above result in eqn. 15 is rigorous only for step-index slabs and uniform curvature spectra. But we expect the same behaviour to be maintained for graded-index fibres with power-law profiles and more general distortions than the ones considered previously. It may be that only the numerical factor is affected.

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